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FORMATION AND EVOLUTION OF SELF-INTERACTING DARK MATTER HALOS

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ABSTRACT

Observations of dark matter dominated dwarf and low surface brightness disk galaxies favor density profiles with a flat-density core, while cold dark matter (CDM) N-body simulations form halos with central cusps, instead. This apparent discrepancy has motivated a re-examination of the microscopic nature of the dark matter in order to explain the observed halo profiles, including the suggestion that CDM has a non-gravitational self-interaction. We study the formation and evolution of self-interacting dark matter (SIDM) halos. We find analytical, fully cosmological similarity solutions for their dynamics, which take proper account of the collisional interaction of SIDM particles, based on a fluid approximation derived from the Boltzmann equation. The SIDM particles scatter each other elastically, which results in an effective thermal conductivity that heats the halo core and flattens its density profile. These similarity solutions are relevant to galactic and cluster halo formation in the CDM model. We assume that the local density maximum which serves as the progenitor of the halo has an initial mass profile $\delta M/M \propto M^{-\varepsilon}$, as in the familiar secondary infall model. If $\varepsilon = 1/6$, SIDM halos will evolve self-similarly, with a cold, supersonic infall which is terminated by a strong accretion shock. Different solutions arise for different values of the dimensionless collisionality parameter, $Q \equiv \sigma \rho_b r_s$, where σ is the SIDM particle scattering cross section per unit mass, ρ_b is the cosmic mean density, and r_s is the shock radius. For all these solutions, a flat-density, isothermal core is present which grows in size as a fixed fraction of r_s. We find two different regimes for these solutions: 1) for $Q < Q_{th} (\simeq 7.35 \times 10^{-4})$, the core density decreases and core size increases as Q increases; 2) for $Q > Q_{th}$, the core density increases and core size decreases as Q increases. Our similarity solutions are in good agreement with previous results of N-body simulation of SIDM halos, which correspond to the low-Q regime, for which SIDM halo profiles match the observed galactic rotation curves if $Q \sim [8.4 \times 10^{-4} - 4.9 \times 10^{-2}] Q_{th}$, or $\sigma \sim [0.56 - 5.6]\, cm^2 g^{-1}$. These similarity solutions also show that, as $Q \to \infty$, the central density acquires a singular profile, in agreement with some earlier simulation results which approximated the effects of SIDM collisionality by considering an ordinary fluid without conductivity, i.e. the limit of mean free path $\lambda_{mfp} \to 0$. The intermediate regime where $Q \sim [18.6 - 231] Q_{th}$ or $\sigma \sim [1.2 \times 10^4 - 2.7 \times 10^4]\, cm^2 g^{-1}$, for which we find flat-density cores comparable to those of the low-Q solutions preferred to make SIDM halos match halo observations, has not previously been identified. Further study of this regime is warranted.

Key words : cosmology: dark matter — cosmology: large-scale structure of universe — galaxies: kinematics and dynamics

I. INTRODUCTION

The cold dark matter (CDM) model provides a successful framework for understanding the formation and evolution of structure in the universe. According to this model, structure forms out of primordial density fluctuations by hierarchical clustering; small objects form first, and then merge to make bigger objects. However, this CDM model has several problems.

There has been a recent concern about the possible discrepancy in the inner density profile of dark-matter dominated cosmological halos between N-body simulations and observations. CDM halos in N-body simulations have a density cusp ($\rho \propto r^{\beta}$ where β ranges from -1 (Navarro, Frenk & White 1997; hereafter "NFW") to -1.5 (Moore et al. 1999; hereafter "Moore profile"), while observations of dwarf and LSB galaxies, which are believed to be dark-matter dominated, indicate flat-density (soft) cores.

Spergel & Steinhardt (2000) suggested that the purely collisionless nature of CDM be replaced by self-interacting dark matter (SIDM) to resolve this discrepancy. The non-gravitational, microscopic interaction (e.g. elastic scattering) of SIDM particles is postulated to be strong enough to produce a soft core by heat conduction. Cosmological N-body simulations which incorporate a finite scattering cross-section σ for the SIDM particles show that this scheme successfully produces soft cores in halos (e.g. Davé et al. 2001; Yoshida et al. 2000b).

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Previous analytical study (Balberg, Shapiro & Inagaki 2002) and estimates based on N-body simulations (Burkert 2000; Kochanek & White 2000) indicate that SIDM cores will become unstable by undergoing gravothermal catastrophe in a Hubble time. This potential instability of SIDM cores means that, if the matter content of the universe is really SIDM particles, we are living in a special epoch where SIDM halos still have soft cores. However, these analyses are based on isolated halos. In the context of the hierarchical clustering scenario, cosmological infall is inevitable. We are led to pose the following question: Can cosmological infall delay the core collapse of SIDM halos?

In this paper we study the formation and evolution of SIDM halos with a proper treatment of cosmological infall. We find that, for a range of infall rates, the collapse (i.e. gravothermal catastrophe) of SIDM cores can be completely inhibited. We also find a new range of values of the scattering cross section which can produce soft cores, separate from and in addition to the range previously identified by N-body simulations. Toward this end, we find a self-similar solution to describe the evolution of SIDM halos for arbitrary degree of collisionality which allows a detailed analytic study.

II. BASIC EQUATIONS

We use a fluid approximation derived from the Boltzmann equation to describe the dynamics of SIDM halos. The basic assumptions we make are as follows: 1) the system has spherical symmetry; 2) infall is radial, cold, and continuous; 3) virialized objects have an isotropic velocity distribution. The first two assumptions have been used before for secondary infall models (e.g. Fillmore & Goldreich 1984). The third assumption comes from the property of CDM halos in N-body simulations: the velocity distribution is isotropic at the center (i.e. radial velocity dispersion is equal to tangential velocity dispersion) and acquires only moderate anisotropy as one goes out to the virial radius.

With these assumptions, the moments of the collisional Boltzmann equation (i.e. valid for arbitrary degree of collisionality) can be shown to reduce to the usual fluid equations for an ideal gas with $\gamma = 5/3$, where γ is the ratio of specific heats. This approach has been widely used in the literature of stellar dynamics (e.g. Bettwieser 1983), especially for the study of gravothermal catastrophe. We note that the particle components in both cases, that of stellar systems (stars) and of SIDM halos (SIDM particles), are weakly collisional, and a corresponding conductive heating term enters the energy equation in both cases, but it has a different origin in each case. For the stellar systems (number of particles $\sim 10^4 - 10^6$), weak, gravitational scattering is responsible for the collisions, while for SIDM halos, nongravitational, microscopic self-interaction is responsible for the collisions.

In summary, we use the following fluid-like equations of conservation of mass, momentum, and energy, respectively:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{r^2 \partial r}\left(r^2 (\rho u)\right) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial r}\left(p + \rho u^2\right) + \frac{2\rho u^2}{r} = -\rho \frac{GM}{r^2}, \tag{2}$$

$$\frac{D}{Dt}\left(\frac{3p}{2\rho}\right) = -\frac{p}{\rho}\frac{\partial}{r^2 \partial r}\left(r^2 u\right) - \frac{\partial}{r^2 \partial r}\left(r^2 f\right), \tag{3}$$

where ρ, $u(\equiv \langle v_r \rangle)$, $p(\equiv \rho \langle v_r - u \rangle^2)$, and f are mass density, bulk radial velocity, pressure, and heat flux, respectively, M is the integrated mass inside radius r, and $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$. For a collisionless system, $f = 0$ and we will call it an "adiabatic" process. For SIDM halos, heat conduction takes the following form:

$$f = -\frac{3ab\sigma}{2}\sqrt{\frac{p}{\rho}}\left(a\sigma^2 + \frac{4\pi G}{p}\right)^{-1}\frac{\partial}{\partial r}\left(\frac{p}{\rho}\right), \tag{4}$$

where a and b are constants of order of unity (Balberg, Shapiro & Inagaki 2002). We consider an elastic scattering case, in which $a = 2.26$ and $b = 1.002$.

III. SELF-SIMILAR EVOLUTION OF SIDM HALOS

We seek a self-similar solution to describe the evolution of SIDM halos in an Einstein-de Sitter universe. This approach makes it possible to analyze properties of the solution in detail, because an exact solution can be found. Previous authors have found self-similar solutions for the evolution of cosmological halos for cold, collisionless dark matter in radial motion (e.g. Fillmore & Goldreich 1984) and for an ordinary, adiabatic, baryonic fluid (e.g. Bertschinger 1985). Our problem differs from these in that our gas is neither collisionless nor in purely radial motion and it differs from an ordinary, adiabatic, baryonic fluid, since our fluid-like conservation equations also contain the SIDM conduction term. On the other hand, we were encouraged to look for a self-similar solution, since the effect of conductivity is to drive a gas towards isothermality, and there are self-similar equilibrium solutions known in the limit of isothermality which have soft cores, such as the "truncated isothermal sphere" (TIS) model proposed by Shapiro, Iliev & Raga (1999).

We start from an adiabatic (i.e. no conduction, or $f = 0$), self-similar solution to find the condition required for the self-similarity of SIDM halo solutions. If the initial overdensity has a scale-free power-law form,

$$\frac{\delta M}{M} \propto M^{-\varepsilon}, \tag{5}$$

where ε is any positive constant, then adiabatic infall will form a self-similarly growing object (Fillmore & Goldreich 1984). A cold, supersonic, radial infall leads to a strong, spherical shock which surrounds a subsonic region of shock-heated gas. The corresponding length scale, e.g. a turnaround radius r_{ta}, grows as

$$r_{ta} \propto t^{\xi}, \tag{6}$$

where

$$\xi = \frac{2}{3}\left(\frac{3\varepsilon+1}{3\varepsilon}\right). \tag{7}$$

In this case, the shock always occurs at the same fixed fraction of r_{ta}, the mean density inside the postshock region is a constant multiple of the time-varying cosmic mean density ρ_b, and the radial profiles of all fluid variables are self-similar. When heat conduction by SIDM particles is introduced, however, an additional length (time) scale is present which breaks the self-similarity of these adiabatic solutions, in general.

There is one particular infall rate, however, which does not break the self-similarity even when heat conduction is introduced. According to the adiabatic solutions, the thermal energy changes according to $\rho\frac{\partial e}{\partial t} \propto r_{ta}^2 t^{-5}$, while the conductive heating term would introduce a change as $\frac{\partial}{r^2\partial r}\left(r^2 f\right) \propto r_{ta}^3 t^{-7}$. To preserve similarity after the addition of heat conduction, therefore, we must have $r_{ta} \propto t^2$, or

$$\xi = 2,\ \varepsilon = \frac{1}{6}. \tag{8}$$

IV. SELF-SIMILAR CDM HALOS ($\varepsilon = 1/6$; NO CONDUCTION)

Let us first examine properties of the adiabatic solution ($f = 0$) for the case of $\varepsilon = 1/6$. We especially focus on the density profile. The density profile of the adiabatic solution within the shock radius r_s agrees quite well with the N-body results for CDM halos (see Fig. 1). The adiabatic solution is well-fit, for example, by an NFW profile with concentration parameter $3 \le c_{NFW} \le 4$ (i.e. fractional deviation $\Delta \le 17\%$ for $0.014 \le r/r_{200} \le r_s/r_{200}$, where r_{200} is the NFW profile radius within which $\langle\rho\rangle = 200\rho_b$, and $r_s \simeq 0.6 r_{200}$). The inner density slope of the similarity solution for $4\times10^{-3} \le r/r_{200} \le 1.4\times10^{-2}$ is -1.27, which is between -1 (NFW) and -1.5 (Moore profile). A natural question is then: why does this adiabatic solution for $\varepsilon = 1/6$ match the CDM N-body halos so well?

The theory of structure formation from density peaks in the Gaussian random noise distribution of initial density fluctuations gives an interesting clue to this correspondence. According to Hoffman & Shaham (1985), local maxima of Gaussian random fluctuations in the density can serve as the progenitors of cosmological structures. For a power-law power spectrum of initial fluctuations, $P(k) \propto k^n$, the initial density profile can be approximated as $\frac{\delta\rho}{\rho} \propto r^{\kappa}$, where $\kappa = 3\varepsilon = n+3$. The value $\varepsilon = 1/6$ corresponds to $\kappa = 1/2$, or $n = -2.5$, while $n = -2.5$ roughly corresponds to galaxy-mass structures in CDM, as shown in Fig. 2.

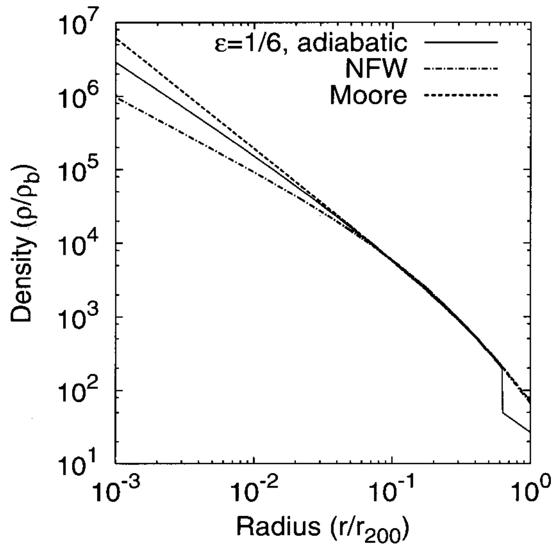

Fig. 1.— Halo density profile for adiabatic solution with $\varepsilon = 1/6$ for standard CDM halos, compared to NFW and Moore profiles. The adiabatic solution, whose inner slope is -1.3, is a good fit to density profiles of CDM halos from N-body simulations.

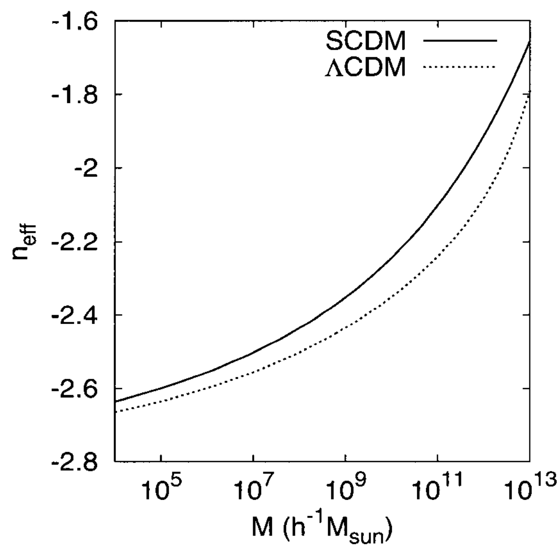

Fig. 2.— Effective index of the power spectrum of initial density fluctuations ($P(k) \propto k^{n_{eff}}$) vs. mass of halos at their typical formation epoch, for flat, cluster-normalized, matter-dominated CDM (SCDM) and COBE-normalized ΛCDM universes.

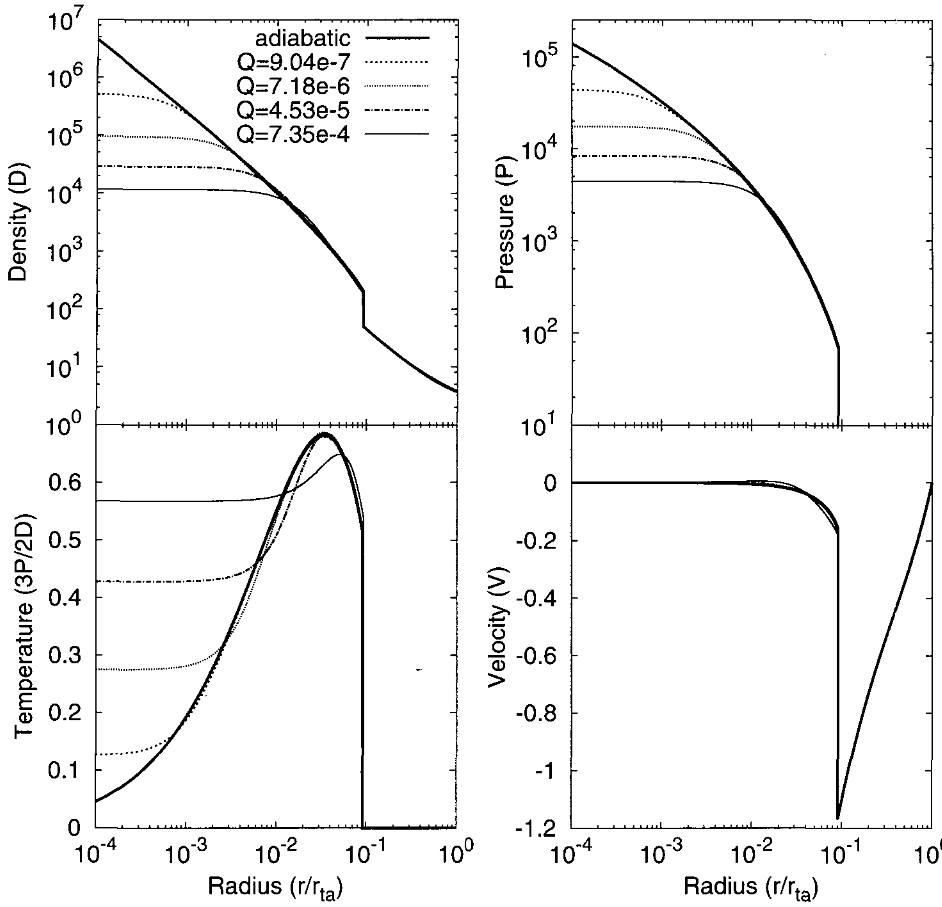

Fig. 3.— Dimensionless profiles for similarity solutions in the low-Q regime, where r_{ta} is the time-varying turnaround radius and the shock radius is $r_s = 0.09 r_{ta}$. Isothermal, flat-density cores exist. As Q increases, core density decreases and core temperature increases. Dimensionless similarity variables follow the definitions in Bertschinger (1985).

V. SELF-SIMILAR SIDM HALOS ($\varepsilon = 1/6$; WITH CONDUCTION)

We find that different similarity solutions of SIDM halos arise for different values of the collisionality parameter, $Q \equiv \sigma \rho_b r_s$. The dimensionless Q is closely related to $P \equiv \sigma \rho v_{rel} \Delta t$, the number of collisions each particle experiences during a time Δt in a local density ρ with relative velocity v_{rel}, sometimes used elsewhere to parameterize the collisionality of SIDM particles (Davé et al. 2001). Roughly speaking, high Q means high collision rate. We also find that there are two different regimes: the low-Q regime and the high-Q regime.

In the low-Q regime, the collision mean free path in the core is larger than the size of the halo. In this regime, the flattening of the core density profile increases as Q increases (see Fig. 3). As Q increases, that is, the central density is lower and the core radius is larger.

In the high-Q regime, the mean free path in the core region is smaller than the size of the halo. The flattening of the core density profile decreases as Q increases, because the mean free path decreases (diffusion limit). As Q increases in this regime, therefore, the core density increases and the core radius shrinks (see Fig. 4). The limiting case of $Q = \infty$ corresponds to the solution with no conduction, which agrees with N-body simulations with infinite cross-section (Yoshida et al. 2000a; Moore et al. 2000). In this limit of maximal collisionality, the nonadiabatic solution approaches the adiabatic solution (see Figs. 3 and 4).

An important aspect of this self-similar solution is that halo cores never enter the gravothermal catastrophe phase, because cosmological infall causes kinetic energy to be pumped continuously into the core to prevent it. As a result, cores grow as a fixed fraction of the shock radius. As mentioned in Section IV, the particu-

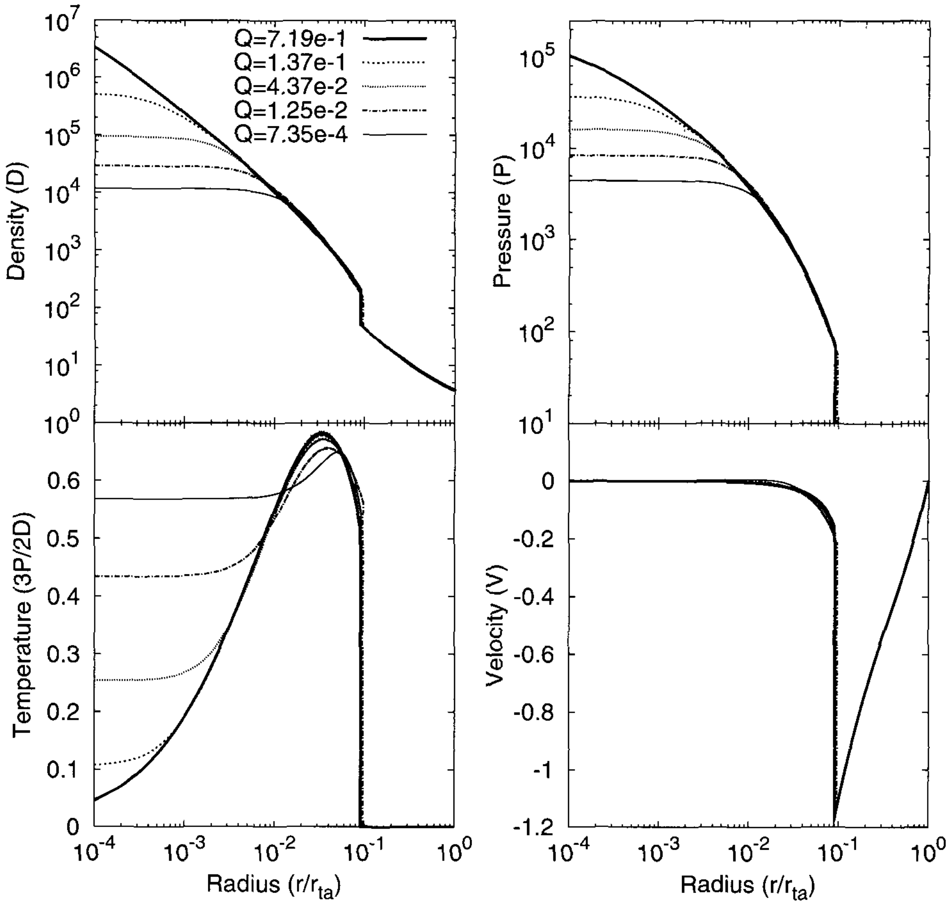

Fig. 4.— Dimensionless profiles for similarity solutions in the high-Q regime. Profiles are indistinguishable from those in Fig. 3, even though Q values are quite different. The effect of Q is reversed from that of low-Q solutions: as Q increases, core density increases and core temperature decreases.

lar value of $\varepsilon = 1/6$ adopted here to make the evolution of SIDM halos self-similar (which, therefore, precludes core collapse), is natural for galaxy formation in a CDM universe, as can be understood in the context of the growth of density peaks. According to this theory, the average initial density profile responsible for the formation of individual galaxy-mass halos in the CDM model corresponds to a value of n_{eff} which is not far from -2.5, the value required to make $\varepsilon = 1/6$. As such, the self-similar behavior reported here for SIDM halos may be a realistic approximation for galactic halos. If the infall rate which builds an individual galactic halo eventually tapers off at late times, however, as reported for N-body simulations by Wechsler et al. (2002), this will break the condition necessary for SIDM self-similarity and may allow gravothermal catastrophe to proceed.

VI. THE ALLOWED RANGE OF SCATTERING CROSS - SECTION FOR AN SIDM UNIVERSE

In a particular universe, we can relate σ to Q if we know the typical formation epochs for halos of different masses, according to the Press-Schechter formalism. We find that a relatively narrow range of Q values,

$$Q \simeq [0.62 - 3.6] \times 10^{-5} \left(\frac{h}{0.70}\right) \left(\frac{\sigma}{5.6cm^2 g^{-1}}\right), \quad (9)$$

characterizes the entire range of halo masses from dwarf-galaxy-mass to cluster-mass, in the currently-favored ΛCDM universe. We also find

$$Q \simeq [1.5 - 6.1] \times 10^{-5} \left(\frac{h}{0.70}\right) \left(\frac{\sigma}{5.6cm^2 g^{-1}}\right) \quad (10)$$

for a flat, cluster-normalized, matter-dominated CDM (i.e. SCDM) universe (See Fig. 5).

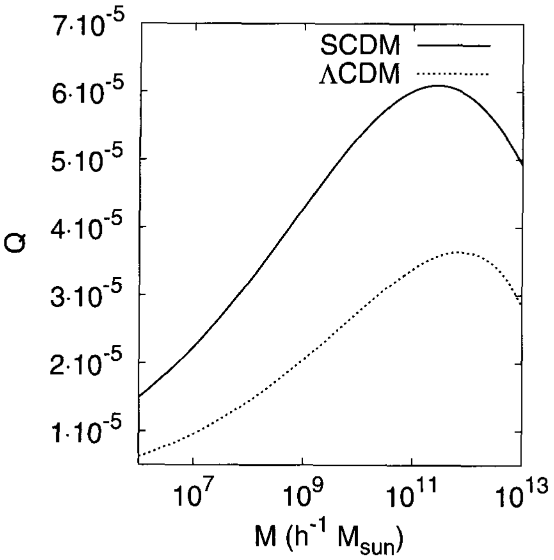

Fig. 5.— The collisionality parameter Q vs. mass of halos at their typical formation epoch for $\sigma = 5.6\,cm^2g^{-1}$ and $h = 0.7$.

The range $\sigma = [0.56 - 5.6]\,cm^2g^{-1}$ is the preferred range of the scattering cross section found by cosmological N-body simulations for the ΛCDM universe to match observed galactic rotation curves (e.g. Davé et al. 2001). The above relation (equ. [9]) with $h = 0.7$ then yields $Q = [6.2\times10^{-7} - 3.6\times10^{-5}]$ for the ΛCDM universe, which is in the low-Q regime. The same profile shapes will occur in SCDM for the same Q-values. If we apply this low-Q solution to SCDM with $h = 0.70$, using equation (10), we get $\sigma = [0.23 - 3.3]\,cm^2g^{-1}$.

As shown in Section V, there also exist high-Q solutions which yield profiles which are quite similar to the low-Q solutions which produce observationally acceptable soft cores. We find that $Q = [1.37\times10^{-2} - 1.7\times10^{-1}]$ in the high-Q regime produces profiles like those in the low-Q regime for $Q = [6.2\times10^{-7} - 3.6\times10^{-5}]$. From the relationship between σ and Q (eqs. [9] and [10]), we predict that $\sigma = [1.2\times10^4 - 2.7\times10^4]\,cm^2g^{-1}$ can also produce acceptable soft cores, therefore, for ΛCDM, while the corresponding σ for SCDM will be about $[5.11\times10^3 - 1.56\times10^4]\,cm^2g^{-1}$.

VII. CONCLUSION

We have found similarity solutions which allow us to describe the dynamical origin and evolution of SIDM halos in a fully cosmological context, analytically, for the first time. Our solutions are based upon fluid-like conservation equations which can be derived by taking moments of the Boltzmann equation, assuming that the particle velocity distribution inside virialized halos is approximately isotropic, as CDM N-body simulations suggest. Our solutions confirm and explain the results of N-body simulations which have attempted to incorporate the collisional effects of SIDM numerically within the CDM model and allow us to extend and generalize those results to a much wider range of parameters and scales than has so far been simulated. Along the way, we have also found a similarity solution for the adiabatic case of standard CDM (i.e. with no SIDM self-interaction) which serves to derive analytically the halo profile of CDM halos found previously by N-body simulation (i.e. intermediate between NFW and Moore profiles), as a product of self-similar cosmological infall.

As seen in our similarity solutions, cosmological infall can affect the dynamics of virialization significantly. In the context of hierarchical clustering, in fact, the collapse of halo cores previously predicted for *isolated* SIDM halos is entirely prevented by cosmological infall, according to these similarity solutions. For realistic mass accretion histories in a CDM universe, therefore, SIDM core collapse will be delayed until this infall becomes negligible. Accordingly, previous analyses (Burkert 2000; Balberg, Shapiro & Inagaki 2002) which predict core collapse in a Hubble time should be re-examined.

By considering the full range of collisionality from $Q = 0$ to $Q = \infty$, we have discovered that the low-Q regime which has been found previously by N-body simulation to produce observationally acceptable soft cores is not unique. We predict that a cross-section of the order of $10^4\,cm^2g^{-1}$ can also produce acceptable soft cores, in addition to the regime of smaller cross-section previously identified. Therefore, we suggest that cosmological N-body simulations be performed which incorporate SIDM particles with $\sigma = [5\times10^3 - 5\times10^4]\,cm^2g^{-1}$ to explore this possibility further.

In reality, the mass accretion rates of CDM halos in N-body simulations are found to depart from the self-similar values over time, declining at late times rather than growing (Wechsler et al. 2002). We have accordingly begun to study the evolution of SIDM halos with a more realistic infall, using a 1-D, spherical hydrodynamics code, and will describe those results elsewhere (Ahn and Shapiro, in preparation).

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REFERENCES

Balberg, S., Shapiro, S. L. & Inagaki, S., 2002, Self-Interacting Dark Matter Halos and the Gravothermal Catastrophe, ApJ, 568, 475

Bertschinger, E., 1985, Self-Similar Secondary Infall and Accretion in an Einstein-de Sitter Universe, ApJ, 58, 39

Bettwieser, E., 1983, A Numerical Method for the Study of the Gravothermal Instability in Star Clusters, MNRAS, 203, 811

Burkert, A., 2000, The Structure and Evolution of Weakly Self-interacting Cold Dark Matter Halos, ApJ, 534, L143

Davé, R., Spergel, D. N., Steinhardt, P. J. & Wandelt, B. D., 2001, Halo Properties in Cosmological Simulations of Self-interacting Cold Dark Matter, ApJ, 547, 574

Fillmore, J. A. & Goldreich, P., 1984, Self-Similar Gravitational Collapse in an Expanding Universe, ApJ, 281, 1

Hoffman, Y. & Shaham, J., 1985, Local Density Maxima - Progenitors of Structure, ApJ, 297, 16

Kochanek, C. S. & White, M., 2000, A Quantitative Study of Interacting Dark Matter in Halos, ApJ, 543, 514

Moore, B., Quinn, T., Governato, F., Stadel, J. & Lake, G., 1999, Cold Collapse and the Core Catastrophe, MNRAS, 310, 1147

Moore, B., Gelato, S., Jenkins, A., Pearce, F. R. & Quilis, V., 2000, Collisional versus Collisionless Dark Matter, ApJ, 535, L21

Navarro, J. F., Frenk, C. S. & White, S. M., 1997, A Universal Density Profile from Hierarchical Clustering, ApJ, 490, 493

Shapiro, P. R., Iliev, I. T. & Raga, A. C., 1999, A Model for the Post-Collapse Equilibrium of Cosmological Structure: Truncated Isothermal Spheres from Top-Hat Density Perturbations, MNRAS, 307, 203

Spergel, D. N. & Steinhardt, P. J., 2000, Observational Evidence for Self-Interacting Cold Dark Matter, PRL, 84, 3760

Wechsler, R. H., Bullock, J. S., Primack, J. R., Kravtsov, A. V. & Dekel, A., 2002, Concentrations of Dark Halos from Their Assembly Histories, ApJ, 586, 52

Yoshida, N., Springel, V., White, S. D. M. & Tormen, G., 2000a, Collisional Dark Matter and the Structure of Dark Halos, ApJ, 535, L103

Yoshida, N., Springel, V., White, S. D. M. & Tormen, G., 2000b, Weakly Self-Interacting Dark Matter and the Structure of Dark Halos, ApJ, 544, L87