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MAGNETIC HELICITY OF SOLAR ACTIVE REGIONS AND ITS IMPLICATIONS

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ABSTRACT

We have studied the magnetic helicity of active regions by using the data from (1) the photoelectric magnetograph of the Okayama Observatory (1983–1995) and (2) the video magnetograph of NAOJ/Mitaka (1992–2000). The latitude distribution of helicity showed a tendency that the regions in the north (south) hemisphere have negative (positive) helicities, respectively, which is already known as the hemispheric sign rule. If we look into the sign of helicity as a function of time, the sign rule was less definite or was reversed sometimes in the sunspot minimum phase. We also studied the relation between the magnetic helicity and the sunspot tilt angles, and found that these two quantities are positively correlated, which is opposite to the expectation of a theoretical model. The implications of this cycle-phase dependence of helicity signs and the correlation between magnetic helicity and sunspot tilt angles are discussed.

Key words : Sun: activity cycle — Sun: magnetic fields — Sun: sunspots

I. INTRODUCTION

The helical nature of solar magnetic fields has attracted great attention in recent years. The most quantitative measure of helical solar magnetic fields is given by direct measurement of magnetic vector with magnetographs. From these data, one can define an index, called magnetic helicity, which measures the sense and magnitude of twist in the magnetic fields. Pevtsov et al. (1995) studied the latitudinal distribution of magnetic helicity based mainly on the magnetograms obtained with the Haleakala Stokes Polarimeter, and found that regions in the northern (southern) hemisphere tend to show negative (positive) helicity, respectively. Abramenko et al. (1996) and Bao & Zhang (1998), by using magnetograms obtained at Huairou observatory and by a different method of analysis, confirmed the hemispheric rule of Pevtsov et al. (1995).

Hagino & Sakurai (2002) have analyzed the helicity of (1) 200 active regions using the data taken with the Solar Flare Telescope (SFT) at Mitaka (Sakurai et al. 1995), and (2) 430 active regions using the photoelectric magnetograph of Okayama Astrophysical Observatory (OAO; Makita et al. 1985). Both instruments have been built and operated by the National Astronomical Observatory of Japan. Their study basically confirmed the hemispheric rule of latitude distribution of helicity. In addition, they found an indication that the sense of hemispheric sign rule of helicity is reversed in the period of sunspot minimum.

In this paper, we will describe technical aspects of our method used in Hagino & Sakurai (2002). Also we will give a preliminary report of our study on the relation between magnetic helicity and sunspot tilt angle. Based on these results, we will discuss the implications of these new findings.

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II. DATA REDUCTION

For simplicity, here we assume that the solar surface coincides with the $z = 0$ plane which is perpendicular to the line-of-sight. The data obtained with a vector magnetograph are three components of the field, namely $B_x(x, y), B_y(x, y)$, and $B_z(x, y)$. The transverse fields B_x and B_y still have a 180° ambiguity in their azimuth. This ambiguity is resolved by the method proposed by Sakurai et al. (1985). First we compute the current-free magnetic field vector based on the observed longitudinal magnetic field B_z, and between the two possible directions of the transverse field we select the one which makes the smaller angle with respect to the current-free vector.

Once the vector magnetic fields are obtained, we use two methods to derive the magnetic helicity.

(a) Fitting Method

In the fitting method, we assume that the observed field can be approximated by a linear force-free field,

$$\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}. \qquad (1)$$

Here α is a constant representing the degree of twist in the magnetic field, and can be used as a proxy to the magnetic helicity (positive α means right-handed twist). For an assumed value of α and the observed longitudinal field B_z, we can compute the linear force-free field whose transverse component is $\boldsymbol{B}_{\mathrm{t,cal}}(\alpha)$. By

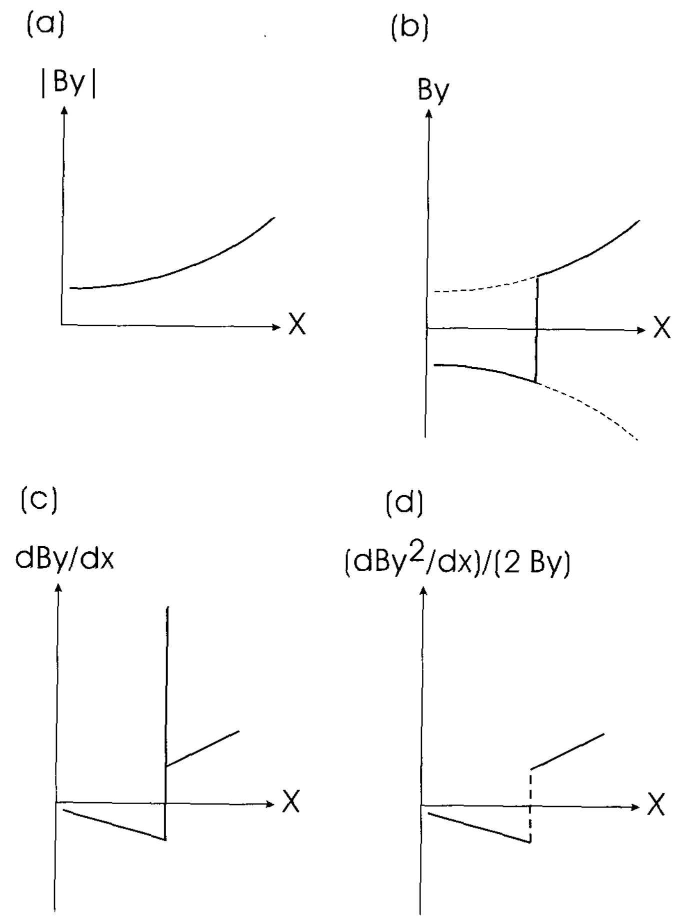

Fig. 1.— The derivatives of B_y with respect to x by the usual method (c) and by the sign-insensitive method (d). Here (b) is the profile of B_y assumed (erroneously) from the profile of $|B_y|$ given in (a).

changing the value of α, we seek the optimum value of α which minimizes the residual

$$R(\alpha) = \frac{\Sigma \left[\boldsymbol{B}_{\rm t,obs}(x,y) - \boldsymbol{B}_{\rm t,cal}(x,y;\alpha)\right]^2}{\Sigma B^2_{\rm t,obs}(x,y)}. \quad (2)$$

The value of α thus determined is designated as $\alpha_{\rm best}$.

(b) Direct Differentiation Method

In the direct differentiation method, we also assume that the observed field can be approximated by a force-free field, and by using the z-component of equation (1) we write

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \equiv J_z = \alpha(x,y) B_z. \quad (3)$$

Here $\alpha(x,y)$ is generally a function of position (x,y). We define the average value of α by

$$\alpha_{\rm av} = \frac{\Sigma J_z {\rm sign}(B_z)}{\Sigma |B_z|}. \quad (4)$$

Here the summation is taken over pixels.

(c) Sign-Insensitive Differentiation of Magnetic Field Vectors

If the determination of the azimuth of the transverse fields is incorrect, the transverse field components may show spurious discontinuities. These are potential source of error in evaluating magnetic helicity, particularly when we use the direct differentiation method. In this respect, the approach taken by Semel & Skumanich (1998) is worth mentioning here. They took advantage of the fact that B_x^2, B_y^2, and $B_x B_y$ are ambiguity-free, and derived the formula for J_z which only involves derivatives of these ambiguity-free quantities.

In order to see how the scheme works, let us assume that the sign of, say, B_y is incorrect in some region (Fig. 1a, b). Then B_y would have a spurious discontinuity, and its derivative $\partial B_y/\partial x$ will have a δ-function

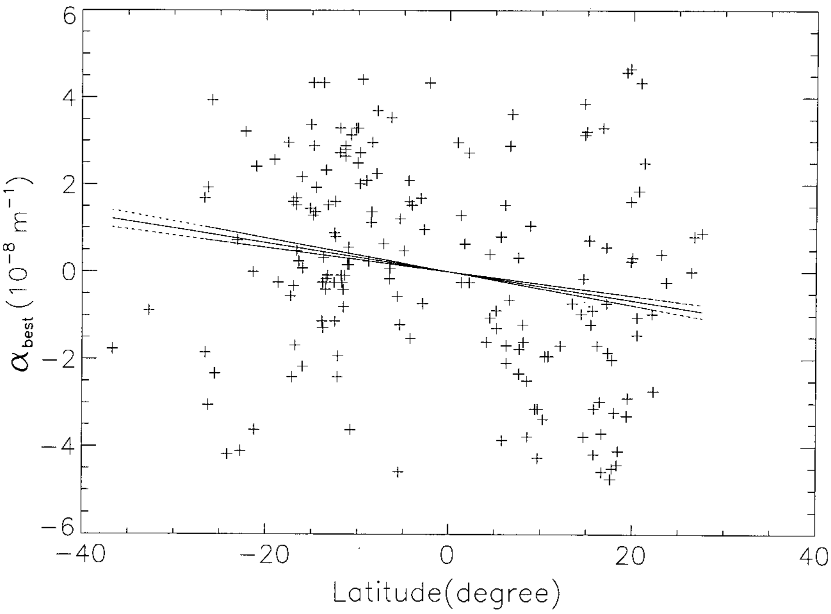

Fig. 2.— The latitude distribution of $\alpha_{\rm best}$ (Hagino and Sakurai 2002). The least-square linear fit and its error bounds are overplotted.

behavior there (Fig. 1c). However, if we write

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_y^2}{\partial x} \Big/ (2B_y)\,, \tag{5}$$

then in the right-hand-side the differentiation only applies to the ambiguity-free B_y^2 which is continuous, and no spurious delta function appears (Fig. 1d). Here we should note that the denominator B_y never vanishes and jumps between positive and negative values.

Starting from

$$\frac{\partial(B_xB_y)}{\partial x} = B_x\frac{\partial B_y}{\partial x} + B_y\frac{\partial B_x}{\partial x}, \tag{6}$$

$$\frac{\partial(B_xB_y)}{\partial y} = B_x\frac{\partial B_y}{\partial y} + B_y\frac{\partial B_x}{\partial y}, \tag{7}$$

$$\frac{\partial B_x^2}{\partial x} = 2B_x\frac{\partial B_x}{\partial x}, \tag{8}$$

$$\frac{\partial B_x^2}{\partial y} = 2B_x\frac{\partial B_x}{\partial y}, \tag{9}$$

$$\frac{\partial B_y^2}{\partial x} = 2B_y\frac{\partial B_y}{\partial x}, \tag{10}$$

$$\frac{\partial B_y^2}{\partial y} = 2B_y\frac{\partial B_y}{\partial y}, \tag{11}$$

$$\tag{12}$$

we can derive

$$B_t^2\frac{\partial B_x}{\partial x} = \frac{B_x}{2}\frac{\partial\left(B_x^2-B_y^2\right)}{\partial x} + B_y\frac{\partial(B_xB_y)}{\partial x}, \tag{13}$$

$$B_t^2\frac{\partial B_y}{\partial y} = -\frac{B_y}{2}\frac{\partial\left(B_x^2-B_y^2\right)}{\partial y} + B_x\frac{\partial(B_xB_y)}{\partial y}, \tag{14}$$

$$B_t^2\frac{\partial B_y}{\partial x} = -\frac{B_y}{2}\frac{\partial\left(B_x^2-B_y^2\right)}{\partial x} + B_x\frac{\partial(B_xB_y)}{\partial x}, \tag{15}$$

$$B_t^2\frac{\partial B_x}{\partial y} = \frac{B_x}{2}\frac{\partial\left(B_x^2-B_y^2\right)}{\partial y} + B_y\frac{\partial(B_xB_y)}{\partial y}. \tag{16}$$

Here B_t is the transverse field strength ($B_t^2 = B_x^2 + B_y^2$). Therefore, we can obtain the formulae for J_z and also for $\nabla_\perp \cdot \boldsymbol{B}$ as

$$\begin{aligned} & B_t^2\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) \\ = & -\frac{1}{2}\left(B_y\frac{\partial}{\partial x} + B_x\frac{\partial}{\partial y}\right)\left(B_x^2 - B_y^2\right) \\ + & \left(B_x\frac{\partial}{\partial x} - B_y\frac{\partial}{\partial y}\right)(B_xB_y)\,, \end{aligned} \tag{17}$$

$$\begin{aligned} & B_t^2\left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y}\right) \\ = & \frac{1}{2}\left(B_x\frac{\partial}{\partial x} - B_y\frac{\partial}{\partial y}\right)\left(B_x^2 - B_y^2\right) \\ + & \left(B_y\frac{\partial}{\partial x} + B_x\frac{\partial}{\partial y}\right)(B_xB_y)\,. \end{aligned} \tag{18}$$

The differentiation only applies to ambiguity-free quantities, and spurious δ-functions do not appear. Of course real jumps in B_x or B_y, if any, remain. Our formula for J_z is simpler than the expression given by Semel & Skumanich (1998).

III. RESULTS

(a) Hemispheric Sign Rule and Solar Cycle Variation

Figure 2 [reproduced from Hagino & Sakurai (2002)] shows the latitude distribution of $\alpha_{\rm best}$, based on the SFT data. Also shown is a least-square linear fit to the data. The negative value of $d\alpha/d\theta$ (θ is the latitude) means the so-called hemispheric sign rule of helicity. Similar results are obtained from the OAO data, or by using $\alpha_{\rm av}$ as a proxy to the magnetic helicity.

Figure 3 [reproduced from Hagino & Sakurai (2002)] shows the yearly variations of $d\alpha_{\rm best}/d\theta$ in the period of 1983–2000, by combining OAO and SFT data. The numerical values of $d\alpha/d\theta$ are listed in Table 1. We can notice that the hemispheric sign rule of helicity ($d\alpha/d\theta < 0$) is not always satisfied. There is a tendency that positive values of $d\alpha/d\theta$ occur in the sunspot minimum phase (around the years 1986 and 1997). The fluctuations are large, and we should carry out statistical tests on the significance of time variability in $d\alpha/d\theta$.

Table 1. The yearly values of the slope $d\alpha/d\theta$ in units of 10^{-9} m^{-1} deg^{-1}.

year	OAO regions	$d\alpha/d\theta$	SFT regions	$d\alpha/d\theta$
1983	11	−0.44		
1984	48	0.15		
1985	27	0.49		
1986	25	0.81		
1987	5	0.78		
1988	24	−0.30		
1989	39	−0.09		
1990	39	−0.04		
1991	89	−0.08		
1992	65	0.03	62	−1.23
1993	30	0.65	26	−0.41
1994	9	−0.27	15	−0.03
1995	19	0.55	9	−2.18
1996			7	−1.15
1997			12	0.44
1998			6	0.70
1999			23	0.22
2000			40	−0.54

(b) Twist and Writhe

The bipolar axis of a sunspot group is statistically tilted in such a way that the preceding sunspot appears closer to the equator (Joy's law). This tilt can be interpreted as representing the helical deformation of the flux tube that makes the sunspot group. A right-handed (left-handed) helical form of the flux tube in the northern (southern) hemisphere will make the sunspot tilt as is observed, respectively. Longcope et al. (1998) argued that, if the initially untwisted flux tube is deformed in its shape into a right-handed helix, the twist of the field lines around the axis of the tube will be left-handed, because the total helicity must remain zero. Here the total helicity is made of the twist around the axis and the 'writhe,' which represents the helical shape of the tube.

Tian et al. (2001) examined the relation between the twist (measured by α) and the writhe (the sunspot tilt angle), based on the data obtained at Beijing Observatory. They found that the two quantities are anti-correlated, as is expected from the argument of Longcope et al. (1998). However, our preliminary analysis based on the SFT data indicates that, on the contrary, the two quantities show weak but positive correlation.

IV. DISCUSSION

We found that the hemispheric sign rule of helicity holds on a time scale of the solar cycle period. On a yearly basis, the slope $d\alpha/d\theta$ shows variations, with a tendency that it is negative in the period of sunspot number maximum and is positive in the period of sunspot number minimum. We also studied the relation between the writhe and twist helicities and found a weak positive correlation among them, which is opposite to what is expected from the theory of Longcope et al. (1998).

If an initially untwisted flux tube is generated at the base of the convection zone, during its ascent through the convection zone it will be influenced by the Coriolis force and by the forces exerted by turbulent convection from the surroundings. The Coriolis force acting on the Ω-shaped ascending loop (specifically the Coriolis force on the downflows from the top of the loop) will tilt the tube axis in the direction consistent with Joy's law. However, this Coriolis effect should be ubiquitous while Joy's law shows large scatter: many regions with tilt angles opposite to Joy's law also exist.

The convective motion has a positive kinetic helicity in the northern hemisphere (in the upper part of the convection zone). If the flux tube diameter is much smaller than the convective eddies, the convection will only deform the external shape of the tube, without directly disturbing the magnetic field inside the tube. Then the tube will attain a positive writhe helicity from the positive convective kinetic helicity, in the northern hemisphere. Because of the conservation of total magnetic helicity, the positive writhe helicity will induce a negative twist helicity inside the tube [the so-called Σ-effect; Longcope et al. (1998)]. The sense of the writhe helicity is consistent with Joy's law of the sunspot tilt angle, and the sense of the twist helicity is

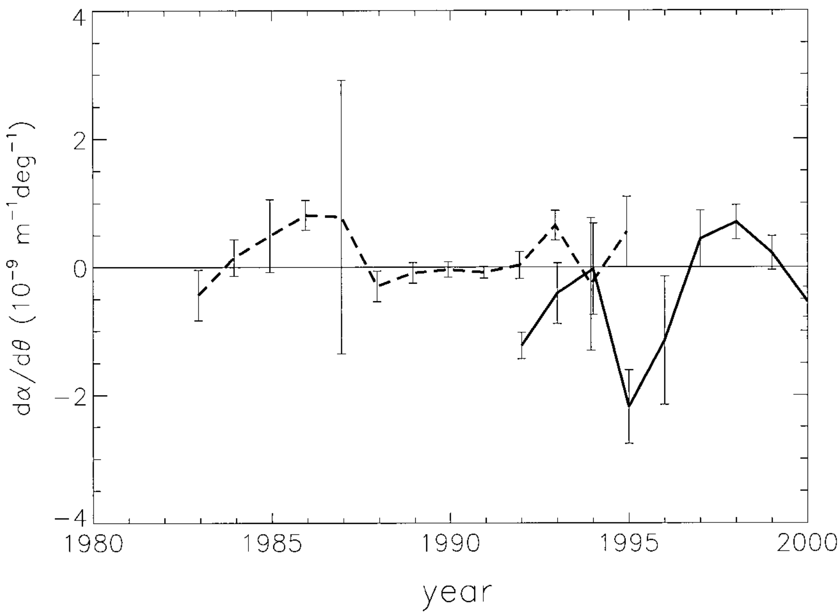

Fig. 3.— Yearly variations of $d\alpha/d\theta$. The dashed line covering 1983–1995 is from OAO, and the solid line covering 1992–2000 is from SFT data.

consistent with the hemispheric sign rule of magnetic helicity. Large dispersions seen in the tilt angles and the magnetic helicity are also a natural consequence of the turbulent convection. However, this scenario expects a negative correlation between the writhe and the twist helicities, which is opposite to what we found.

The positive correlation between the writhe and the twist helicities can be explained if the eddies deforming the flux tube are of the sizes comparable to the flux tube diameter. Such convective eddies will twist the flux tube both externally and internally. The flux tube will increase its diameter as it ascends, and the convective eddy size will be smaller near the surface layers. Therefore, the origin of the writhe and twist can be located in a shallow layer near the surface. However, if the kinetic helicity of the convection is injected into the flux tube, the tube will show positive writhe as well as positive twist statistically, in the northern hemisphere. At least the latter is not the case, which means that the Σ-effect is not the main source of the magnetic helicity seen in active regions.

Our view is that the flux tubes are not untwisted when they are created at the base of the convection zone. The large dispersion in the magnetic helicity might already be there in the generation process, but it can also be created by stochastic forces from convective eddies during their ascent. The positive correlation between the twist and the writhe supports such processes in the upper part of the convection zone where the flux tubes have expanded to a size comparable to the size of convective eddies surrounding them. In the same argument, we suppose that the sunspot tilt is mainly due to the Coriolis force acting on the Ω-shaped flux tube rising through the convection zone. The dispersion of the tilt angles must be due to random forces from convective eddies. The statistical Σ-effect might be responsible for part of the sunspot tilt.

Our finding on the time variation of $d\alpha/d\theta$, if literally interpreted, implies that the flux tubes are twisted, in the northern hemisphere, into a left-handed sense near sunspot maximum and into a right-handed sense near sunspot minimum. This means that a dynamo working in the convection zone must reverse its sense of twist given to the generated fields. We should note here, however, that the reaction of the Coriolis force on the Ω-shaped loops will tend to create a negative twist in the northern hemisphere. Therefore, an alternative interpretation is that the dynamo always create flux tubes with positive magnetic helicity in the northern hemisphere. In the sunspot minimum phase those flux tubes emerge as they are, but in the sunspot maximum phase the flux tubes are strongly affected by the Coriolis force and infected by positive writhe and negative twist. The influence of the Coriolis force is larger when the rising motion of the flux tube is faster, namely the magnetic field is stronger and hence the buoyancy force is stronger. Such a condition might be favorably found in the sunspot maximum phase. On the other hand, in the activity maximum the sunspots appear near the equator, and therefore only smaller Coriolis force can be available.

An over-twisted flux tube may be subject to kink instability. The sense of the kink (or the writhe) is to reduce the twist and to increase the writhe. Therefore, if the flux tube in the northern hemisphere is twisted in a left-handed screw and creates a kink, the expected writhe (or sunspot tilt angle) will also be left-handed, which is opposite to the observed sunspot tilt.

Although we have many pieces of new observational evidences, we have not yet arrived at a consistent picture. We will emphasize that the solar-cycle variation of magnetic helicity is particularly important in constraining the properties of dynamo process, and further accumulation of data is of crucial importance.

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