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SIMULATIONS IN OPTION PRICING MODELS APPLIED TO KOSPI200

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ABSTRACT. Simulations on the nonlinear partial differential equation derived from Black-Scholes equation with transaction costs are performed. These numerical experiments using finite element methods are applied to KOSPI200 in 2002 and the option prices obtained with transaction costs are closer to the real prices in market than the prices used in Korea Stock Exchange.

1. INTRODUCTION

In the option pricing models of the seminal studies of Black and Scholes [2] and Merton [11], the call option is completely and continuously replicated by a stock and riskless asset portfolio. These models for option pricing assume the frictionless markets without transaction costs. These option pricing models are not applicable in the presence of transaction costs on trading the underlying asset, in recent years many researchers have attempted to develop option pricing models containing transaction costs. This research was begun by Leland [10] and extended by Boyle and Vorst [3], Hoggard, Whalley and Wilmott [8], Avellaneda and Panas [1], Toft [14], Whalley and Wilmott [15], and Henrotte [6]. The first five of these suppose hedging takes place at given discrete time intervals and the last two assume flexible but prescribed trading rules. These involve a band around the ideal value of Δ, within which the number of assets actually held in the portfolio is allowed to vary.

Leland [10] showed that the price of the option should be induced by the Black-Scholes price with a modified volatility, which depends on the transaction costs, the original volatility, and the time interval between successive adjustments of the portfolio. On work of Merton [12] and Shen [13], Boyle and Vorst [3] put Leland's work into the binomial framework of Cox, Ross, and Rubinstein [4] and derived self-financing strategies completely replicating the final payoffs to short and long positions in put

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and call options, assuming proportional transaction costs on trades in the asset and no transaction costs on trades in the bonds.

As a different direction, the global-in-time models illustrated by the model of Hodges and Neuberger [7] and Davis, Panas, and Zariphopoulou [5] achieve an element of optimality since they are based on the approach of utility maximization. These models are slow to compute since they usually result in three- or four-dimensional free boundary problems. In a recent paper Kim [9], he showed the generalized model of Hoggard, Whalley and Wilmott [8] and Henrotte [6] which are concentrated on the analysis of the transaction costs and bandwidth rehedging policy.

In this paper, we obtain numerical solutions of the nonlinear partial differential equations arising in option pricing models with transaction costs and compare these results with real option prices traded in KOSPI200. In section 2, we introduce the nonlinear partial differential equation [9] for option price with transaction costs and bandwidth and solve this nonlinear equation numerically with finite element method using piecewise quadratic polynomial basis functions. In section 3, we compare the result of section 2 with KOSPI200 stock index option prices traded in Korea.

2. Option pricing equation and its numerical solutions

In this section we introduce the option pricing nonlinear partial differential equation [9]. To derive the option pricing equation that has option price $u(t, S)$ with transaction costs, $K(v, S)$, and the bandwidth, Λ, which is a measure of the maximum expected risk in the rehedging portfolio, we need the following assumptions.

- The portfolio is revised every δt where δt is a finite, fixed and small interval.

$$\delta S = \mu S \delta t + \sigma S \phi \delta t^{1/2}$$

where ϕ is drawn from a standardized normal distribution and μ is the stock price's instantaneous expected return and σ is the instantaneous variance of stock price's return.

- Short selling is allowed and the assets are divisible.
- The risk-free interest rate r and the asset volatility σ are known, deterministic functions of time over the life of the option as constant.
- No arbitrage opportunities (The hedged portfolio has an expected return equal to that from a bank deposit)
- The constant dividend yield is η and this dividend is taxed at rate τ. Suppose v is the number of shares traded, we can find the number of assets we trade

$$v = \sigma S \frac{\partial^2 u}{\partial S^2} \phi \delta t^{1/2}$$

from the portfolio

$$\Pi = u - \Delta S$$

where $\Delta = \partial u/\partial S$. The transaction costs $K(v,S)$ is expressed as the sum of three terms, a fixed cost, a cost proportional to volume traded and a cost proportional to the value traded.

$$K(v,S) = k_1 + k_2|v| + \left(\sum_{i=1}^{n}(\zeta_i - \zeta_{i-1})\,U(|v|\,S - x_i)\right)|v|\,S$$

where k_i and ζ_i are constant, and x_i represents the level of the amount $|v|\,S$ which has the proportional constant ζ_i and U is the Heaviside function as

$$U(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

We introduce the nonlinear partial differential equation derived from the option pricing model with transaction costs and bandwidth [9].

Theorem 1. *The option price u with the transaction costs and the bandwidth satisfies the nonlinear partial differential equation*

$$\frac{\partial u(t,S)}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u(t,S)}{\partial S^2} + (r-\eta)(1-\tau)\,S\frac{\partial u(t,S)}{\partial S} - r(1-\tau)\,u(t,S)$$
$$(1) \qquad - \frac{\sigma^2 S^4 \Gamma^2}{\Lambda}\left(k_1 + \left(k_2 + S\sum_{i=1}^{n}(\varsigma_i - \varsigma_{i-1})\,U\left(\Lambda^{\frac{1}{2}} - x_i\right)\right)\frac{\Lambda^{\frac{1}{2}}}{S}\right) = 0$$

where σ, r, η and τ are constants and Γ is the option's gamma $\partial^2 u/\partial S^2$.

We try to find numerical solutions of call option price u satisfied the equation (1). Suppose the stock price S moves from 0 to a and T is the time to maturity of the option. We have the mixed boundary conditions of the equation (1) as following.

$$\begin{cases} u(T,S) = \max(S - X, 0) \\ u(t,0) = 0 \\ u_S(t,a) = 1 \end{cases}$$

where X is the strike price. By using differences for the differential in time, the equation (1) becomes

$$\frac{u(t_m,S) - u(t_{m-1},S)}{\Delta t} + \frac{1}{2}\sigma^2 S^2 u''(t_{m-1},S) + (r-\eta)(1-\tau)\,Su'(t_{m-1},S)$$
$$- r(1-\tau)\,u(t_{m-1},S) - \frac{k_2}{\Lambda^{1/2}}\sigma^2 S^3\left(u''(t_{m-1},S)\right)^2$$
$$(2) \qquad - \left(\frac{k_1}{\Lambda} + \frac{1}{\Lambda^{1/2}}\sum_{i=1}^{n}(\varsigma_i - \varsigma_{i-1})\,U\left(\Lambda^{\frac{1}{2}} - x_i\right)\right)\sigma^2 S^4\left(u''(t_{m-1},S)\right)^2 = 0$$

where

$$\frac{\partial u(t,S)}{\partial S} = u'(t,S)\ , \quad \frac{\partial^2 u(t,S)}{\partial S^2} = u''(t,S)$$

and $\Delta t = t_i - t_{i-1}$ $(0 = t_0 < t_1 < \cdots < t_{M-1} < t_M = T)$. Define three notations for option prices at different time node points and for the simplication,

$$\begin{aligned} V(S) &:= u(t_{m-1}, S) \\ \overline{V}(S) &:= u(t_m, S) \\ \Upsilon_n &:= \sum_{i=1}^{n} (\varsigma_i - \varsigma_{i-1}) U\left(\Lambda^{\frac{1}{2}} - x_i\right). \end{aligned}$$

With these notations the equation (2) can be rewritten as following.

$$\begin{aligned} &\left(\frac{1}{\Delta t} + r(1-\tau)\right) V(S) - \frac{1}{2}\sigma^2 S^2 V''(S) \\ &- (r-\eta)(1-\tau) SV'(S) + \frac{k_2}{\Lambda^{1/2}}\sigma^2 S^3 \left(V''(S)\right)^2 \\ &+ \left(\frac{k_1}{\Lambda} + \frac{1}{\Lambda^{1/2}}\Upsilon_n\right)\sigma^2 S^4 \left(V''(S)\right)^2 - \frac{1}{\Delta t}\overline{V}(S) = 0. \end{aligned}$$

S is the only variable of the function V since t is fixed from now on. Let $C_c[0,a]$ be the class of continuous functions with compact support on $[0,a]$. Define the set Φ of functions as

$$\Phi := \{\Psi(S) \in C_c[0,a] : \Psi(S) \text{ is second order differentiable function}\}.$$

We multiply the function $\Psi(S)$ $(\in \Phi)$ to both sides of the above equation and integrate on $[0,a]$

$$\begin{aligned} &\left(\frac{1}{\Delta t} + r(1-\tau)\right)\int_0^a V(S)\Psi(S)dS - \frac{1}{2}\sigma^2 \int_0^a S^2 V''(S)\Psi(S)dS \\ &- (r-\eta)(1-\tau)\int_0^a SV'(S)\Psi(S)dS + \frac{k_2}{\Lambda^{1/2}}\sigma^2 \int_0^a S^3 \left(V''(S)\right)^2 \Psi(S)dS \\ &+ \left(\frac{k_1}{\Lambda} + \frac{1}{\Lambda^{1/2}}\Upsilon_n\right)\sigma^2 \int_0^a S^4 \left(V''(S)\right)^2 \Psi(S)dS - \frac{1}{\Delta t}\int_0^a \overline{V}(S)\Psi(S)dS = 0. \end{aligned} \tag{3}$$

Because equation (3) is satisfied for all functions $\Psi(S)$, equation (2) and (3) have the same solution $V(S)$. Now we choose admissible basis functions and test functions $\Psi_i(S)$ as two following cases.

Case I. i is even.

$$\Psi_i(S) = \begin{cases} 0 & \text{if } S < (i-2)h \\ \frac{1}{2h^2}(S-(i-2)h)(S-(i-1)h) & \text{if } (i-2)h \le S < ih \\ \frac{1}{2h^2}(S-(i+1)h)(S-(i+2)h) & \text{if } ih \le S < (i+2)h \\ 0 & \text{if } (i+2)h \le S \end{cases}$$

where $h = S_i - S_{i-1}$ $(0 = S_0 < S_1 < \cdots < S_{N-1} < S_N = a)$ and N is an even number.

Case II. i is odd.

$$\Psi_i(S) = \begin{cases} 0 & \text{if } \ S < (i-1)\,h \\ -\dfrac{1}{h^2}\,(S-(i-1)\,h)\,(S-(i+1)\,h) & \text{if } \ (i-1)\,h \le S < (i+1)\,h \\ 0 & \text{if } \ (i+1)\,h \le S. \end{cases}$$

Let $V_h(S)$ be the approximate solution of $V(S)$.

$$V(S) \simeq V_h(S) = \sum_{j=1}^{N} \alpha_j^{m-1} \Psi_j(S).$$

Note that the solution at $j = 0$ can be obtained directly from the second boundary condition. α_j^{m-1} represents the numerical solution of the equation (1) at $(m-1)$th time step and jth node point of stock price. Now we calculate all integration terms of the equation (3). Each term can be calculated in three cases that the node points are even numbers, odd numbers and an end node point. For the first integration term,

$$\int_0^a V(S)\Psi(S)dS$$
$$= \sum_{j=1}^{N} \alpha_j^{m-1} \int_0^a \Psi_j(S)\Psi_i(S)dS$$

Case I. i is even:

$$= -\frac{h}{15}\alpha_{i-2}^{m-1} + \frac{2h}{15}\alpha_{i-1}^{m-1} + \frac{8h}{15}\alpha_i^{m-1} + \frac{2h}{15}\alpha_{i+1}^{m-1} - \frac{h}{15}\alpha_{i+2}^{m-1}$$

Case II. i is odd:

$$= \frac{2h}{15}\alpha_{i-1}^{m-1} + \frac{16h}{15}\alpha_i^{m-1} + \frac{2h}{15}\alpha_{i+1}^{m-1}$$

Case III. i is N:

$$= -\frac{h}{15}\alpha_{N-2}^{m-1} + \frac{2h}{15}\alpha_{N-1}^{m-1} + \frac{4h}{15}\alpha_N^{m-1}.$$

Because the second integration term involves the third boundary condition, the first part used in integration by parts vanishes except at end node point of S.

$$\int_0^a S^2 V''(S)\Psi(S)dS$$

Case I. i is even:

$$\begin{aligned} = &-\frac{h}{30}\left(4+5i^2\right)\alpha_{i-2}^{m-1} + \frac{4h}{15}\left(1+5i^2\right)\alpha_{i-1}^{m-1} \\ &- \frac{h}{15}\left(4+35i^2\right)\alpha_i^{m-1} + \frac{4h}{15}\left(1+5i^2\right)\alpha_{i+1}^{m-1} \\ &- \frac{h}{30}\left(4+5i^2\right)\alpha_{i+2}^{m-1} \end{aligned}$$

Case II. i is odd:

$$
\begin{aligned}
&=\frac{4h}{15}\left(1+5i^2\right)\alpha_{i-1}^{m-1}-\frac{8h}{15}\left(1+5i^2\right)\alpha_i^{m-1}\\
&+\frac{4h}{15}\left(1+5i^2\right)\alpha_{i+1}^{m-1}
\end{aligned}
$$

Case III. i is N:

$$
\begin{aligned}
&=a^2-\frac{h}{30}\left(4+5N^2\right)\alpha_{N-2}^{m-1}\\
&+\frac{4h}{15}\left(1+5N^2\right)\alpha_{N-1}^{m-1}-\frac{h}{30}\left(4+35N^2\right)\alpha_N^{m-1}.
\end{aligned}
$$

We perform the third term of equation (3) in a similar way.

$$
\int_0^a SV'(S)\Psi(S)\,dS
$$

Case I. i is even:

$$
\begin{aligned}
&=\frac{h}{30}\left(-4+5i\right)\alpha_{i-2}^{m-1}-\frac{2h}{15}\left(-2+5i\right)\alpha_{i-1}^{m-1}-\frac{4h}{15}\alpha_i^{m-1}\\
&+\frac{2h}{15}\left(2+5i\right)\alpha_{i+1}^{m-1}-\frac{h}{30}\left(4+5i\right)\alpha_{i+2}^{m-1}
\end{aligned}
$$

Case II. i is odd:

$$
=-\frac{2h}{15}\left(-2+5i\right)\alpha_{i-1}^{m-1}-\frac{8h}{15}\alpha_i^{m-1}+\frac{2h}{15}\left(2+5i\right)\alpha_{i+1}^{m-1}
$$

Case III. i is N:

$$
\begin{aligned}
&=\frac{h}{30}\left(-4+5N\right)\alpha_{N-2}^{m-1}-\frac{2h}{15}\left(-2+5N\right)\alpha_{N-1}^{m-1}\\
&+\frac{h}{30}\left(-4+15N\right)\alpha_N^{m-1}.
\end{aligned}
$$

The next two parts including $\int_0^a S^3\left(V''(S)\right)^2\Psi(S)dS$ and $\int_0^a S^4\left(V''(S)\right)^2\Psi(S)dS$ of the integration containing the non-linear property of equation (1) should be calculated carefully. First of all, we find the second derivatives of all basis functions and multiply their square values to the followings

$$
\int_0^a S^3\Psi_i(S)dS \quad \text{and} \quad \int_0^a S^4\Psi_i(S)dS
$$

where i is the main node point. Thus the first part is expressed as following.

$$
\int_0^a S^3\left(V''(S)\right)^2\Psi(S)dS
$$

Case I. i is even:

$$=\frac{1}{15}\left(4-6i+5i^3\right)\left(\alpha_{i-2}^{m-1}-2\alpha_{i-1}^{m-1}+\alpha_i^{m-1}\right)^2$$
$$+\frac{1}{15}\left(-4-6i+5i^3\right)\left(\alpha_i^{m-1}-2\alpha_{i+1}^{m-1}+\alpha_{i+2}^{m-1}\right)^2$$

Case II. i is odd:

$$=\frac{4}{15}i\left(3+5i^2\right)\left(\alpha_{i-1}^{m-1}-2\alpha_i^{m-1}+\alpha_{i+1}^{m-1}\right)^2$$

Case III. i is N:

$$=\frac{1}{15}\left(4-6N+5N^3\right)\left(\alpha_{N-2}^{m-1}-2\alpha_{N-1}^{m-1}+\alpha_N^{m-1}\right)^2.$$

Similarly, we have the second part mentioned above in three cases.

$$\int_0^a S^4\left(V''(S)\right)^2\Psi(S)dS$$

Case I. i is even:

$$=\frac{h}{105}\left(-48+112i-84i^2+35i^4\right)\left(\alpha_{i-2}^{m-1}-2\alpha_{i-1}^{m-1}+\alpha_i^{m-1}\right)^2$$
$$+\frac{h}{105}\left(-48-112i-84i^2+35i^4\right)\left(\alpha_i^{m-1}-2\alpha_{i+1}^{m-1}+\alpha_{i+2}^{m-1}\right)^2$$

Case II. i is odd:

$$=\frac{4h}{105}\left(3+42i^2+35i^4\right)\left(\alpha_{i-1}^{m-1}-2\alpha_i^{m-1}+\alpha_{i+1}^{m-1}\right)^2$$

Case III. i is N:

$$=\frac{h}{105}\left(-48+112N-84N^2+35N^4\right)\left(\alpha_{N-2}^{m-1}-2\alpha_{N-1}^{m-1}+\alpha_N^{m-1}\right)^2.$$

Since the last term of the equation (3) is evaluated from the first boundary condition, we skip the calculation steps. From all three cases the numerical solutions of the equation (1) using iterative methods can be found easily and rapidly.

As a different way, put option's boundary conditions are formulated as following.

$$\begin{cases} u\left(T,S\right)=\max\left(X-S,0\right) \\ u\left(t,0\right)=Xe^{-r(T-t)} \\ u\left(t,a\right)=0. \end{cases}$$

The numerical solutions of the put option price with transaction costs in equation (1) can be obtained in a very similar way to the procedure in the call option pricing.

3. Numerical experiments for option prices with real data

In this section we compare numerical solutions computed in section 2 with KOSPI200 stock index option price traded in Korea. KOSPI200 option price is represented by "point" and its price is 100,000 Won per one point contract. We choose the data that started on September 13, 2002 and ended on December 12, 2002 and suppose that the bandwidth, Λ, of the rehedging position is the exercise price for each underlying asset price. Suppose there are no dividend and tax in KOSPI200 stock index option, so $\eta = \tau = 0$. We use the 91-date CD(certificate of deposit) yield for risk-free interest rate and apply 0.3% brokerage commission as transaction costs.

We perform the evaluations of the option prices every 10 days from starting date of the KOSPI200 stock index option and use the closing price of the option during the day as option price. If the day chosen for data is not a working day, the option price of the next working day is taken. To simplify the notations of tables, we adopt the following abbreviations.

- $\triangleright$ Day : deal date of the option
- $\triangleright$ EP : exercise price for option
- $\triangleright$ AP : underlying asset price i.e. KOSPI200 stock index
- $\triangleright$ CDR : risk-free interest rate i.e. 91-date CD rate
- $\triangleright$ RP : real KOSPI200 stock index option price traded i.e. closing price
- $\triangleright$ TP : theoretical price provided by Korea Stock Exchange
- $\triangleright$ NS : numerical solution of equation (1) computed in section 2

In Table 1, we compare KOSPI200 stock index call option prices with theoretical prices provided by Korea Stock Exchange and our numerical solutions calculated in section 2. The absolute value of maximum error for theoretical prices, $\|\mathrm{RP} - \mathrm{TP}\|_\infty = 2.17$, is larger than that for numerical solutions, $\|\mathrm{RP} - \mathrm{NS}\|_\infty = 0.415$, on September 13, 2002. The total errors for call option are $\|\mathrm{RP} - \mathrm{TP}\|_1 = 9.07$ and $\|\mathrm{RP} - \mathrm{NS}\|_1 = 1.82$. These facts show that numerical solutions are closer to real traded option prices than theoretical prices.

In Table 2, we compare KOSPI200 stock index put option prices with theoretical prices and numerical solutions. The absolute value of maximum error for theoretical prices, $\|\mathrm{RP} - \mathrm{TP}\|_\infty = 2.7$, is larger than that for numerical solutions, $\|\mathrm{RP} - \mathrm{NS}\|_\infty = 1.15$, on October 4, 2002. The total errors for put option are $\|\mathrm{RP} - \mathrm{TP}\|_1 = 9.92$ and $\|\mathrm{RP} - \mathrm{NS}\|_1 = 6.02$. We also know that numerical solutions are closer to real traded option prices than theoretical prices.

In Table 1 and 2, it is shown that numerical solutions give the better approximation to real option prices than theoretical prices provided by Korea Stock Exchange and numerical solutions of call option prices on KOSPI200 stock index are closer to real prices than those of put option prices. On the maturity, 104,624,716 contracts on KOSPI200 stock index call option were traded but only 72,499,076 contracts on put option traded. The volume of contracts traded on maturity is expected to affect the accuracy of numerical solution of option price. Numerical solutions computed here can be provided as the theoretical prices for KOSPI200 stock index option prices in Korea Stock Exchange.

Table 1. Comparison on call option prices with numerical solutions

Day	EP	AP	CDR	Volatility	RP	TP	NS
09-13	90	90.3	4.80%	33.24%	6.3	7.3	6.64884594
09-23	85	85.18	4.82%	33.95%	5.55	7.72	5.96464573
10-04	82.5	81.91	4.86%	32.74%	4.5	4.51	4.77272173
10-14	77.5	77.27	4.88%	34.79%	4.6	2.88	4.54069822
10-23	82.5	82.77	4.94%	36.08%	5.15	3.55	4.87008560
11-04	85	85.04	4.94%	36.93%	4.45	2.68	4.33982184
11-12	82.5	82.74	4.92%	34.83%	3.6	3.79	3.64902607
11-22	87.5	87.97	4.89%	33.53%	3.05	2.89	3.19504184
12-02	92.5	92.69	4.90%	32.29%	2.1	1.65	2.23847062

Table 2. Comparison on put option prices with numerical solutions

Day	EP	AP	CDR	Volatility	RP	TP	NS
09-13	90	90.3	4.80%	33.24%	6.1	4.77	5.27634767
09-23	85	85.18	4.82%	33.95%	5.9	4.3	4.87853316
10-04	82.5	81.91	4.86%	32.74%	5.75	4.83	4.59589328
10-14	77.5	77.27	4.88%	34.79%	4.8	7.5	4.15003742
10-23	82.5	82.77	4.94%	36.08%	5.15	6.02	4.03093637
11-04	85	85.04	4.94%	36.93%	4.45	6.27	3.85084925
11-12	82.5	82.74	4.92%	34.83%	3.6	3.45	3.06375846
11-22	87.5	87.97	4.89%	33.53%	2.42	2.83	2.47804241
12-02	92.5	92.69	4.90%	32.29%	1.99	2.11	1.93102257

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