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A Combined Model of Trip Distribution, Mode Choice and Traffic Assignment

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교통분포, 수단선택 및 교통할당의 결합모형

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In this paper, we propose a parametric optimization approach to simultaneously determining trip distribution, mode choice, and user-equilibrium assignment. In our model, mode choice decisions are based on a binomial logit model and passenger and cargo demands are divided into appropriate mode according to the user equilibrium minimum travel time. Underlying network consists of road and rail networks combined and mode choice available is auto, bus, truck, passenger rail, and cargo rail. We provide an equivalent convex optimization problem formulation and efficient algorithm for solving this problem. The proposed algorithm was applied to a large scale network examples derived from the National Intermodal Transportation Plan (2000-2019).

Keywords: user-equilibrium traffic assignment, binomial logit model, transportation planning model, Frank-Wolf algorithm.

1. Introduction

Traffic analysis usually involves four phases of estimation. The four phases are trip generation, trip distribution, mode choice, and traffic assignment. Trip generation determines the trip productions at origins and trip attractions at destinations. Trip distribution next determines the split of total trip production from a origin to other destination nodes. Mode choice decision calculates the number of traveler between origin and destination cities employing specific mode. In most literature, passenger car and train are two alternatives for mode choice decision. The final phase is the traffic assignment, in that, the user-equilibrium link flows are estimated assuming the results from all the previous phase are given as input. Models and algorithms in the four phases can be found in Sheffi(1985).

Usually mode choice decisions are made based on some utility function depending on the minimum travel time between each OD pair. Mode choice actually fixes demand matrix for the user-equilibrium traffic assignment problem. Sequential application of mode choice and traffic assignment results in the discrepancies of minimum path cost when the mode choice is made and when the fixed demand traffic assignment problems is solved. The fixed demand traffic assignment problem is solved using a convex combination method of Frank and Wolf(1956). Software packages like EMME/2 also implement

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Frank-Wolf algorithm. Variational inequality formulation of traffic assignment problem is found in Dafermos(1980).

To reduce such discrepancies, some of the four phases are combined in a simultaneous model. The papers by Safwat and Magnanti (1988), Abrahamsson and Lundqvist (1999), Florian *et al.* (2000), and Maruyama *et al.* (1999) show combined models in which binary mode choice decision is modeled. In this research we considered the situation where multiple mode choices are available to passengers and cargo shipments. We extend the binary mode choice decision to a multiple mode choice for two different demand classes, passengers and cargo.

The model developed in this paper is inspired during a research project funded by Korea Research Institute for Human Settlements (KRIHS). We developed a bilevel programming model for selecting minimum social cost investment project that builds additional road and rail network by 2020 in Korea. Previous attempt to estimate social cost which include traffic, accidents, operating, environments, and maintenance of road and rail system applied a sequential approach in that a fixed demand traffic assignment problem is solved based on the input from mode choice. This approach shows substantial discrepancies of social cost evaluated at mode choice calculation and fixed demand traffic assignment.

The remainder of the paper is organized as follows. In the next section, we provide notations, constraints, user-equilibrium conditions, and equivalent optimization problem formulation of the combined model. The first-order necessary optimality condition is derived. We also prove that the proposed model satisfies user-equilibrium and binary logit function constraints. Section 3 presents a variant of convex combination method in that shortest path and transportation problems are solved to obtain a descent direction. Simplified forms of transportation subproblems are introduced. A summary of the entire algorithm including step length calculation and convergence test are given. Section 4 describes computational results for four cases derived from the National Intermodal Transportation Plan(2000-2019). Some conclusions are presented in the last section.

2. Model Formulation

In this paper, we formulate a practical model for simultaneously determining trip distributions, mode split and traffic assignment based on exogenously given passenger and cargo trip generations. We denote $O_i(D_j)$ as the trip amount generated from(to) origin i(destination j) and $\overline{O}_i(\overline{D}_j)$ as the cargo demand of origin i(destination j). Underlying network consists of road and rail links. Each directed link a is connecting two nodes (k, l) and node k is the tail node and l is the head node. l_a denotes the length of link a. There are two kinds of nodes considered. One is centroid node which has origin or destination trip requirement and the other is transshipment node which does not have any demand. Modes available for passengers traveling on this network are auto(au), bus(bus), and rail(pr), and for cargo shipments, truck(tr) and rail(cr). These five modes are grouped into sets $M_p=\{au, bus, pr\}$ for modes for passengers, $M_c=\{tr, cr\}$ for cargo, $M_r=\{au, bus, tr\}$ for modes using road network, and $M_t=\{pr, cr\}$ for using rail network. Set $M=M_p\cup M_c=M_r\cup M_t$ denotes all five modes.

The mode selection behavior of the passenger (Lee, 1998) is described as a binomial logit model and the probability of choosing mode m among M_p is given as

$$P_{ij}^m=\frac{e^{-\theta_1 u_{ij}^m+\xi_{ij}^m}}{e^{-\theta_1 u_{ij}^{au}+\xi_{ij}^{au}}+e^{-\theta_1 u_{ij}^{bus}+\xi_{ij}^{bus}}+e^{-\theta_1 \overline{c}_{ij}^{rail}+\xi_{ij}^{pr}}} \quad (1)$$

Similarly, the probability of choosing a cargo mode from M_c is given as

$$P_{ij}^m=\frac{e^{-\theta_2 u_{ij}^m+\xi_{ij}^m}}{e^{-\theta_2 u_{ij}^{tr}+\xi_{ij}^{tr}}+e^{-\theta_2 \overline{c}_{ij}^{rail}+\xi_{ij}^{cr}}} \quad (2)$$

Here, u_{ij}^m denotes the equilibrium travel time between OD pair (i, j) employing mode m on road network. $\xi_{ij}^m=\alpha^m d_{ij}+\beta^m$ is exogenously given parameter for OD pair (i, j) and represents cost proportional to the distance d_{ij} between OD pair (i, j). Here, parameter α_m models cost of traveling a unit distance by mode m and β_m is a mode-specific parameter. The parameters θ_1 and θ_2 are positive constants and make the utility of traveling from i to j being a decreasing function of travel time u_{ij}. For modes pr and cr on the rail network, $\overline{c}_{ij}^{rail}$ is used instead of u_{ij}^m to represent minimum travel time between OD pair i and j. Thus, $e^{-\theta_1 u_{ij}^m+\xi_{ij}^m}$ as a whole represents utility of traveling from i to j by mode m consisting of costs proportional to the minimum travel time and the minimum distance between i and j.

If the number of travelers from i to j using mode m is g_{ij}^m, then by (1) and (2), $P_{ij}^m=g_{ij}^m/\sum_{m\in M_p} g_{ij}^m$ for

$m \in M_p$ and $P^m_{ij} = g^m_{ij} / \sum_{m \in M_c} g^m_{ij}$ for $m \in M_c$. Note that at equilibrium, $u^{au}_{ij} = u^{bus}_{ij} = u^{tr}_{ij}$ because these three modes are using road network and experience same delay.

On the road link a, link volume x_a is given as

$$x_a = \sum_{ijk} \delta_{ijka} (f^{au}_{ijk} + f^{bus}_{ijk} + f^{tr}_{ijk})$$

Here δ_{ijka} denotes an incidence relation that if arc a is in the path k connecting OD pair (i, j), $\delta_{ijka} = 1$, and otherwise $\delta_{ijka} = 0$. f^m_{ijk} denotes the amount of travel using path k connecting OD pair (i, j) by mode m. In each link, $t_a(\cdot)$ denotes a volume-delay function which has a form of monotonic increasing function of volume x_a. A popular volume-delay function is standard U.S. Bureau of Public Roads (BPR) function

$$t_a(x_a) = t^0_a \left[1 + \alpha \left(\frac{x_a}{c_a} \right)^\beta \right]$$

where t^0_a, α, β are parameters and c_a denotes the capacity of link a. On the rail network, the volume-delay function is represented by a simpler model where travel time crossing any link is link length l_a divided by average speed v_{train} of a train. Thus, on rail network, the travel time on path k between i and j is given as $\bar{c}^{rail}_{ijk} = \sum_a \delta_{ijka} l_a / v_{train}$, and the minimum travel time between i and j is $\bar{c}^{rail}_{ij} = \min_k \{ \bar{c}^{rail}_{ijk} \}$.

Using the above definitions, Wardrop's equilibrium condition on the road network is formulated as follows.

$$\text{If } f^m_{ijk} > 0, \ \bar{c}_{ijk} = u^m_{ij} \quad \forall i, j, m in M_r \qquad (3a)$$

$$\text{If } f^m_{ijk} = 0, \ \bar{c}_{ijk} \geq u^m_{ij} \quad \forall i, j, m \in M_r \qquad (3b)$$

where $\bar{c}_{ijk} = \sum_a \delta_{ijka} t_a(x_a)$ denotes the travel time of path k connecting i and j on road network. This condition says that at user equilibrium, if the flow on path k connecting i and j is positive, the travel time on path k is same as the minimum travel time between i and j. This implies that any path change by a single user cannot decrease the user's travel time.

Trip distributions satisfy the following constraints regarding the total generated trips from origin i and the total trips ending at destination j.

$$\sum_j \sum_{m \in M_p} g^m_{ij} = O_i \quad \forall i \qquad (4a)$$

$$\sum_i \sum_{m \in M_p} g^m_{ij} = D_j \quad \forall j \qquad (4b)$$

$$\sum_j \sum_{m \in M_c} g^m_{ij} = \bar{O}_i \quad \forall i \qquad (4c)$$

$$\sum_i \sum_{m \in M_c} g^m_{ij} = \bar{D}_j \quad \forall j \qquad (4d)$$

Equivalent minimization problem determining the trip distribution, mode choice, and traffic assignment are as follows. The variables inside parentheses on the right of the equations denote Langrangian dual variables.

Minimize

$$\begin{aligned} z(\boldsymbol{x}, \boldsymbol{g}) = & \sum_a \int_0^{x_a} t_a(\omega)\, d\omega + \sum_{ij} (g^{br}_{ij} + g^{cr}_{ij}) \bar{c}^{rail}_{ij} \\ & + \sum_{ij} \sum_{m \in M_p} \frac{1}{\theta_1} g^m_{ij} (\ln g^m_{ij} - \xi^m_{ij}) \qquad (5a) \\ & + \sum_{ij} \sum_{m \in M_c} \frac{1}{\theta_2} g^m_{ij} (\ln g^m_{ij} - \xi^m_{ij}) \end{aligned}$$

$$\text{s.t.} \quad g^m_{ij} = \sum_k f^m_{ijk} \quad \forall i, j, m \in M_r \quad (u^m_{ij}) \qquad (5b)$$

$$x_a = \sum_{ijk} \delta_{ijka} (f^{au}_{ijk} + f^{bus}_{ijk} + f^{tr}_{ijk}) \quad \forall a \qquad (5c)$$

$$\sum_j \sum_{m \in M_p} g^m_{ij} = O_i \quad \forall i \quad (\alpha_i) \qquad (5d)$$

$$\sum_i \sum_{m \in M_p} g^m_{ij} = D_j \quad \forall j \quad (\beta_j) \qquad (5d)$$

$$\sum_j \sum_{m \in M_c} g^m_{ij} = \bar{O}_i \quad \forall i \quad (\bar{\alpha}_i) \qquad (5f)$$

$$\sum_i \sum_{m \in M_c} g^m_{ij} = \bar{D}_j \quad \forall j \quad (\bar{\beta}_j) \qquad (5g)$$

$$g^m_{ij} \geq 0, \quad \forall i, j, m \in M \qquad (5h)$$

$$f^m_{ijk} \geq 0, \quad \forall i, j, k, m INM. \qquad (5i)$$

Using Lagrangian variables, Lagrangian L is defined as follows.

$$\begin{aligned} L = & \sum_a \int_0^{x_a} t_a(\omega)\, d\omega + \sum_{ij} (g^{br}_{ij} + g^{cr}_{ij}) \bar{c}^{rail}_{ij} \\ & + \sum_{ij} \sum_{m \in M_p} \frac{1}{\theta_1} g^m_{ij} (\ln g^m_{ij} - \xi^m_{ij}) \\ & + \sum_{ij} \sum_{m \in M_c} \frac{1}{\theta_2} g^m_{ij} (\ln g^m_{ij} - \xi^m_{ij}) \\ & + \sum_{ij} \sum_{m \in M_r} u^m_{ij} (g^m_{ij} - \sum_k f^m_{ijk}) \qquad (6) \\ & + \sum_i \alpha_i \left[\sum_j \sum_{m \in M_p} g^m_{ij} - O_i \right] \\ & + \sum_j \beta_j \left[\sum_i \sum_{m \in M_p} g^m_{ij} - D_j \right] \\ & + \sum_i \bar{\alpha}_i \left[\sum_j \sum_{m \in M_c} g^m_{ij} - \bar{O}_i \right] \\ & + \sum_j \bar{\beta}_j \left[\sum_i \sum_{m \in M_c} g^m_{ij} - \bar{D}_j \right]. \end{aligned}$$

The first order necessary optimality condition is minimizing Lagrangian L under the nonnegativity constraints

(5h) and (5i), and summarized as follows.

$$\frac{\partial L}{\partial f_{ijk}^m}=\sum_a t_a(x_a)\delta_{ijka}-u_{ij}^m\geq 0 \quad \forall i,j,k,m\in M_r \quad (7a)$$

$$f_{ijk}^m\frac{\partial L}{\partial f_{ijk}^m}=0 \quad \forall i,j,k,m\in M_r \quad (7b)$$

$$\frac{\partial L}{\partial g_{ij}^{au}}=\frac{1}{\theta_1}(\ln g_{ij}^{au}+1-\xi_{ij}^{au})+u_{ij}^{au}+\alpha_i+\beta_j\geq 0 \quad (7c)$$

$$\frac{\partial L}{\partial g_{ij}^{bus}}=\frac{1}{\theta_1}(\ln g_{ij}^{bus}+1-\xi_{ij}^{bus})+u_{ij}^{bus}+\alpha_i+\beta_j\geq 0 \quad (7d)$$

$$\frac{\partial L}{\partial g_{ij}^{pr}}=\frac{1}{\theta_1}(\ln g_{ij}^{pr}+1-\xi_{ij}^{pr})+\overline{c}_{ij}^{rail}+\alpha_i+\beta_j\geq 0 \quad (7e)$$

$$\frac{\partial L}{\partial g_{ij}^{tr}}=\frac{1}{\theta_2}(\ln g_{ij}^{tr}+1-\xi_{ij}^{tr})+u_{ij}^{tr}+\overline{\alpha}_i+\overline{\beta}_j\geq 0 \quad (7f)$$

$$\frac{\partial L}{\partial g_{ij}^{cr}}=\frac{1}{\theta_2}(\ln g_{ij}^{cr}+1-\xi_{ij}^{cr})+\overline{c}_{ij}^{rail}+\overline{\alpha}_i+\overline{\beta}_j\geq 0 \quad (7g)$$

$$g_{ij}^m\frac{\partial L}{\partial g_{ij}^m}=0, \quad \forall i,j,m\in M \quad (7h)$$

$$\frac{\partial L}{\partial \alpha_i}=0,\ \frac{\partial L}{\partial \beta_j}=0,\ \frac{\partial L}{\partial \overline{\alpha}_i}=0,\ \frac{\partial L}{\partial \overline{\beta}_j}=0. \quad (7i)$$

For $g_{ij}^m>0$, by constraint (7h), $\frac{\partial L}{\partial g_{ij}^m}=0$. In this case, we can solve (7c)-(7g) for g_{ij}^{au}, g_{ij}^{bus}, g_{ij}^{pr}, g_{ij}^{tr}, and g_{ij}^{cr}. We have

$$g_{ij}^{au}=\exp\left[-1+\xi_{ij}^{au}-\theta_1(u_{ij}^{au}+\alpha_i+\beta_j)\right] \quad (8a)$$

$$g_{ij}^{bus}=\exp\left[-1+\xi_{ij}^{bus}-\theta_1(u_{ij}^{bus}+\alpha_i+\beta_j)\right] \quad (8b)$$

$$g_{ij}^{pr}=\exp\left[-1+\xi_{ij}^{pr}-\theta_1(\overline{c}_{ij}^{rail}+\alpha_i+\beta_j)\right] \quad (8c)$$

$$g_{ij}^{tr}=\exp\left[-1+\xi_{ij}^{tr}-\theta_2(u_{ij}^{tr}+\overline{\alpha}_i+\overline{\beta}_j)\right] \quad (8d)$$

$$g_{ij}^{cr}=\exp\left[-1+\xi_{ij}^{cr}-\theta_2(\overline{c}_{ij}^{rail}+\overline{\alpha}_i+\overline{\beta}_j)\right]. \quad (8e)$$

Let $A_i=\frac{e^{-1-\theta_1\alpha_i}}{O_i}$ and $B_j=\frac{e^{-\theta_1\beta_j}}{D_j}$. Then, we have

$$g_{ij}^{au}=A_iB_jO_iD_j\ e^{-\theta_1u_{ij}^{au}+\xi_{ij}^{au}}$$

and similarly,

$$g_{ij}^{bus}=A_iB_jO_iD_j\ e^{-\theta_1u_{ij}^{bus}+\xi_{ij}^{bus}}$$

$$g_{ij}^{pr}=A_iB_jO_iD_j\ e^{-\theta_1\overline{c}_{ij}^{rail}+\xi_{ij}^{pr}}.$$

Also, let $\overline{A}_i=\frac{e^{-1-\theta_2\overline{\alpha}_i}}{\overline{O}_i}$ and $\overline{B}_j=\frac{e^{-\theta_2\overline{\beta}_j}}{\overline{D}_j}$. Then, we have

$$g_{ij}^{tr}=\overline{A}_i\ \overline{B}_j\ \overline{O}_i\ \overline{D}_je^{-\theta_2u_{ij}^{tr}+\xi_{ij}^{tr}}$$

and

$$g_{ij}^{cr}=\overline{A}_i\ \overline{B}_j\ \overline{O}_i\ \overline{D}_je^{-\theta_2\overline{c}_{ij}^{rail}+\xi_{ij}^{cr}}.$$

To see that the solution satisfying the conditions (7a)-(7i) actually satisfies binomial logit function,

$$\frac{g_{ij}^{au}}{g_{ij}^{au}+g_{ij}^{bus}+g_{ij}^{pr}}=\frac{A_iB_jO_iD_je^{-\theta_1u_{ij}^{au}+\xi_{ij}^{au}}}{A_iB_jO_iD_j(e^{-\theta_1u_{ij}^{au}+\xi_{ij}^{au}}+e^{-\theta_1u_{ij}^{bus}+\xi_{ij}^{bus}}+e^{-\theta_1\overline{c}_{ij}^{rail}+\xi_{ij}^{pr}})}$$

$$=\frac{e^{-\theta_1u_{ij}^{au}+\xi_{ij}^{au}}}{(e^{-\theta_1u_{ij}^{au}+\xi_{ij}^{au}}+e^{-\theta_1u_{ij}^{bus}+\xi_{ij}^{bus}}+e^{-\theta_1\overline{c}_{ij}^{rail}+\xi_{ij}^{pr}})}=P_{ij}^{au}$$

Similarly for P_{ij}^m, $m\in M-\{au\}$. This proves that optimal solution deriving from conditions (7c)-(7h) satisfy binomial logit functions (1) and (2). Also, to see that the optimal solution satisfy Wardrop's user equilibrium condition, we see that from (7a) and (7b), whenever $f_{ij}^m>0$, $\sum_a t_a(x_a)\delta_{ijka}=\overline{c}_{ijk}=u_{ij}^m$. Hence, the optimal link flows satisfy the user-equilibrium conditions (3a)-(3b). Therefore we proved

Theorem 1. The solution of the first order necessary condition (7a)-(7i) satisfy Wardrop's user equlibrium conditions (3a)-(3b) and binomial logit function (1) and (2).

3. Solution Algorithm

During a linearization algorithm based on Frank and Wolf(1963), we first need to find a descent direction. By linearizing the objective function (5a), at a feasible solution (f_{ijk}^m, g_{ij}^m) we have the following linear programming problem in terms of auxiliary variables (y_{ijk}^m, z_{ij}^m).

Minimize

$$\sum_{ijk}\overline{c}_{ijk}(y_{ijk}^{au}+y_{ijk}^{bus}+y_{ijk}^{tr})+\sum_{ij}\sum_m \overline{d}_{ij}{}^m z_{ij}^m \quad (9a)$$

s.t

$$\sum_j\sum_{m\in M_p}z_{ij}^m=O_i \quad \forall i \quad (9a)$$

$$\sum_i\sum_{m\in M_p}z_{ij}^m=D_j \quad \forall j \quad (9b)$$

$$\sum_j\sum_{m\in M_c}z_{ij}^m=\overline{O}_i \quad \forall i \quad (9c)$$

$$\sum_i \sum_{m \in M_c} z_{ij}^m = \overline{D}_j \quad \forall j \tag{9d}$$

$$-z_{ij}^{au} + \sum_k y_{ijk}^{au} = 0 \quad \forall i,j \tag{9e}$$

$$-z_{ij}^{bus} + \sum_k y_{ijk}^{bus} = 0 \quad \forall i,j \tag{9f}$$

$$-z_{ij}^{tr} + \sum_k y_{ijk}^{tr} = 0 \quad \forall i,j \tag{9g}$$

$$z_{ij}^m \geq 0, \quad \forall i,j, m \in M \tag{9h}$$

$$y_{ijk}^m \geq 0, \quad \forall i,j,k, m \in M. \tag{9i}$$

Where current link flow $\overline{x}_a = \sum_i \sum_j \sum_k \delta_{ijka} \sum_{m \in M_r} f_{ijk}^m$ and $\overline{c}_{ijk} = \sum_a t_a(\overline{x}_a)\delta_{ijka}$ denotes the length of the path k connecting OD pair (i,j) at the current iteration. $\overline{d}_{ij}^m$, $m \in M$ is defined as follows.

$$\overline{d}_{ij}^{au} = \frac{1}{\theta_1}(\ln g_{ij}^{au} + 1) - \frac{1}{\theta_1}\xi_{ij}^{au}$$

$$\overline{d}_{ij}^{bus} = \frac{1}{\theta_1}(\ln g_{ij}^{bus} + 1) - \frac{1}{\theta_1}\xi_{ij}^{bus}$$

$$\overline{d}_{ij}^{pr} = \overline{c}_{ij}^{rail} + \frac{1}{\theta_1}(\ln g_{ij}^{pr} + 1) - \frac{1}{\theta_1}\xi_{ij}^{pr}$$

$$\overline{d}_{ij}^{tr} = \frac{1}{\theta_2}(\ln g_{ij}^{tr} + 1) - \frac{1}{\theta_2}\xi_{ij}^{tr}$$

$$\overline{d}_{ij}^{cr} = \overline{c}_{ij}^{rail} + \frac{1}{\theta_2}(\ln g_{ij}^{cr} + 1) - \frac{1}{\theta_2}\xi_{ij}^{cr}.$$

Note that above problem is two separate problems which can be separable with respect to passenger demand and cargo demand. The first problem is stated as

Minimize

$$\sum_{ijk} \overline{c}_{ijk}(y_{ijk}^{au} + y_{ijk}^{bus}) + \sum_{ij} \sum_{m \in M_p} \overline{d}_{ij}^m z_{ij}^m \tag{10a}$$

$$\text{s.t.} \quad \sum_j \sum_{m \in M_p} z_{ij}^m = O_i \quad \forall i \tag{10b}$$

$$\sum_i \sum_{m \in M_p} z_{ij}^m = D_j \quad \forall j \tag{10c}$$

$$-z_{ij}^m + \sum_k y_{ijk}^m = 0 \quad \forall i,j, m \in \{au, bus\} \tag{10d}$$

$$z_{ij}^m \geq 0, \quad \forall i,j, m \in M_p \tag{10e}$$

$$y_{ijk}^m \geq 0, \quad \forall i,j,k, m \in M_p. \tag{10f}$$

This problem can be simplified as follows. Let $\overline{c}_{ijm} = \min_k\{\overline{c}_{ijk}\}$. Note that $\overline{c}_{ijm}$ can be found applying any shortest path algorithm to the network with link cost $t_a(\overline{x_a})$. Then, in the above LP problem, at optimality, if $\overline{c}_{ijk} > \overline{c}_{ijm}$, the corresponding $y_{ijk}^{au} = 0$ and $y_{ijk}^{bus} = 0$. And $y_{ijm}^{au} = \sum_k y_{ijk}^{au}$, $y_{ijm}^{bus} = \sum_k y_{ijk}^{bus}$. Also, note that by (9f)-(9g), $\sum_k y_{ijk}^{au} = z_{ij}^{au}$, $\sum_k y_{ijk}^{bus} = z_{ij}^{bus}$. Hence, denoting $\overline{c}_{ij} = \overline{c}_{ijm}$, $\forall i,j$ we can change

$$\sum_i \sum_j \sum_k \overline{c}_{ijk} y_{ijk}^{au} = \sum_i \sum_j \overline{c}_{ij} \sum_k y_{ijk}^{au} = \sum_i \sum_j \overline{c}_{ij} z_{ij}^{au}.$$

Thus, we can reformulate (10) as

Minimize

$$\sum_{ij} (\overline{c}_{ij} + \overline{d}_{ij}^{au}) z_{ij}^{au} + \sum_{ij} (\overline{c}_{ij} + \overline{d}_{ij}) z_{ij}^{bus} + \sum_{ij} \overline{d}_{ij}^{pr} z_{ij}^{pr} \tag{11a}$$

$$\text{s.t.} \quad \sum_j \sum_{m \in M_p} z_{ij}^m = O_i, \quad \forall i \tag{11b}$$

$$\sum_i \sum_{m \in M_p} z_{ij}^m = D_j, \quad \forall j \tag{11c}$$

$$z_{ij}^m \geq 0, \quad \forall i,j, m \in M_p. \tag{11d}$$

As in Florian and Nguyen(1978), we can delete terms from both objective and constraint functions. Define $\Omega_{ij}^{au} = 1$ if, cost coefficient of z_{ij}^{au} is the minimum among the cost coefficients of z_{ij}^{au}, z_{ij}^{bus}, z_{ij}^{pr}. Similarly for Ω_{ij}^{bus} and Ω_{ij}^{pr}, we have

Minimize

$$\sum_{ij} \Omega_{ij}^{au} (\overline{c}_{ij} + \overline{d}_{ij}^{au}) z_{ij}^{au} + \sum_{ij} \Omega_{ij}^{bus} (\overline{c_{ij}} + \overline{d}_{ij}^{bus}) z_{ij}^{bus} + \sum_{ij} \Omega_{ij}^{pr} \overline{d}_{ij}^{pr} z_{ij}^{pr} \tag{12a}$$

$$\text{s.t.} \quad (\sum_j \Omega_{ij}^{au} z_{ij}^{au} + \Omega_{ij}^{bus} z_{ij}^{bus} + \Omega_{ij}^{pr} z_{ij}^{pr}) = O_i, \quad \forall i \tag{12b}$$

$$\sum_i (\Omega_{ij}^{au} z_{ij}^{au} + \Omega_{ij}^{bus} z_{ij}^{bus} + \Omega_{ij}^{pr} z_{ij}^{pr}) = D_j, \quad \forall j \tag{12c}$$

$$z_{ij}^m \geq 0, \quad \forall i,j, m \in M_p. \tag{12d}$$

Above transportation problem has the property that for each (i,j), there is only one terms both in objective and constraints functions. Hence, the usual application of transportation problem algorithm solves (12) efficiently and the second problem is similarly formulated as the following transportation problem.

Minimize

$$\sum_{ij} \Omega_{ij}^{tr} (\overline{c}_{ij} + \overline{d}_{ij}^{tr}) z_{ij}^{tr} + \sum_{ij} \Omega_{ij}^{cr} \overline{d}_{ij}^{cr} z_{ij}^{cr} \tag{13a}$$

$$\text{s.t.} \quad \sum_j (\Omega_{ij}^{tr} z_{ij}^{tr} + \Omega_{ij}^{cr} z_{ij}^{cr}) = \overline{O}_i, \quad \forall i \tag{13b}$$

$$\sum_i (\Omega_{ij}^{tr} z_{ij}^{tr} + \Omega_{ij}^{cr} z_{ij}^{cr}) = \overline{D}_j, \quad \forall j \tag{13c}$$

$$z_{ij}^m \geq 0, \quad \forall i,j, m \in M_c. \tag{13d}$$

New values of link flows $y_a^n = \sum_i \sum_j \sum_k \delta_{ijka} \sum_{m \in M_r} y_{ijk}^m$ are obtained by assigning z_{ij}^m demands onto the shortest path found in the beginning of iteration.

After determine the descent direction, the step length λ_n could be found by applying any line search algorithm with the constraint $0 \le \lambda_n \le 1$ with the minimizing function $z(\boldsymbol{x}+\lambda(\boldsymbol{y}-\boldsymbol{x}), \boldsymbol{g}+\lambda(\boldsymbol{z}-\boldsymbol{g}))$ which is (5a) evaluated at $(\boldsymbol{x}+\lambda(\boldsymbol{y}-\boldsymbol{x}), \boldsymbol{g}+\lambda(\boldsymbol{z}-\boldsymbol{g}))$. This function is a one-dimensional function of variable λ. Note that since the derivative evaluation of (5a) is easily done, bisection algorithm can be used as a line search algorithm.

Denoting x_a^n and $g_{ij}^{m,n}$ as the values of x_a and g_{ij}^m attained at iteration n, then the update $(x_a^{n+1}, g_{ij}^{m,n+1})$ in terms of the iteration n value $(x_a^n, g_{ij}^{m,n})$ and the descent direction $(y_a^n - x_a^n, z_{ij}^m - g_{ij}^{m,n})$ is summarized as

$$(\boldsymbol{x}^{n+1}, \boldsymbol{g}^{n+1}) = (\boldsymbol{x}^n + \lambda_n(\boldsymbol{y}^n - \boldsymbol{x}^n), \boldsymbol{g}^n + \lambda_n(\boldsymbol{z}^n - \boldsymbol{g}^n)).$$

Here $\boldsymbol{x}^n$ and $\boldsymbol{g}^n$ are the vector representation of (x_a^n) and $(g_{ij}^{m,n})$ at iteration n, respectively.

We summarize the proposed algorithm as follows.

Step 1 (Initialization). Begin with $f_{ijk}^m = 0$.

Step 2 (Convergence Test). If $\sum_a \frac{|x_a^{n+1} - x_a^n|}{x_a^n} + \sum_{ijm} \frac{|g_{ij}^{m,n+1} - g_{ij}^{m,n}|}{g_{ij}^{m,n}} < \varepsilon$, stop. Otherwise go to Step 3.

Step 3 (Descent Direction). Solve transportation problems (12) and (13).

Step 4 (Step Length). Solve a line search problem with the constraint $0 \le \lambda_n \le 1$.

Step 5 (Update). $(\boldsymbol{x}^{n+1}, \boldsymbol{g}^{n+1}) = (\boldsymbol{x}^n + \lambda_n(\boldsymbol{y}^n - \boldsymbol{x}^n), \boldsymbol{g}^n + \lambda_n(\boldsymbol{z}^n - \boldsymbol{g}^n))$. Go back to Step 2.

4. Computational Results

Above described algorithm was implemented on a PC using Microsoft Visual C++ 6.0 software. For the line search algorithm, we used bisection method since the derivative evaluation of (5a) is easy. The shortest path algorithm inside Frank-Wolf algorithm was modified SPLIB described in Cherkassky *et al.* (1996). An implementation using heap data structure can be obtained from http://www.avglab.com/andrew/soft.html. For the stopping criteria, we used $\varepsilon = 10^{-3}$ and the maximum number of iteration used was 300. All 300 iterations are finished within 3 minutes on a PC with Pentium IV 1.6 MHz processor.

For the test of the algorithm, we construct four imaginary networks derived from the National Intermodal Transportation Plan(2000-2019). The first case is the existing network at the end of The Third Comprehensive National Territorial Plan in 1999. This network consists of 1,859 nodes (132 centroid, 1,729 transshipment nodes) and 5,386 links (4,634 road links and 752 rail links). Additional cases are constructed by adding the road and rail links planned in NITP(2000-2019) to the network of case 1. Actual projects included in NITP(2000-2019) consist of 23 highway construction projects and 28 rail network construction projects. These 51 projects are aggregated into 10 groups based on investment regions and transportation modes (Lee *et al.* 2000).

The second case considered in this paper is the network that adds 230 road links to the network of case 1 and corresponds to scenarios 1-5 in Lee *et al.*(2000). These 230 road links are all highway road links planned in NITP(2000-2019). The third case adds 118 rail links to the network of case 1 and corresponds to scenarios 6-10 in Lee *et al.*(2000). The final case includes both road and rail links in case 2 and case 3. The network considered in case 1 is illustrated in Figure 1.

External parameters for the volume-delay functions and trip production and attraction volumes are also from Lee *et al.*(2000). Parameters describing mode split (1) and (2) are summarized in Table 1. These values are from Lee(1998) and present Won-equivalent utilities experienced by travellers using specific mode between each OD pair. For example, when a traveller rides a bus from origin to destination apart by d_{ij} km(suppose $d_{ij} < 200$ km) and the bus trip takes u_{ij} minutes, then the utility measured in Won is $\exp^{\frac{-6.1332}{1000}u_{ij} + \frac{-0.1921}{1000}50d_{ij}}$. Actual derivation of these parameters are found in Lee(1998).

Table 2 summarizes actual computation carried on four cases described above. The column label '# links' and underlying column labels 'road' and 'rail' show the number of links in road and rail part of the network. Average user-equilibrium minimum travel time between each OD pair is recorded under the column label 'avg travel time'. Since case 2 adds 230 road links to the network of case 1, average travel time between each OD pair decreases by 10.2%. Comparing case 1 and case 3, average travel time

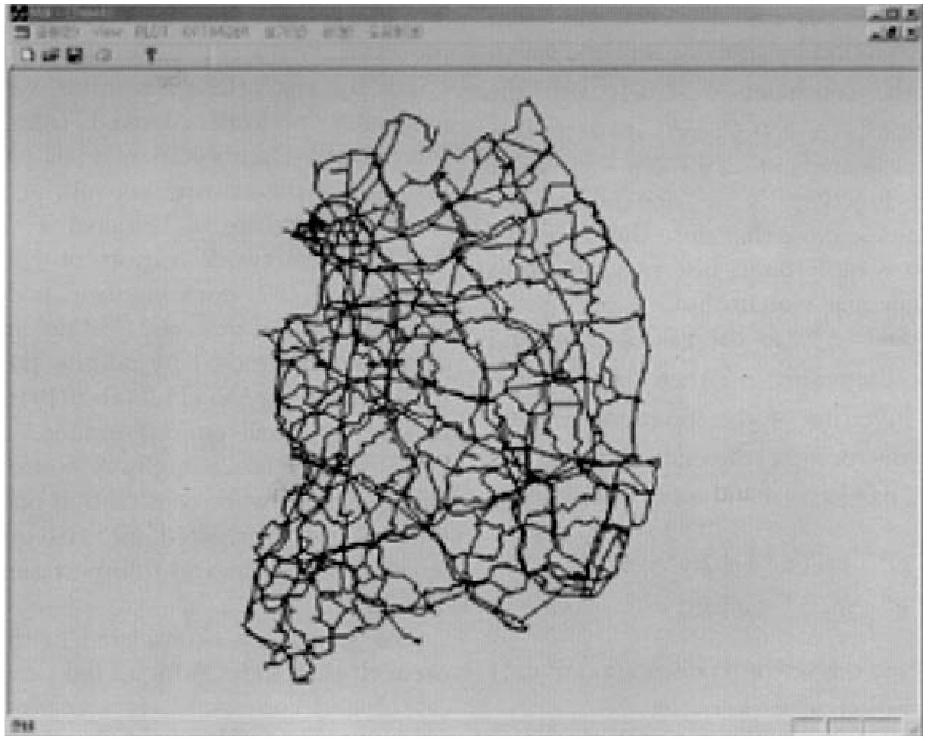

Figure 1. Map showing the existing network in Case 1.

using rail network decreases by 8.1%. Similar reductions in travel distances are observed in the column 'avg travel distance'. Since the volume-delay function of rail link has constant speed independent of carried load, the values in the column 'rail' under 'avg travel time' column are same in cases 3 and 4.

The distribution of passenger and cargo demands among au, bus, pr, tr, and cr modes are summarized in the column 'distribution of passenger and cargo demand'. Columns 'au', 'bus', and 'pr' divide passenger load and cargo demand split is recorded under the columns 'tr' and 'cr'.

Comparing case 1 and case 2, the proportion of passenger rail shrinks by 38% in the column 'pr' and increase of auto mode by 21%. Increase of auto mode in total passenger demand reflects the reduction of the minimum travel time and the minimum travel distance after construction of new road network links. But, increase in auto mode can result in increased accident, operating, or environment costs. Case 3 under the column 'pr' shows that construction of 118 rail links results in 7.8% and 0.5% increases in passenger and cargo rail modes.

Case 2 and case 3 also demonstrate that construction

Table 1. Parameters for (1) & (2) from Lee(1998)

mode	parameter per 1000 travel minutes		parameter per 1000 km		unit cost per km (WON)		β^m
	passenger (θ_1)	cargo(θ_2)	passenger	cargo			
au	6.1332		−0.1921		100		0.97
bus	6.1332		−0.1921		≤200 km	50	
					>200km	45.52	
pr	6.1332		−0.1921		≤100 km	3,800	-2.0765
					>100km	37.96	
tr		0.3176		-0.0118	452.54		
cr		0.3176		-0.0118	38.31		-3.1780

Table 2. Computational result for 4 cases

case	# links		avg travel time		avg travel distance		avg speed on road links	distribution of passenger and cargo demand				
	road	rail	road	rail	road	rail		au	bus	pr	tr	cr
1	4634	752	563.89	296.99	104.01	251.28	51.20	46.44	47.58	5.98	33.93	66.07
2	4864	752	506.63	296.99	64.10	251.28	54.41	56.55	39.80	3.65	65.93	34.07
3	4634	870	552.12	270.51	104.01	245.88	51.20	46.23	47.32	6.45	33.57	66.43
4	4864	870	513.72	270.51	64.10	245.88	54.41	56.45	39.72	3.83	65.48	34.51

of new road links increases the share of auto and bus portion among passenger demand more greater than the investment of new rail links. Investigating utility function parameters, we found that Won-equivalent utility for rail is currently much lower than the utility for road. Also, the minimum travel distance on rail network is much longer than on the road network. Considering uncertainties related with the highway traffic congestion, θ_1 and θ_2 parameters need to be recalculated in order to reflect the advantage of rail mode.

5. Conclusion

We proposed a combined trip generation, mode choice, and trip assignment model where the mode choice decisions are executed among two classes of demand and five transportation modes. The combined model uses binomial logit functions and entropy minimizing term in the objective function.

We derived the first order necessary condition for a minimum and developed an efficient convex combination algorithm similar to Frank-Wolf algorithm (1956). Our particular direction-finding linear program relies on the shortest path algorithm and transportation subproblems for passenger and cargo demands.

We tested our algorithm using four cases derived from NITP(2000-2019). From our computation, we found that addition of new road or rail links to the existing network increases shares of road or rail transport mode among passenger and cargo demands. Also, we found that utility function models of rail mode requires update of parameters in order to reflect environment friendly and on- timedness of rail network.

This model can be incorporated into a social overhead capital(SOC) investment projects that estimates traffic, accidents, operating, environment, and maintenance costs based on user equilibrium traffic load on the planned network.

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