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PID Autotuning Algorithm with an Asymmetric Self-oscillation

비대칭 자기 진동에 대한 PID 자동동조 알고리듬

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Abstract

We use the saturation nonlinear feedback element to generate self-oscillation in order to find an ultimate gain and period of linear plant. The use of saturation nonlinear feedback element can improve accuracy of an ultimate gain and period of unknown linear plant. An ultimate gain and period of linear plant can be used to tune a PID controller parameters. It is possible that an asymmetric oscillation can be occurred under the special circumstances such as with static load disturbance. We analyze an asymmetric self-oscillation. As the results of an analysis, we propose a method to find an ultimate gain and period of linear plant under the asymmetric self-oscillation.

요 약

선형 플랜트의 극한 이득과 주기를 찾기 위해 포화함수를 이용하여 자기 진동을 발생 시켰다. 포화함수 사용으로 극한이득과 주기의 정확성을 높였다. 발견한 극한 이득과 극한 주기는 PID 제어기 값을 구하는데 사용하였다. 정적인 부하 왜란 등이 있는 경우 비대칭 진동이 발생할 수 있다. 발생하는 비대칭 자기 진동을 분석하였고 분석결과로부터 극한 이득과 주기를 찾는 방법을 제안하였다.

Keyword : asymmetric oscillation, saturation function, ultimate gain, ultimate period

I. Introduction

In spite of significant advances of modern control theory, PID controllers have been widely used in the industries since field engineers are familiar with the structure of PID controllers. Many methods to find PID control parameters, i.e., proportional gain, integral time, and derivative time, have been suggested and commercialized[1]. Automatically setting the PID controller parameters called autotuning of PID controllers has been received much attention because of reducing a start-up time and easy to use; simply pushing a single button to tune a controller's parameters. One of simple autotuning method is the use of relay as a test

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signal[2]. The relay feedback generates the stable limit cycle which is a robust limit cycle, since the closed-loop system is a nonlinear system. From the observation of the limit cycle in the plant output, one can calculate the ultimate gain and period of an unknown plant which is correspond to one point identification in the Nyquist plot of the unknown plant. From the one point information in the Nyquist plot of an unknown plant, various PID controller design methods, e.g., Ziegler-Nichols tuning formula and phase margin and gain margin, can be applied to find controller parameters[3,4,5,6,7,8]. Since the method used the describing function approximation to calculate ultimate gain and period is based on the assumption of which the plant output contains only fundamental frequency components, it is desirable that the output of unknown plants does not contain the high frequency components. However the relay feedback easily excites the high frequency components, since relay feedback acts like a step function, and results in containing the unwanted frequency components in the plant output. Consequently, it is possible that there is an error between the ultimate gain and period calculated using the describing function and the real one due to the presence of high harmonics of the plant output. Moreover, most of actuators have nonlinear characteristics in the limit case, it is expected that plant output can be distorted with relay feedback, therefore it is desirable that a slope of actuator input signal is less than that of actuator's. To remove the high frequency components, the paper[9] proposes the slope bounded saturation function instead of relay feedback element to generate the self-sustained oscillation in the plant output. It was demonstrated that the use of saturation functions reduces the excitation of high frequency component of plant. The work[9] assumes that there is a symmetric oscillation in the plant output and uses the describing function to find the ultimate gain and period. But it is possible that there is an asymmetric oscillation caused by the static load disturbances, the use of asymmetric relay, and non-zero constant reference input[10,11]. We can not use the describing function approximation to find the ultimate gain and period in the case of the asymmetric oscillation, since the describing function approximation used in [9] was required a symmetric oscillation. In this paper, we analyze the asymmetric oscillation with a saturation nonlinear element. In particular, we consider the presence of static load disturbances in the closed-loop system, since static load disturbances are usually unknown, while asymmetric nonlinear elements and non-zero constant reference input are known in advance. From the analysis of asymmetric oscillation, we give a method to restore the symmetric oscillation using biased saturation nonlinear element. Once a symmetric oscillation is restored in the plant output, we can use the our previous work results[9] to tune a PID controller. We illustrate the performance of our method via an example.

II. The analysis and autotuning for asymmetric oscillation with a saturation nonlinear element

2.1 Symmetric self-oscillation with a saturation function nonlinear element

The analysis of symmetric oscillation with the saturation function nonlinearity had been studied in the paper[9]. We briefly describe the symmetric oscillation for the completeness. Consider the feedback system without static load disturbance as given in Fig. 1 to analyze the symmetric oscillation.

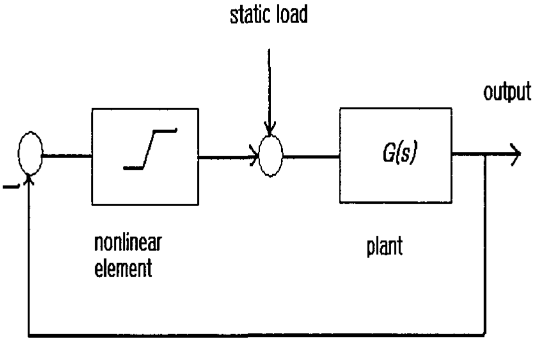

그림 1. 비 선형 요소가 들어 있는 폐회로 시스템
Fig. 1. A closed-loop system with nonlinear element

The describing function method is a popular tool to analyze periodic solutions for linear time-invariant dynamic system with nonlinear feedback element[12,13]. To have a self-oscillation in the plant output, the following harmonic balance equation should have a non-trivial solution[12,13]

$$1 + G(jw) \cdot N(a) = 0 \tag{1}$$

where $G(jw)$ is the frequency response of plant, a is the amplitude of fundamental component of the plant output, and $N(a)$ is a describing function of nonlinear element. Note that the equation (1) can be derived under the assumption of symmetric oscillation. For the saturation function shown in Fig. 2 as a nonlinear element defined by

$$sat(x) = \begin{cases} -d & x < -\frac{d}{s} \\ sx & -\frac{d}{s} \le x \le \frac{d}{s}, \\ d & x > \frac{d}{s} \end{cases}$$

(2)

the describing function of the saturation nonlinear element is given by

$$N_{sat}(a) = \begin{cases} s & \text{if } d > sa \\ \frac{2s}{\pi}\left[\arcsin\left(\frac{d}{sa}\right) + \frac{d}{sa}\sqrt{1-\left(\frac{d}{sa}\right)^2}\right] & \text{if } d \le sa \end{cases}$$

(3)

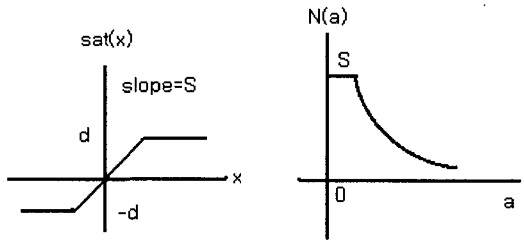

그림 2. 포화 함수형 비선형 요소 및 묘사함수
Fig. 2. Saturation nonlinear element and its describing function

The existence condition for non-trivial solution of the equation (1) is $s > |1/G(jw_1)|$, where $|G(jw_1)|$ is the absolute value of $G(jw)$ at the intersection point with the negative real axis in the Nyquist plot of $G(jw)$ and w_1 is the frequency of the plant output at that point. It means that the slope of saturation function should be a large enough to sustain a symmetric oscillation in the closed-loop system. One way to find the good slope to have a self-oscillation is described in [9]. We can calculate the ultimate gain defined by $\frac{1}{|G(jw_1)|}$ and ultimate period from the observation of magnitude and period of the plant output using the equation (1) and (3).

2.2 The analysis of asymmetric oscillation and autotuning with the saturation function nonlinear element

It is possible that there is an asymmetric oscillation in the output caused by static load disturbances, the use of asymmetric relay, and non-zero constant reference input[10,11]. When there is an asymmetric oscillation in the plant output, the output of the saturation function nonlinear element is shown with $t_1 - t_2 \neq t_4 - t_5$ in the Fig. 3. Note that when $t_1 - t_2 = t_4 - t_5$, the plant output has a symmetric one.

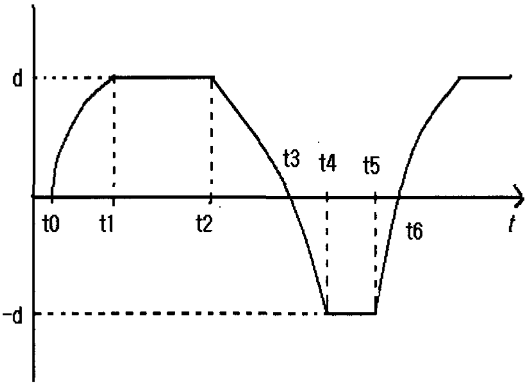

그림 3. 플랜트 출력이 비대칭인 경우의 포화함수의 출력.
Fig. 3. The output of saturation function with asymmetric plant output

We assume that there is an asymmetric oscillation caused by an unknown static load disturbance in the plant output. The output $y(t)$ can be represented by the following Fourier series. $y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkwt}$, where a_k are Fourier series coefficient. We can also represent the output of nonlinear element, $\psi(-y)$, in Fourier series with the same frequency as $\psi(-y) = \sum_{k=-\infty}^{\infty} c_k e^{jkwt}$ where c_k are the Fourier series coefficients. Since $y(t)$ is the output of the plant, $G(s)$, and $\psi(-y) + l$ is an input of plant, the following equation should hold

$$d(p)y(t) - n(p)(\psi(-y) + l) = 0 \qquad (4)$$

where $p = \frac{d()}{dt}$, n(s) and d(s) are the numerator and denominators polynomials of G(s). Using the following relation,

$$d(p) \sum_{k=-\infty}^{\infty} a_k e^{jkwt} = \sum_{k=-\infty}^{\infty} d(jkw) a_k e^{jkwt}$$
$$n(p) \sum_{k=-\infty}^{\infty} c_k e^{jkwt} = \sum_{k=-\infty}^{\infty} c(jkw) c_k e^{jkwt}$$

the equation (4) can be rewritten as

$$\sum_{k=-\infty}^{\infty} [d(jkw) a_k - n(jkw)(c_k + l_k)] e^{jkwt} = 0 \qquad (5)$$

where $l_k = l$ for $k=0$, and $l_k = 0$ for $k \geq 1$. Since e^{jwkt} are orthogonal for each k, the equation (5) is equivalent to $G(jkw)(c_k + l_k) - a_k = 0$. Then the first order harmonic balance approximation is given by

$$G(0)(\hat{c}_0 + l) - a_0 = 0 \qquad (6)$$

$$G(jw)\,\hat{c}_1 - \frac{a}{2j} = 0 \qquad (7)$$

where a_0 is the d.c. value of the plant output,

$$\hat{c}_o = w/2\pi \int_0^{2\pi/w} \psi(-(a_o + a \sin wt))dt,$$

$$\hat{c}_1 = w/2\pi \int_0^{2\pi/w} \psi(-(a_o + a \sin wt)e^{-jwt})dt,$$

a and w are the amplitude and frequency of the first harmonic of the plant output, respectively. Note that c_k is the Fourier coefficient for infinite dimensional case, while $\hat{c}_k$ is the Fourier coefficient for finite dimensional case which is an approximation of infinite dimensional case. It can be verified that for symmetric case, $a_0 = 0 \Rightarrow \hat{c}_0 = 0$. One can derive the equation (1) from the equation (7). When $a_0 \neq 0$ in the plant output, we can not use the equation (1) to find the ultimate gain. One way to find out one point information in the Nyquist plot of the plant is to solve the complex equations (6) and (7). However, it is very complicated to solve the equation (7). The other method is the cancellation of a static load disturbance to restore a symmetric oscillation, since an asymmetric oscillation comes from a static load disturbance. Note that it is studied for relay feedback case in [10]. However, we do not know the static load disturbance in advance. We need to estimate a static load disturbance. Since we know the a_o from the observation of the plant output and can calculate the $\hat{c}_o$ from the observation of the output of saturation function nonlinear element, we can calculate l from the equation (6) under the assumption of knowing of $G(0)$. Note that the plant d.c. gain $G(0)$ can be found using various methods. The step response method is one of simple methods. Then the unknown static load disturbance is given by $l=\frac{a_o}{G(0)}-\hat{c}_o$, where

$$\begin{aligned}\hat{c}_o &= d_1(t_2-t_1)-d_1(t_5-t_4)+2\Big(\int_{t_0}^{t_1} S_1(a_0+a\sin wt)\\ &\quad dt+\int_{t_3}^{t_4} S_1(a_0+a\sin wt)dt\Big)\\ &= (d_1(t_2-t_1)-d_1(t_5-t_4)+2(S_1a_0(t_1-t_0)\\ &\quad -(aS_1/w)(\cos wt_1-\cos wt_0)+S_1a_0(t_4-t_3)\\ &\quad -(aS_1/w)(\cos wt_4-\cos wt_3)))/(t_6-t_0)\end{aligned} \tag{8}$$

where S_1 is the slope of the saturation function. Once we calculate the unknown static load disturbance, we can restore the symmetric oscillation by using an asymmetric saturation function nonlinear element defined by

$$sat_{bias}(x) = \begin{cases} -d+y_0 & x < -\frac{d}{S_1} \\ S_1x+y_0 & -\frac{d}{S_1} \le x \le \frac{d}{S_1} \\ d+y_0 & x > \frac{d}{S_1} \end{cases}$$

with $y_0=-l$.

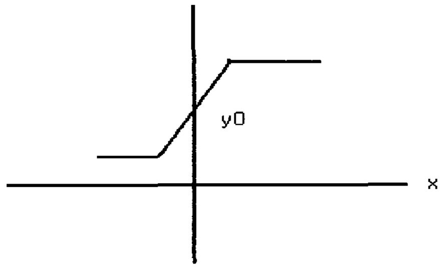

그림 4. 바이어스 값이 y0인 비대칭 포화함수.
Fig. 4. Asymmetric saturation function nonlinear element with bias value=y0

In fact, the biased saturation nonlinear element cancels the static load disturbance. After restoring a symmetric oscillation in the plant output, we can calculate the ultimate gain, $k_c=N_{sat}(a)$, where $N_{sat}(a)$ can be obtained using the equation (3). Once we know the ultimate gain and period, we can design the PID controller[2,9]. Suppose that the desired phase margin is ϕ_m for the closed-loop system and the structure of PID controller is given by $G_c(s)=k(1+sT_d+\frac{1}{sT_i})$ where k, T_d, and T_i are proportional gain, differential time, and integral time, respectively. Since the PID parameters of $G_c(\cdot)$ are to be chosen such that the following equation is satisfied

$$\angle G_c(jw_1)+\angle G(jw_1)=-\pi+\phi_m \tag{9}$$

where w_1 is a ultimate frequency. We can calculate $\angle G_c(jw_1)$ from the equation (9). Following the design method of [1],

$$T_d=\frac{\tan(\phi_m)+\sqrt{4\alpha+\tan^2(\phi_m)}}{2w_1}$$

$$T_i=T_d/\alpha$$

where a typical value of α = 0.25 . Since loop transfer function with the PID controller has unit gain at w_1, $|\ G_c(jw_1)G(jw_1)\ |\ =\ 1$.

$$\Rightarrow |G_c(jw_1)| = \left|\frac{1}{G(jw_1)}\right| = k_1$$

$$\Rightarrow \frac{k}{\cos(\phi_m)} = k_1$$

$$\Rightarrow k = k_1\cos(\phi_m)$$

We can design similarly for the amplitude margin specification, and it can be shown

$$T_d = \frac{\sqrt{4\alpha}}{2w_1}$$
$$T_i = T_d/\alpha$$
$$k = (1/A_m)k_1$$

where A_m is a desired amplitude margin.

III. Example

Consider the plant given by $G(s) = 1/(1+4s)e^{-3s}$ and static load disturbance is equal to 0.75. The Nyquist plot of $G(s)$ is shown in Fig. 5.

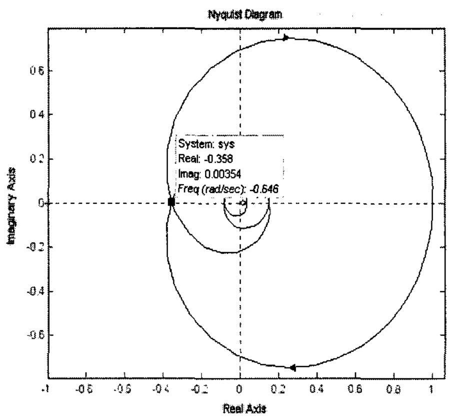

그림 5. $G(s)=1/(1+4s)e^{-3s}$의 Nyquist 선도
Fig. 5. The Nyquist plot of $G(s)=1/(1+4s)e^{-3s}$

It can be observe that there is asymmetric oscillation in the plant output and the output of saturation nonlinear element in the Fig. 6.

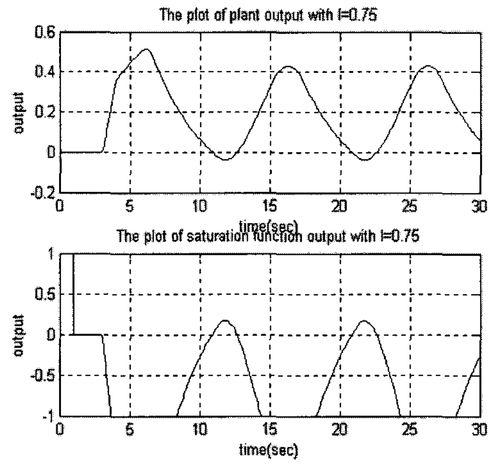

그림 6. $l=0.75$ 인 경우의 플랜트 출력 및 포화함수 출력.
Fig. 6. The plot of plant output and the output of saturation nonlinear element with $l=0.75$.

One can observe that $a_0\approx0.21$, $a\approx0.23$, and the period=10[sec] from the plant output. One can verify that $N_{sat}(a)=4.46$ and ultimate gain = 4.46 using the equation (3). From Fig. 5, one can observe that the real ultimate gain=2.8. The error of estimate value of ultimate gain to the real one is around 45%. To restore the symmetric oscillation, we need to find c_0. Using the equation (6), we can find $c_0\approx-0.63$. Since $G(0)$=1, we can calculate $l=0.72$ from the equation (6). We use the saturation nonlinear element with $y_0=-0.72$ in Fig. 4. The plot of plant output with the biased saturation function is shown in the Fig. 7. The plant output restores the symmetric oscillation in Fig. 7. From Fig. 7, it can be verified that $a=0.45$, the ultimate period=9.8 [sec]. Using the equation (3), one can verify that $N_{sat(a)}=2.7$, the ultimate gain=2.7. Since the real ultimate gain = 2.8 and the ultimate period= 9.7[sec], the results is close to the real one.

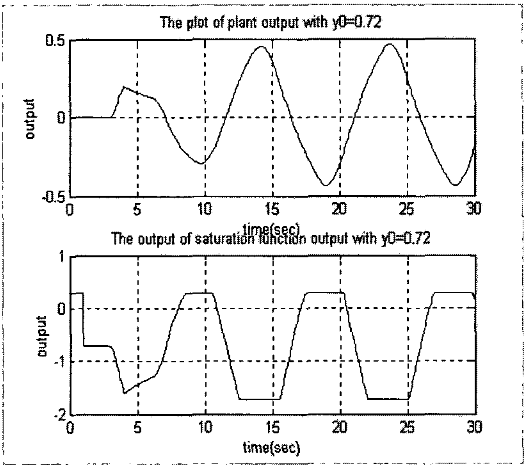

그림 7. $y_0=-0.72$인 경우의 플랜트 출력 및 포화함수 출력

Fig. 7. The plot of plant output and output of saturation nonlinear element with $y_0=-0.72$.

Ⅳ. Conclusion

We consider the asymmetric oscillation caused by a static load disturbance in the plant output. The asymmetric oscillation with a saturation nonlinear element was analyzed by using a first order harmonic balance equation. We estimate a static load disturbance from the harmonic balance equation and propose a method to restore symmetric oscillation by using an estimate of static load disturbance and biased saturation nonlinear element which is used the cancellation of static load disturbance. After restoring the symmetric oscillation, we can use the same tuning method used in the symmetric oscillation. A performance our method is verified in the example.

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