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Modification of Existing Similarity Coefficients by Considering an Operation Sequence Ratio in Designing Cellular Manufacturing Systems

Yong Yin†
Department of Public Policy and Social Studies
Yamagata University, 1-4-12, Kojirakawa-cho, Yamagata-shi, 990-8560, JAPAN
Tel: +81-23-628-4281, E-mail: yin@human.kj.yamagata-u.ac.jp

Kazuhiko Yasuda
Graduate School of Economics and Management
Tohoku University, Kawauchi, Aoba-ku, Sendai, 980-8576, JAPAN
Tel: +81-22-217-6296, E-mail: yasuda@econ.tohoku.ac.jp

Abstract. An operation sequence of parts is one of the most important production factors in the design of cellular manufacturing systems. Many similarity coefficient method (SCM) based approaches have been proposed to solve cell formation problems in the literature. However, most of them do not consider the operation sequence factor. This study presents an operation sequence ratio (OSR) and modifies some existing similarity coefficients using the OSR to solve cell formation problems considering operation sequences. The computational results show that the OSR ratio is useful and robust in solving cell formation problems with operation sequences.

Keywords: cell formation, cellular manufacturing, group technology, similarity coefficient

1. INTRODUCTION

Group technology (GT) is a manufacturing philosophy that exploits similarities in product designs and manufacturing processes. The objective of GT is to increase production efficiency by processing part families within machine cells. GT leads to a lot of advantages such as reduction of material handling times, costs, labors, paper works, in-process inventories, production lead times, and increase of machine utilizations (Ham *et al.*, 1985).

One application of GT to production is the cellular manufacturing (CM). Cell formation (CF) is a vital aspect in the design of a CM system. CF identifies similar parts and groups them into part families which are manufactured by a cluster of dissimilar machines. The main objective of CF is to construct machine cells, identify part families, and allocate part families to machine cells so as to minimize inter-cellular movements of parts.

Numerous methods have been proposed to identify machine cells and their associated part families. These methods can be grouped into classification and coding systems, and clustering methods. Production flow analysis (PFA) is the first clustering method which was used by Burbidge (1971) to rearrange rows and columns of a machine part incidence matrix by trial and error until a satisfactory solution is found. The ranked-order clustering algorithm (ROC) introduced by King (1980) is an example of an analytical approach.

Extension reviews of various approaches for CF are available in the literature (Kumar and Vannelli, 1983; Wemmerlov and Hyer, 1986; Chu and Pan, 1988; Lashkari and Gunasingh, 1990; Reisman *et al.*, 1997; Selim *et al.*, 1998). Wemmerlov and Johnson (1997) employed a mail survey methodology and provided implementation experiences and performance achievements in 46 user firms. Miltenburg and Zhang (1991) carried out a comparative study of nine well-known algorithms.

The remainder of this paper is organized as follows. In section 2, we discuss the background and definition of the operation sequence ratio. In section 3, a two-stage

† : Corresponding Author

heuristic algorithm is presented. In section 4, some traditional similarity coefficients are used and numerical examples are provided to illustrate the solution procedure. Finally, the conclusions are given in section 5.

2. PROBLEM FORMULATION

In the design of cellular manufacturing systems, many production factors should be involved when cells are created. They are machine requirements, machine setup times, utilizations, workloads, alternative routings, machine capacities, operation sequences, setup costs and cell layout (Wu and Salvendy, 1993). Due to the complexity of the cell formation problem, it is impossible to consider all production factors. A few approaches considering different factors have been developed. In this paper, we propose an operation sequence ratio to modify existing similarity coefficients with consideration of the operation sequence factor.

2.1 Background

An operation sequence of parts is an important manufacturing factor in the design of a cellular manufacturing system. The operation sequence is defined as an ordering of machines on which parts are sequentially processed (Vakharia and Wemmerlov, 1990). Choobineh (1988), Sarker and Xu (1998) emphasized the importance of the operation sequence factor. Choobineh (1988) indicated that machine requirements and operation sequences are the most relevant production factors. However, many methods developed so far usually focused on the machine requirements factor, the operation sequence factor is ignored by these methods. Since the achievement of GT in a production system is affected by the choice of machines and impact of material flows, only consideration of machine requirements can not reflect the impact of material flows.

Sarker and Xu (1998) presented a brief review of cell formation methods in consideration of the operation sequence. A number of operation sequence-based similarity/dissimilarity coefficients are discussed in their paper. They presented four cell formation methods: mathematical programming, network analysis, materials flow analysis method, and heuristics.

Choobineh (1988) presented a two-stage procedure for the design of a cellular manufacturing system with operation sequence. The first stage uses a similarity coefficient to form part families. In the second stage, an integer programming model is developed to obtain machine cells.

Vakharia and Wemmerlov (1990) proposed a similarity coefficient with operation sequences to integrate the intracell flow in a cell formation problem by using a clustering methodology.

Logendran (1991) developed an algorithm to form the cells by evaluating the intercell and intracell moves with the operation sequences. He also indicated the impact of operation sequences and cell layouts in a cell formation problem.

Wu and Salvendy (1993) considered a network analysis method by using an undirected graph (network) to model a cell formation problem with operation sequences.

Sarker and Xu (1998) pointed out that a new operation sequences based similarity coefficient can be developed. Since a number of efficient similarity coefficients have been proposed and applied to cell formation problems, the purpose of this paper is to extend existing similarity coefficients to solve cell formation problems with operation sequences in order to utilize these existing similarity coefficients and give an alternative to "reinventing the wheel" for the common consideration of operation sequences based cell formation problems. We use an operation sequence ratio to achieve this purpose.

2.2 Definition

A number of similarity coefficients have been proposed in the literature to solve cell formation problems. Sarker and Islam (1999) presented the performance of some of most commonly used similarity coefficients. Most similarity coefficients ignore the actual impact of material flows. To overcome this deficiency, we propose an operation sequence ratio to extend the existing similarity coefficients to consider operation sequences.

Among the similarity coefficients, the one that comes first and is most important in numerical taxonomy is Jaccard similarity coefficient which was firstly used by McAuley (1972) to form machine cells in a single linkage clustering technique. In this paper, we use Jaccard similarity coefficient to interpret proposed operation sequence ratio.

2.2.1 The Jaccard similarity coefficient

Jaccard similarity coefficient is defined between two machines in terms of the number of parts that visit each machine. It is often expressed as follows:

$$S_{ik} = \frac{a}{a+b+c}, \quad 0 \le S_{ik} \le 1 \qquad (1)$$

where

S_{ik} : similarity between machine i and machine k.
a : the number of parts processed by both machines .
b : the number of parts processed by machine i and not k.
c : the number of parts processed by machine k and not i.

The Jaccard similarity coefficient is simple and easy to calculate, so it is widely used by many clustering algorithms.

2.2.2 Definition of the operation sequence ratio (OSR)

In order to extend the existing similarity coefficients to solve cell formation problems with operation sequences, we use an operation sequence ratio OSR_{ik} to modify the existing similarity coefficients. We define the new similarity coefficients as follows:

$$S_{ik}' = S_{ik} * OSR_{ik} \tag{2}$$

S_{ik}' is the modified similarity coefficient that considers operation sequences and S_{ik} is an existing similarity coefficient. The value of the ratio OSR_{ik} varies from 0 to 1, it is defined as follows:

$$OSR_{ik} = \frac{X_{ik}}{D_{ik}}, \quad (0 \leq OSR_{ik} \leq 1) \tag{3}$$

The denominator D_{ik} indicates the number of possible produced movements of parts between machines i and k. The numerator X_{ik} is the number of actual direct movements of parts between machines i and k.

$$X_{ik} = \sum_{j=1}^{P} x_{ik}^{j} \qquad D_{ik} = \sum_{j=1}^{P} d_{ik}^{j}$$

where

x_{ik}^{j} the number of times that part j moves between machines i and k directly.

d_{ik}^{j} the number of possible produced movements of part j between machines i and k.

P the number of parts in the system.

x_{ik}^{j} is the number of actual direct movements under the constraint that machine $k(i)$ is the immediate successor of machine $i(k)$ in the operation sequence of part j. However, d_{ik}^{j} is a measure of possibility. If part j visits machines i and k one time, then $d_{ik}^{j}=1$ even though machine $k(i)$ is not the immediate successor of machine $i(k)$, and we say there is a possibility that part j moves from machine $i(k)$ to machine $k(i)$. Whereas, $x_{ik}^{j}=1$ only under the condition that machine $k(i)$ is the immediate successor of machine $i(k)$.

The operation sequences of parts can be classified into two types: the part visits a machine only one time or several times in its' process routing. We discuss the operation sequence ratio OSR_{ik} in both types.

(a) parts visit a machine only one time

In this case, the operation sequence ratio is calculated as follows:

$$X_{ik} = \sum_{j=1}^{P} x_{ik}^{j} \qquad D_{ik} = a$$

where

$$x_{ik}^{j} = \begin{cases} 1 & \text{if part } j \text{ is used by both machines } i, k \text{ and } k(i) \text{ is the immediate successor of } i(k); \\ 0 & \text{otherwise.} \end{cases}$$

The Jaccard similarity coefficient is modified as the following equation.

$$S_{ik}' = S_{ik} * OSR_{ik} = \frac{a}{a+b+c} * \frac{X_{ik}}{a} = \frac{X_{ik}}{a+b+c} \tag{4}$$

(b) parts visit a machine several times

Since the part can visit a machine several times, the definition of the possible produced intermachine move ments d_{ik}^{j} becomes complicated. The relevant parameter for defining d_{ik}^{j} is the number of times part j visits each machine. We use this parameter to establish d_{ik}^{j} as follows.

n_i^j the number of times part j visits machine i.

$n^j = Min(n_i^j,\ n_k^j)$.

$$e_i^j = \begin{cases} 2 & \text{if both the first and last operations of part } j \text{ are performed on machine } i; \\ 1 & \text{else if either first or last operation of part } j \text{ is performed on machine } i; \\ 0 & \text{otherwise.} \end{cases}$$

$i(k)$ indicates either machine i or k, and is determined as table 1.

$$e_{i(k)}^{j} = \begin{cases} e_i^j & \text{if } i(k) = i; \\ e_k^j & \text{if } i(k) = k. \end{cases} \tag{1}$$

Finally, d_{ik}^{j} is formulated as table 2.

The proposed operation sequence ratio modifies existing similarity coefficients shown in equation (2). Hence, the modified similarity coefficients have flexibility to solve the problems with operation sequences.

Table 1. The determination of machine $i(k)$.

	$n_i^j < n_k^j$	$n_k^j < n_i^j$	$n_k^j = n_i^j$		
			$e_i^j=2$	$e_k^j=2$	Otherwise
$i(k)$	i	k	i	k	either i or k

Table 2. The formulation of possible produced inter-machine movements d_{ik}^{j}.

	$n_i^j \neq n_k^j$			$n_i^j = n_k^j$		
	$e_{i(k)}^j=2$	$e_{i(k)}^j=1$	$e_{i(k)}^j=0$	$e_{i(k)}^j=2$	$e_{i(k)}^j=1$	$e_{i(k)}^j=0$
d_{ik}^j	$2n^j-2$	$2n^j-1$	$2n^j$	$2n^j-2$	$2n^j-1$	$2n^j-1$

2.2.3 Computing example

We use a computing example to illustrate the definition of the proposed operation sequence ratio. Assume there are six parts and their process routings are as follows.

part 1 (p1) : *m* 4, *m* 2, *m* 1, *m* 2, *m* 1,
part 2 (p2) : *m* 1, *m* 3, *m* 4, *m* 2, *m* 5,
part 3 (p3) : *m* 1, *m* 2, *m* 1, *m* 2, *m* 1,
part 4 (p4) : *m* 1, *m* 2, *m* 1, *m* 3, *m* 1, *m* 4, *m* 2,
part 5 (p5) : *m* 4, *m* 1, *m* 4, *m* 5, *m* 4,
part 6 (p6) : *m* 4, *m* 1, *m* 2, *m* 5, *m* 4,

where $mi\,(i = 1, 2, ..., 5)$ represents machine i

We calculate the similarity between machines 1 and 2. From above operational data, we construct machine-part matrix as in the following Figure 1. The elements in the matrix represent the operation sequences of parts.

Machine/Part

	p1	p2	p3	p4	p5	p6
m 1	3, 5	1	1, 3, 5	1, 3, 5	2	2
m 2	2, 4	4	2, 4	2, 7		3

Figure 1. A computing example

Part 1 (p1) includes both machines 1 (m1) and 2 (m2) twice. The last operation is processed on machine 1. Hence, the coefficient with part 1 is computed as follows:

$n^1 = n_1^1 = n_2^1 = 2$; $e_1^1 = 1$, $e_2^1 = 0$; $i(k) =$ either 1 or 2.
Finally, $d_{12}^1 = 2n^1 - 1 = 3$ and $x_{12}^1 = 3$.

Similarly, for part 3 (p3) and part 4 (p4), the coefficients are computed as follows:

$n^3 = Min(n_1^3, n_2^3) = n_2^3 = 2$; $e_1^3 = 2$, $e_2^3 = 0$; $i(k) = 2$.
Finally, $d_{12}^3 = 2n^3 = 4$ and $x_{12}^3 = 4$.

$n^4 = Min(n_1^4, n_2^4) = n_2^4 = 2$; $e_1^4 = 1$, $e_2^4 = 1$; $i(k) = 2$.
Finally, $d_{12}^4 = 2n^4 - 1 = 3$ and $x_{12}^4 = 2$.

The other results are given as follows:

$d_{12}^2 = 1$, $x_{12}^2 = 0$; $d_{12}^5 = 0$, $x_{12}^5 = 0$; $d_{12}^6 = 1$, $x_{12}^6 = 1$.

Hence,

$$X_{12} = \sum_{j=1}^{6} x_{12}^j = 10, \quad D_{12} = \sum_{j=1}^{6} d_{12}^j = 12$$

and

$$OSR_{ik} = 10/12$$

Since the Jaccard similarity coefficient between machines 1 and 2 (S_{12})=5/6, the modified Jaccard similarity coefficient is as follows:

$$S_{12}' = S_{12} * OSR_{12} = (5/6) * (10/12) = 25/36$$

2.3 A comparison

To illustrate the superiority of the modified similarity coefficient, we compare the performances between the modified Jaccard and original Jaccard similarity coefficient. Since average linkage clustering (ALC) algorithm is the most robust algorithm regardless of similarity coefficients (Tarsuslugil and Bloor, 1979; Seifoddini, 1989; Vakharia and Wemmerlov, 1995), we use ALC to obtain machine cells. The initial input data is shown in Figure 2. The entries in the figure represent the numbers of parts and machines. The machines are arranged in an operational order.

Part number	machine number
p1	4, 2
p2	4, 2
p3	1, 2, 4
p4	5, 1, 7, 1, 3
p5	6, 7, 3, 5
p6	3, 6, 7, 5
p7	6, 3, 7, 5
p8	3, 6, 1, 5, 1
p9	1, 4, 2

Figure 2. Part-machine operational sequence data

The obtained machine groups (MG) and part families (PF) by using original Jaccard similarity coefficient are given as follows:

MG-1 : *m* 4, *m* 2, *m* 1
MG-2 : *m* 3, *m* 6, *m* 7, *m* 5

PF-1 : *p*9, *p*3, *p*2, *p*1
PF-2 : *p*8, *p*7, *p*5, *p*6, *p*4

The solution matrix is shown in Figure 3.

Machine/ Part

	9	3	2	1	8	7	5	6	4
m4	2	3	1	1					
m2	3	2	2	2					
m1	1	1			3,5				2,4
m3					1	2	3	1	5
m6					2	1	1	2	
m7						3	2	3	3
m5					4	4	4	4	1

Figure 3. Solution matrix by using Jaccard similarity coefficient

The obtained machine groups (MG) and part families (PF) by using modified Jaccard similarity coefficient are given as follows:

MG-1 : $m\,4$, $m\,2$
MG-2 : $m\,1$, $m\,3$, $m\,6$, $m\,7$, $m\,5$

PF-1 : $p9$, $p3$, $p2$, $p1$
PF-2 : $p8$, $p7$, $p5$, $p6$, $p4$

The solution matrix is shown in Figure 4.

Machine/ Part

	9	3	2	1	8	7	5	6	4
m4	2	3	1	1					
m2	3	2	2	2					
m1	1	1			3,5				2,4
m3					1	2	3	1	5
m6					2	1	1	2	
m7						3	2	3	3
m5					4	4	4	4	1

Figure 4. Solution matrix by using modified Jaccard similarity coefficient

Figures 3 yields 7 intercell movements by parts 4 and 8. However, Figure 4 yields only 2 intercell movements by parts 3 and 9. The modified Jaccard similarity coefficient illustrates its superiority over the original coefficient.

3. SOLUTION PROCEDURE

There are two kinds of solution procedures in the literature for solving cell formation problems by using similarity coefficients. The first one is mathematical programming procedures such as the p-median model (Kusiak, 1987). The second one is heuristic algorithms, which seek sub-optimal solutions. Single linkage clustering (SLC), complete linkage clustering (CLC), and average linkage clustering (ALC) are well-known SCM based heuristic algorithms. In this paper, we present a two-stage heuristic algorithm, which utilizes the proposed operation sequence ratio. The assumptions for the heuristic are (a) each machine is assigned to one and only one machine-cell, (b) the desired number of cells is predefined, and (c) the cell size is also predefined to prevent production of large cells. The details of the heuristic are introduced as follows.

3.1 Stage 1

The objective of stage 1 is to obtain basic machine cells. At first, compute operation sequence ratios between machine pairs, and construct a similarity matrix according to equation (2). Then, group two machines into a machine cell, and revise the similarity matrix. A cell size constraint is checked in the procedure and the procedure continues until the predefined number of machine cells has been obtained.

An average similarity coefficient is used for revising the similarity matrix. The coefficient is defined to evaluate the similarity between two machine cells f and g, and it is described as follows:

$$\overline{S}_{fg} = \frac{\sum_{i \in f} \sum_{k \in g} S_{ik}'}{NM_f NM_g} \tag{5}$$

where NM_f and NM_g are the numbers of machines in machine cells f, and g, respectively.

The general procedure of the proposed heuristic algorithm is presented as follows:

Step 1. Compute operation sequence ratios between machine pairs.
Step 2. Produce the similarity matrix S_{ik}'.
Step 3. Join two machines that have the highest value into a new machine cell.
Step 4. Revise the similarity coefficients between the new machine cell and the rest of machines (machine cells) in the similarity matrix by equation (5).
Step 5. Find two machines (machine cells) that have the highest value in the similarity matrix.
Step 6. Check the cell size constraint.
if (the constraint is satisfied)
{ join two machines (machine groups) into a new machine group.
go to step 7. }
else
{ remove two highest value machines from the similarity matrix.
go back to step 5. }
Step 7. Check the constraint with the number of cells.
if (the predefined number of cells has been obtained)
stop.
else go back to step 4.

The flow chart of the first stage is shown in Figure 5. After finishing the first stage of the algorithm, we obtain the basic machine cells that satisfy the cell number constraint. To solve cell formation problems, we need to decide part families for each machine cell.

3.2 Stage 2

In a general cell formation problem, the concept of a

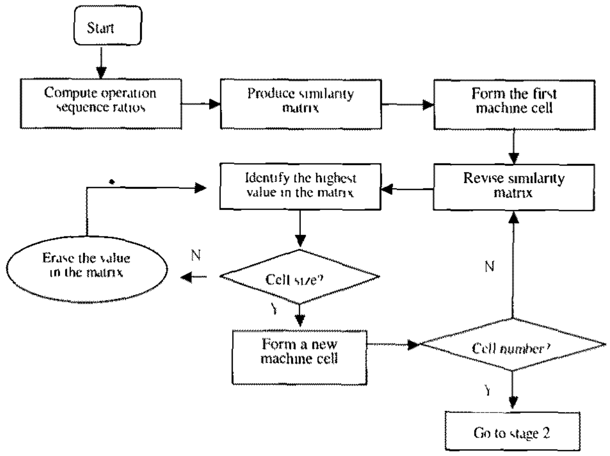

Figure 5. Flow chart of stage 1

bottleneck machine is defined as a machine that has to process parts assigned to other machine cells. In this stage, bottleneck machines are identified, a bottleneck machine set *BM* is formed, and the number of the entries in the set is defined as *N*. For each entry bm_{nc} in the set:

- n index of bottleneck machine ($n = 1, \ldots, N$)
- c index of cell ($c = 1, \ldots, C$)
- bm_{nc} the n^{th} bottleneck machine which was assigned to cell c in the stage 1.

We also define the total number of the intercell movements in the system as follows:

$$T = \sum_{j=1}^{P} T_j \qquad (6)$$

where

T_j the number of intercell movements of part j.

The details of stage 2 are presented as follows:

Initialize: set iteration number $I = 0$.

Step 1. Allocate part j to the cell in which produced intercell movements are minimum ($j = 1, \ldots, P$). Calculate the number of intercell movements ($T(I)$) in the system by equation (6).

Step 2. Identify bottleneck machines and form the bottleneck machine set *BM*.

Step 3. Create a new matrix $B(B_{nc'})$ ($n = 1, \ldots, N$; $c' = 1, \ldots, C$).

Initialize: set elements of the matrix to 0s;

set $n = 1$

loop 1{ (loop 1 begins here)

Initialize: set $c' = 1$

loop (loop 2 begins here)

move bm_{nc} from cell c to cell c' ($c \neq c'$)

calculate the number of intercell movements ($T'(I)$) by equation (6)

set $B_{nc'} = T(I) - T'(I)$

set $c' = c' + 1$

if ($c' \leq C$) return to the top of loop 2;

else exit loop 2. (loop 2 ends here)

}

set $n = n + 1$

if ($n \leq N$) return to the top of loop 1;

else exit loop 1. (loop 1 ends here)

}

Step 4. Find the element that bears the highest value $B^h_{nc'}$ in the matrix B.

if ($B^h_{nc'} > 0$)

{ check the cell size constraint

if (the constraint is satisfied)

{ reassign bottleneck machine n to cell c', $I = I + 1$

go back to step 1. }

else

{ set $B^h_{nc'} = 0$ in the matrix B,

go back to the top of step 4. } }

else stop.

The flow chart of the above procedure is shown in Figure 6.

4. SELECTED SIMILARITY COEFFICIENTS AND EXPERIMENTATION

Some commonly known similarity coefficients are selected from literature to illustrate the usage of the proposed operation sequence ratio. Three examples are tested by the proposed algorithm. The algorithm has been coded in C++ and implemented on a Pentium II based PC.

4.1 Similarity coefficients

Table 3 presents definitions and ranges for the selected similarity coefficients in this study.

All the similarity coefficients presented in this study can be described by using the variables a, b, c, d, Where a is the number of parts processed on both machines i and k; b is the number of parts processed only on machines i ; c is the number of parts processed only on machine k ; and d is the number of parts processed on neither machine i nor machine k.

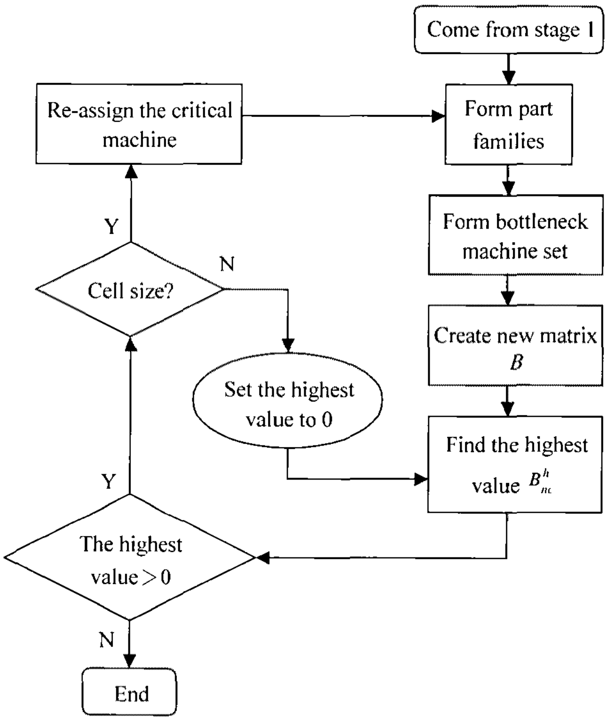

Figure 6. Flow chart of stage 2

4.2 Numerical examples

4.2.1 Example 1

Example 1 is presented to illustrate the procedure of the proposed algorithm. The initial machine-part matrix is given in Figure 7 with 5 machines and 11 parts. The entry in the matrix indicates the operation sequences of parts. In the matrix, parts 1 and 2 visit machines 1 and 4 twice, respectively.

We use Jaccard similarity coefficient as an example to solve this problem. From equation (1), Jaccard similarity coefficients between machine pairs are calculated and shown in Figure 8.

Machine/ Part

	1	2	3	4	5	6	7	8	9	10	11
m1	1,3				2	1	2	1		3	
m2		2		2	1			1			2
m3	2		1		3	3		2		2	
m4		1,3		1			1		2		1
m5			2			2		3		1	

Figure 7. Initial machine-part matrix of the example

Table 3. Definitions and ranges of the selected similarity coefficients

Coefficient	Definition S_{ik}	Range
1. Jaccard	$a/(a+b+c)$	0–1
2. Hamann	$[(a+d)-(b+c)]/[(a+d)+(b+c)]$	-1–1
3. Yule	$(ad-bc)/(ad+bc)$	-1–1
4. Simple matching	$(a+d)/(a+b+c+d)$	0–1
5. Sorenson	$2a/(2a+b+c)$	0–1
6. Rogers and Tanimoto	$(a+d)/[a+2(b+c)+d]$	0–1
7. Sokal and Sneath	$2(a+d)/[2(a+d)+b+c]$	0–1
8. Rusell and Rao	$a/(a+b+c+d)$	0–1
9. Baroni–Urbani and Buser	$[a+(ad)^{1/2}]/[a+b+c+(ad)^{1/2}]$	0–1
10. Phi	$(ad-bc)/[(a+b)(a+c)(b+d)(c+d)^{1/2}]$	-1–1
11. Ochiai	$a/[(a+b)(a+c)]^{1/2}$	0–1
12. Relative matching	$[a+(ad)^{1/2}]/[a+b+c+d+(ad)^{1/2}]$	0–1
13. Dot–product	$a/(b+c+2a)$	0–1
14. Kulczynski	$1/2[a/(a+b)+a/(a+c)]$	0–1
15. MaxSC	$max\ [a/(a+b), a/(a+c)]$	0–1
16. Sokal and Sneath 2	$a/[a+2(b+c)]$	0–1
17. Sokal and Sneath 4	$1/4[a/(a+b)+a/(a+c)+d/(b+d)+d/(c+d)]$	0–1

	m1	m2	m3	m4	m5
m1		0.10	0.71	0.10	0.43
m2			0.10	0.67	
m3					0.67
m4					

Figure 8. Jaccard similarity coefficients of the example

Operation sequence ratios and modified Jaccard similarity coefficients between machine pairs are also computed by equations (3) and (2), respectively. The results are given in Figures 9 and 10, respectively. Similarity coefficients in Figure 10 are used to form machine groups.

	m1	m2	m3	m4	m5
m1		1.00	0.83	1.00	0.33
m2				1.00	
m3					1.00
m4					

Figure 9. Operation sequence ratios

	m1	m2	m3	m4	m5
m1		0.10	0.59	0.10	0.14
m2				0.67	
m3					0.67
m4					

Figure 10. Modified Jaccard similarity coefficients

After finishing the first stage, we obtain basic machine cells that satisfy the predefined constraint. In this example, we set the number of cells $C=2$ and cell size $S=3$. The result of stage 1 is represented as follows:

MG-1: *m1*, *m3*, *m5*
MG-2: *m2*, *m4*

In stage 2 of the algorithm, we decide the part family corresponding each machine cell. After finishing step 1 of the stage 2, we obtain the basic part families as follows:

PF-1: *p1*, *p3*, *p5*, *p6*, *p7*, *p8*, *p10*
PF-2: *p2*, *p4*, *p9*, *p11*

The initial machine-part matrix is then changed into Figure 11, the intercell movements $T(I)$ are 2 generated by parts 5 and 7.

Step 2 is to identify bottleneck machines. Two bottleneck machines (m2, m4) are found in this problem. Step 3 and 4 check whether bottleneck machines could be moved to other machine cells in order to decrease the intercell movements. In this example, no more intercell

Machine/ Part

	1	3	5	6	7	8	10	2	4	9	11
m1	1,3		2	1	2	1	3				
m3	2	1	3	3		2	2				
m5		2		2		3	1				
m2			1					2	2	1	2
m4					1			1,3	1	2	1

Figure 11. Final machine-part incidence matrix

movements could be decreased and matrix 6 is the final solution of this example.

All other 16 similarity coefficients give the same machine groups and part families as the Jaccard's.

4.2.2 Example 2

The problem used by Selvam and Balasubramanian (1985) is considered in this example. The input data is given in Figure 12. The entries in the figure represent the numbers of parts and machines. The machines are arranged in an operational order.

For this example, we set the number of cells $C=2$ and cell size $S=3$. All 17 kinds of similarity coefficients give the same results as in Figure 13. Figure 13 yields 0 intercell movements. The final solution is the same as the one provided by Selvam and Balasubramanian (1985).

Part number	machine number
p1	1, 2, 3, 4, 5
p2	6, 7, 8
p3	8, 9, 10, 9
p4	1, 2, 3, 5
p5	6, 8, 9, 10

Figure 12. Part-machine operational sequence data of example 2.

Cell	Machines	Parts
1	1, 2, 3, 4, 5	1, 4
2	6, 7, 8, 9, 10	2, 3, 5

Figure 13. Solution of example 2.

4.2.3 Example 3

Example 3 is given by Wu and Salvendy (1993) with 15 machines and 22 part types. The initial data is shown in Figure 14.

We set the number of cells $C=4$ and cell size $S=5$ for example 3. All 17 kinds of similarity coefficients give the same results as in Figure 15. Figure 15 produced 8 intercell movements. The solution is the same as the one

Part number	machine number
p01	5,7,2,7
p02	4,5,7
p03	4,7
p04	2,3,10,11,
p05	14,8,10,
p06	2,3,11,
p07	8,9,
p08	1,13,
p09	1,15,12,13,
p10	15,1,15,4,
p11	15,12,13,12,
p12	6,9,8,14,
p13	6,8,
p14	14,6,13,
p15	6,8,9,
p16	6,9,11,
p17	9,14,
p18	3,11,
p19	11,3,10,
p20	4,5,13,
p21	1,13,3,
p22	10,11,10,

Figure 14. Part-machine operational sequence data of example 3.

Cell	Machines	Parts
1	1,13,12,15	8,9,10,11,21
2	2,3,11,10	4,6,18,19,22
3	4,5,7	1,2,3,20
4	6,8,9,14	5,7,12,13,14,15,16,17

Figure 15. Solution of example 3.

given by Wu and Salvendy (1993).

5. CONCLUSIONS

Operation sequences of parts is one of the most important manufacturing factors in the design of cellular manufacturing systems. In spite of the fact that a lot of researchers emphasize consideration of operation sequences based similarity coefficient in the cell formation procedure, none of them have presented the utilization of the traditional existing similarity coefficients.

In this paper, we introduce an operation sequence ratio which can be used to extend existing similarity coefficients to solve cell formation problems with operation sequences. 17 commonly known similarity coefficients are selected from literature to illustrate the usage of the proposed operation sequence ratio. A comparison and three numerical examples are provided to demonstrate the effectiveness of the proposed operation sequence ratio and the heuristic algorithm. The computational results show that the developed operation sequence ratio is robust in modifying existing similarity coefficients.

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