

# The Problem of Disjunctive Causal Factors: In Defense of the Theory of Probabilistic Causation<sup>1)</sup>

Joonsung Kim

Department of Philosophy  
University of Wisconsin-Madison

**[Abstract]** The problem of disjunctive causal factors is generalized as follows. Suppose that there are no factors of the kind considered so far that need to be held fixed in background contexts. Nevertheless, it is still possible that within the background contexts, each disjunct of a disjunctive causal factor  $X \vee W$  confers a different probability on an effect factor in question. So a problem arises of how we identify a single causally significant probability of the effect factor in the presence of the disjunctive causal factor, assuming that each disjunct of the disjunctive causal factor confers a different probability on the effect factor. In this paper, I first introduce an experiment in which disjunctive causal factors seem to pose a problem for the theory of probabilistic causation. Second, I show how Eells' solution to the problem of disjunctive causal factors meets the problem that arises in the experiment. Third, I examine Hitchcock's arguments against Eells' solution, arguing that Hitchcock misconstrues Eells' solution, and disregards the feature of the theory of probabilistic causation such that a factor is a causal factor for another factor relative to a population  $P$  of a population type  $Q$ .

**[Keyword]** Background Contexts, Counterfactual Factors, Disjunctive Causal Factors, Populations, Probabilistic Causation, Probability, Propensity, Random Experiment, Eells, Hitchcock

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## Introduction

Consider a causal claim "Smoking  $X$  causes lung cancer  $Y$ ." Assume that indeterminism holds in the world. Then, the theory of probabilistic causation, which is developed by Suppes (1970), Cartwright ([1979] 1983), Skyrms (1980), Eells and Sober (1983) and Eells (1991), explicates the causal claim as follows:

$X$  is a positive causal factor for  $Y$  if and only if  $X$  has positive probabilistic significance for  $Y$  in every background context  $K_i$ , i.e.,  $Pr(Y/X\&K_i) > Pr(Y/-X\&K_i)$ , for all  $i$ .

This explication of the causal claim is based on two conditions. First,  $X$  is a positive causal factor for  $Y$  relative to a population  $P$ . A population  $P$  is a token population that exemplifies a population type, or kind  $Q$ , and can exemplify many different population types  $Q_1, Q_2, \dots, Q_n$ . The causal significance of  $X$  for  $Y$  depends on which type  $Q$  we consider a population  $P$  as exemplifying. Consider, for example, an actual human population that exemplifies both the type  $Q_1$  and the type  $Q_2$ . Then, smoking  $X$  has positive causal significance for getting lung cancer  $Y$  relative to a token population  $P$  exemplifying a population type  $Q_1$  (e.g., human beings). But  $X$  may not have positive causal significance for  $Y$ , relative to a token population  $P$  exemplifying type  $Q_2$  (e.g., a type given by a description of everyone's actual complex causal conditions) (Eells 1991, p.25). So the theory of probabilistic causation is called three-place theory  $\langle X, Y, P \rangle$  or four-place theory  $\langle X, Y, P, Q \rangle$  of probabilistic causation. Second, all the factors, causally independent of  $X$  (in the sense that  $X$  is not a cause of the factors) and causally relevant to  $Y$ , are held fixed in background contexts  $K_i$ . Suppose, for example, that a genetic factor  $F$  is, independently of  $X$ , causally relevant to  $Y$ . Then,  $F$  is positively held fixed in a background context  $K_1$  and negatively held fixed in a background context  $K_2$  as follows:  $Pr(Y/X\&K_1) > Pr(Y/-X\&K_1)$  and  $Pr(Y/X\&-K_2) < Pr(Y/-X\&-K_2)$ . This is called *the condition of contextual unanimity*. This condition is worth noticing.

Consider, for example, the case in which a genetic factor  $Z$  is a common causal factor for both smoking  $X$  and getting lung cancer  $Y$ . Then,  $Pr(Y/X) > Pr(Y/-X)$ . This positive probabilistic relevance of  $X$  to  $Y$  is not relative to background contexts in which all the factors causally independent of  $X$  and causally relevant to  $Y$  should be held fixed. But let us hold  $Z$ , which is causally independent of  $X$  and causally relevant to  $Y$ , positively fixed in a background context. Then,  $Pr(Y/X\&Z) = Pr(Y/-X\&Z)$ . Thus, by holding fixed  $Z$  in the background context, we can see that the positive causal relevance of  $X$  to  $Y$  disappears, and  $X$  is spuriously correlated to  $Y$ .

The theory of probabilistic causation is generalized on the basis of the above two conditions as follows:  $X$  is a positive, neutral or negative causal factor for  $Y$  if and only if  $Pr(Y/X\&K_i) >, =, < Pr(Y/-X\&K_i)$  relative to *each* of the background contexts  $K_i$  in a population  $P$  of a population type  $Q$ . Otherwise,  $X$  is mixed for  $Y$ . In other words,  $X$  is a positive, neutral or negative factor for  $Y$  if and only if  $X$  has "unanimously" positive, neutral or negative probabilistic significance for  $Y$ . Otherwise,  $X$  is mixed for  $Y$ . So the theory of probabilistic causation is called the "unanimity theory"<sup>2)</sup>. The theory of probabilistic causation is also called the "binary theory" in the sense that it explicates causal relevance in terms of the *qualitative* comparison between the probability of  $Y$  in the presence of  $X$  and the probability of  $Y$  in the absence of  $X$  (Humphreys (1989), Hitchcock (1993)).

It is claimed that disjunctive causal factors seem to pose a problem for the theory of probabilistic causation. Suppose that there are no factors of the kind considered so far that need to be held fixed in background contexts. Nevertheless, it is still possible that within the background contexts, each disjunct of a disjunctive causal factor  $X \vee W$  confers a different probability on an effect factor in question. So a problem arises of how we identify a single causally significant probability of the effect factor in the presence of the disjunctive causal factor, assuming that each disjunct of the disjunctive causal factor confers a different probability on the effect factor. This problem is

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2) In what follows, the theory of probabilistic causation refers to the unanimity theory and vice versa.

*called the problem of disjunctive causal factors.*

Eells (1988, 1991), who champions the unanimity theory, proposes a solution to the problem by using the idea of causal interaction, and holding certain "counterfactual factors" fixed in background contexts. Hitchcock (1993) introduces an experimental situation, and claims that Eells' solution to the problem of disjunctive causal factors is not tenable in the experimental situation.

In this paper, I shall first introduce an experiment (Humphreys 1989<sup>3</sup>), Hitchcock 1993) in which a disjunctive causal factor seems to pose a problem for the theory of probabilistic causation. Second, I shall show how Eells' solution to the problem of disjunctive causal factors meets the problem in the experiment. Third, I shall examine Hitchcock's arguments against Eells' solution, arguing that Hitchcock misconstrues Eells' solution, and disregards the feature of the theory of probabilistic causation such that a factor is a causal factor for another factor relative to a population  $P$  of a population type  $Q$ .

## 1. The Problem of Disjunctive Causal Factors

Let us look at an experiment in which the problem of disjunctive causal factors arises (Hitchcock 1993, pp.340-43). A medical team assesses the causal significance of a medicine for patients' recovery. The medical team divides the study group of patients into three treatment groups. The medical team provides the first group with placebo  $A$ . The medical team provides the second group with a moderate dose  $B$ . The medical team provides the third group with a strong dose  $C$ . Suppose that the probability of recovery  $Y$  given each of  $A$ ,  $B$ ,  $C$  is as follows:

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3) Humphreys (1989, pp.41-42) first introduces this kind of experiment in order to show that there is a problem with the qualitative comparison between the probability of  $Y$  in the presence of  $X$  and the probability of  $Y$  in the absence of  $X$ .

$$\Pr(Y/ A) = 0.2$$

$$\Pr(Y/ B) = 0.4$$

$$\Pr(Y/ C) = 0.9$$

The medical team wants to know whether the moderate dose is a positive causal factor for patients' recovery. The standard theory of probabilistic causation requires the team to compare  $\Pr(Y/B)$  with  $\Pr(Y/-B)$ . Since  $-B$  is equivalent to  $AvC$ , we need to assess  $\Pr(Y/ AvC)$ . Suppose that  $\Pr(A)$  is equal to  $\Pr(C)$ .  $\Pr(Y/-B) = [\Pr(A)\Pr(Y/A) + \Pr(C)\Pr(Y/C)] / [\Pr(A) + \Pr(C)]$ . Then  $\Pr(Y/-B) = (0.5)(0.2) + (0.5)(0.9) = 0.55$ . So  $\Pr(Y/B) = 0.4 < \Pr(Y/-B) = 0.55$ . So  $B$  is a negative causal factor for  $Y$ , according to the theory of probabilistic causation I introduced in the last section. Again, suppose that  $\Pr(A)$  is 0.6 and  $\Pr(C)$  is 0.1. Then, contrary to the previous case,  $B$  is a positive causal factor for  $C$ , according to the theory of probabilistic causation.

Hitchcock (1993) poses three problems for the unanimity theory. First, we need the values of  $\Pr(A)$  and  $\Pr(C)$  in order to assess  $\Pr(Y/ AvC)$ . But it is not certain how these values should be determined. Second, some values of  $\Pr(A)$  and  $\Pr(C)$  provide us with seemingly counter-intuitive causal information. In the above case, a moderate dose  $B$  is a negative causal factor for patients' recovery  $Y$ , according to the theory of probabilistic causation. This conflicts with our intuition that the moderate dose seemingly has positive causal significance for patients' recovery. Third, the causal significance of  $B$  for  $Y$  depends on the ratio of  $\Pr(A)$  to  $\Pr(C)$ . But it seems odd that objective causal significance of  $B$  for  $Y$  depends on the ratio of  $\Pr(A)$  to  $\Pr(C)$ .

The three problems arise since  $-B$  is equivalent to the disjunctive causal factor  $AvC$ , and the two disjuncts  $A$  and  $C$  of the disjunctive causal factor confer different probabilities on  $Y$ . As Eells says, a disjunction is "maximally probabilistically inhomogeneous for a factor in question if (1) all of its disjuncts confer different probabilities on the factor in question and (2) all of its disjuncts are probabilistically homogeneous<sup>4</sup> for the factor in

question”(Eells 1991, p.148). So the problem arises of how to identify a single causally significant probability of a factor  $Y$  in the presence of the disjunctive causal factor  $A \vee C$  when each disjunct  $A$  and  $C$  confers different probabilities on the factor  $Y$ . This is the problem of disjunctive causal factors. In section 2, I shall show how Eells' solution to the problem of disjunctive causal factors meets the first problem. In section 3, I shall criticize Hitchcock's arguments against the theory of probabilistic causation that the second and the third problems render it untenable, arguing that Hitchcock misconstrues Eells' solution, and disregards the feature of the theory of probabilistic causation.

## 2. Causal Interaction and Disjunctive Causal Factors

Eells (1988, pp.189-209; 1991, pp.144-68) provides a solution to the problem of disjunctive causal factors by using the idea of causal interaction, and holding "counterfactual factors" fixed in background contexts. In order to understand Eells' solution, it is helpful first to see Eells' solution to the special case of casual interaction. Let us consider the following case of causal interaction. Two relations of probabilistic relevance below show that,  $F$  being held positively and negatively fixed,  $X$  does not have unanimous causal significance for  $Y$ , but is mixed for  $Y$ .

$$\begin{aligned} Pr(Y/ X\&F) &= 0.2 < Pr(Y/-X\&F) = 0.9, \\ Pr(Y/ X\&-F) &= 0.6 > Pr(Y/-X\&-F) = 0.4 \end{aligned}$$

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4) A set of all the disjuncts is maximally specific for the factor in question, so that there are no further factors that should be considered causally relevant to the factor in question.

	-Y	-Y	-Y	
0.9	Y	-Y	-Y	0.6
0.2		Y	Y	
Y			Y	0.4
X&F	-X&F	X&-F	-X&-F	

<Figure>

In the above figure, consider probabilistic significance of  $X\&-F$  for  $Y$ . If  $-X\&F$  is sufficiently probable, then  $X\&-F$  will have negative probabilistic significance for  $Y$ , since  $Pr(Y/ X\&-F)$  would then be lower than  $Pr(Y/ -(X\&-F))$ . On the other hand, if  $X\&F$  or  $-X\&-F$  is sufficiently probable, then  $X\&-F$  will have positive probabilistic significance for  $Y$ , since  $Pr(Y/ X\&-F)$  would then be greater than  $Pr(Y/ -(X\&-F))$ . Thus, it seems that the theory of probabilistic causation has the consequence that whether  $X\&-F$  has positive, negative or neutral causal significance for  $Y$  depends on the probabilities of the other combined factors. But causal significance of  $X\&-F$  for  $Y$  seemingly should not depend on the overall probabilities of the other combined factors, for "different individuals who are  $X\&-F$  may have different propensities to distribute themselves among the other combined factors *were they not  $X\&-F$* " (Eells 1988, p.197). If so, then a problem arises of how we identify a single causally significant probability in  $-(X\&-F)$  for  $Y$  in the special case of causal interaction, assuming that there are no factors of the kind considered so far that need to be held fixed in background contexts.

In order to solve the problem, Eells considers, as interactive factors, the *different propensities* the individuals may have to distribute themselves in the other combined factors were not  $X\&-F$ .  $X\&-F$  interacts with the different propensities with regard to  $Y$ . The different propensities are expressed as counterfactuals. In the above case of interaction, different individuals have three kinds of propensities, which are expressed as the following three kinds

of counterfactuals:

- (1) The individuals who would have  $X\&F$  if they did not have  $X\&-F$ .
- (2) The individuals who would have  $X\&F$  if they did not have  $X\&-F$ .
- (3) The individuals who would have  $X\&-F$  if they did not have  $X\&-F$ .

Let us hold each of the three kinds of individuals (1), (2), (3) in three background contexts,  $K_1$ ,  $K_2$ ,  $K_3$ . Then, we can have the following three two-way probability comparisons:

$$\begin{aligned} Pr(Y/ K_1\& (X\&-F)) &= 0.6 > 0.2 = Pr(Y/ K_1\& -(X\&-F)) \\ Pr(Y/ K_2\& (X\&-F)) &= 0.6 < 0.9 = Pr(Y/ K_2\& -(X\&-F)) \\ Pr(Y/ K_3\& (X\&-F)) &= 0.6 > 0.4 = Pr(Y/ K_3\& -(X\&-F)) \end{aligned}$$

The probabilities on the left side depend not on counterfactual factors but on the way an individual is. That is, they depend on the background contexts in which all the factors, independent of  $X\&-F$ , causally relevant to  $Y$  are already held fixed. But the probabilities on the right side depend on counterfactual factors. Notice that the counterfactuals  $K_1$ ,  $K_2$ ,  $K_3$  are *factors* that refer to the different *propensities* different individuals would have to distribute themselves in the other combined factors were they not  $X\&-F$ . So  $-(X\&-F)$  together with each of the three counterfactual factors specifies each of  $X$  and  $F$ , which gives us a unique causally significant probability for  $Y$ . As the above three probabilistic relations show,  $X\&-F$  is causally mixed for  $Y$ .

Let us see how this method provides a solution to the problem of disjunctive causal factors in the experiment introduced in section 1. In the experiment of the dosage of medicine, the theory of probabilistic causation seemingly has the consequence that whether  $B$  has positive, negative or neutral causal significance for  $Y$  depends on the probability of  $Y$  in the presence of  $B$  and the probability of  $Y$  in the absence of  $B$ . The absence of  $B$ , i.e.,  $-B$ , is equivalent to  $A\vee C$ , since  $A$ ,  $B$ ,  $C$  are mutually exclusive and collectively exhaustive. So whether  $B$  has positive, negative or neutral causal significance for  $Y$  depends



on the probability of  $Y$  in the presence of  $B$  and the probability of  $Y$  in the presence of  $AvC$ . The problem arises of how a single probability of  $Y$  in the presence of the disjunctive causal factor  $AvC$  is identified, since each disjunct  $A$  and  $C$  is supposed to confer a different probability on  $Y$ . As Eells showed in the above case of interaction, we consider, as interactive factors, the different propensities the patients may have to distribute themselves in  $A$  and  $C$  were not  $B$ . That is,  $B$  interacts with the different propensities with  $Y$ . The different patients have two kinds of propensities, which are expressed as two kinds of counterfactuals as follows:

- (1) The patients who would have  $A$  if they did not have  $B$ .
- (2) The patients who would have  $C$  if they did not have  $B$ .

Let these two kinds of patients (1) and (2) be in turn  $K_1$ , and  $K_2$ . Holding each of the two factors fixed provides us with two background contexts. Then, we have the following probability comparisons:

$$\begin{aligned} Pr(Y/ K_1 \& B) &= 0.4 > 0.2 = Pr(Y/ K_1 \& (AvC)) \\ Pr(Y/ K_2 \& B) &= 0.4 < 0.9 = Pr(Y/ K_2 \& (AvC)) \end{aligned}$$

The probabilities on the left side depend not on counterfactual factors but on the background contexts in which all the factors, independent of  $B$ , causally relevant to  $Y$  are already held fixed. The probabilities on the right side depend on counterfactual factors.  $AvC$  together with the counterfactual factors specifies each of  $A$  and  $C$ , which gives us a unique causally significant probability for  $Y$ . As the above probabilistic relations show,  $B$  is causally *mixed* for  $Y$ .

At this point, assume that there are no determinate factors. That is, a specific factor  $A$  or  $C$ , in which they would have distributed themselves if the patients did not have  $B$ , is not determined. So if the patients did not have  $B$ , they would distribute themselves *with different probabilities* between  $A$  and  $C$ . Let  $r$  range over the different probability functions for  $A$  and  $C$  such that

$r(AvC)=1$ . Let  $s$  range over the different probability function for  $B$  such that  $s(B)=1$ , which in fact does not need to be considered in this experiment. Then, for each  $r$  and  $s$ , let  $Kr,s$  be the factor: If the patients did not have  $B$  (i.e., the patients had  $AvC$ ), then they would distribute themselves between  $A$  and  $C$  with probabilities given by  $r$  and if the patients had  $B$ , then they would distribute themselves in  $B$  with probabilities given by  $s$ . Note again that  $r(A)$  and  $r(C)$  indicate *propensities* with which individuals would *indeterministically* distribute themselves in  $A$  and  $C$  if  $\neg B$  had been the case for them.<sup>5)</sup>

Assuming that  $Kr,s$  is held fixed in a background context, single probabilities of  $Y$  in the presence of  $AvC$  are identified (i.e.  $Pr(Y / Kr,s \& (AvC))$ ) as follows:

$$Pr(Y / Kr,s \& (AvC)) = Pr(A / Kr,s \& (AvC))Pr(Y / A \& Kr,s \& (AvC)) + Pr(C / Kr,s \& (AvC))Pr(Y / C \& Kr,s \& (AvC)).$$

Each of  $A$  and  $C$  implies  $AvC$ , so the conjunct  $AvC$  can be eliminated from the second conditional probabilities on the right side.  $Kr,s$  can be also eliminated from the second conditional probabilities on the right side, since each of  $A$  and  $C$  is supposed to specify everything causally relevant to  $Y$ . Therefore,

$$Pr(Y / Kr,s \& (AvC)) = Pr(A / Kr,s \& (AvC))Pr(Y/A) + Pr(C / Kr,s \& (AvC))Pr(Y/C)$$

Considering the meaning of  $Kr,s$ ,  $Pr(A / Kr,s \& (AvC))$  and  $Pr(C / Kr,s \& (AvC))$  are identified with  $r(A)$  and  $r(C)$ , so that

$$Pr(Y / Kr,s \& (AvC)) = r(A) Pr(Y/A) + r(C) Pr(Y/C).$$

This is Eells' solution to the first problem of how  $Pr(A)$  and  $Pr(C)$  should be

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5) See Eells (1991, Ch.1) for the detailed discussion concerning the relation between propensity (causal tendency) and probability as hypothetical relative frequency.

determined. Thus, single  $Pr(Y/ Kr,s \ \& \ (AvC))$ 's are identified, depending on  $r(A)$  and  $r(C)$ . In this experiment,  $B$  is *mixed* for  $Y$ , since the relation between  $Pr(Y/ B)$  and  $Pr(Y/ Kr,s \ \& \ (AvC))$  may be different depending on  $r(A)$  and  $r(C)$ . Suppose that  $r(A) = p^1, p^2, p^3, \dots, p^m$  and  $r(C) = q^1, q^2, q^3, \dots, q^n$ .  $p^1, p^2, p^3, \dots, p^m$  are possible and different values of  $r(A)$  and  $q^1, q^2, q^3, \dots, q^n$  are possible and different values of  $r(C)$ . Then, we can combine  $r(A)$  and  $r(C)$  in  $m \times n$  ways as follows:  $\langle p^1, q^1 \rangle, \langle p^1, q^2 \rangle, \dots, \langle p^1, q^n \rangle, \langle p^2, q^1 \rangle, \dots, \langle p^2, q^n \rangle, \dots, \langle p^m, q^1 \rangle, \dots, \langle p^m, q^n \rangle$ . So it is not difficult to see that, depending on each of the ordered couples, the qualitative comparison between  $Pr(Y/ B)$  and  $Pr(Y/ Kr,s \ \& \ (AvC))$  may be different since  $B$  interacts with each of the ordered couples with regard to  $Y$ :  $Pr(Y/ B) > Pr(Y/ Kr,s \ \& \ (AvC))$ ,  $Pr(Y/ B) = Pr(Y/ Kr,s \ \& \ (AvC))$  or  $Pr(Y/ B) < Pr(Y/ Kr,s \ \& \ (AvC))$ . Thus, we can see that  $B$  is *mixed* for  $Y$ .

### 3. Chance Set-ups and Causal Significance

Hitchcock (1993, pp.345-6) poses two problems for Eells' solution to the problem of disjunctive causal factors as follows. Suppose that the patients had not been in a group  $B$ . Then, they possibly would have been equally likely to be in a group  $A$  and in a group  $C$ . So probability 0.5 is assigned to  $r(C)$  and  $r(A)$ . Then,  $Pr(Y/ -B) = Pr(Y/ AvC) = r(C) Pr(Y/C) + r(A) Pr(Y/A) = (0.5)(0.2) + (0.5)(0.9) = 0.55$ . This probability value, 0.55, is larger than  $Pr(Y/B) = 0.4$ . So a modest dose of medicine  $B$  has negative causal significance for patients recovery  $Y$ . This result conflicts with our intuition that a modest dose of medicine has positive causal significance for patients' recovery. This is the first alleged problem with Eells' solution. Also, according to Eells' solution, the causal significance of  $B$  for  $C$  is sensitive to the procedure by which the patients were assigned to the three groups  $A, B, C$ . This is the second alleged problem with Eells' solution.

Let us turn to the first alleged problem. The experiment in question aims to

answer the question of *how* (e.g., positively or negatively) *B* is *causally relevant to Y in the experimental situation*. This question does not presuppose that *B* is positively causally relevant to *Y*. The question concerns what causal significance *B* has for *Y*. Eells' solution provides an answer to this question such that *B* is *negatively causally relevant to Y* in the experimental situation. Let us see why, contrary to Hitchcock, this solution should appeal to our intuition. Causal significance of *B* for *Y* is assessed by the comparison between the probability of *Y* in the presence of *B* and the probability of *Y* in the absence of *B*. Imagine, for example, a world in which only *B* is a medicine for *Y*. In that world, the absence of *B* indicates that there is not any other medicine to treat *Y*. Then, *B* is certainly positively causally relevant to *Y*. In that world, our intuition is that *B* is positively causally relevant to *Y*.<sup>6)</sup> If so, then let us see what the absence of *B* in the experiment at issue is. The absence of *B* is only *A* and *C*. Unlike the previous imaginary world, even though the patients are not treated with *B*, they can be treated with *A* or *C*. Let us focus just on our ordinary intuition. Assume that there are only three kinds of medicine, and the alternative to *B* is only *A* or *C* in the experiment. Then, it seems rather odd that *B* is still positively causally relevant to *Y* in the comparison between the probability of *Y* in the presence of *B* and the probability of *Y* in the absence of *B*. Note that Eells' solution uses counterfactual factors (strictly speaking, propensities of individuals counterfactually expressed). For example, if individuals had not been in group *B*, then they would have had *propensities* (with different probabilities) to be distributed in *A* or *C*. As Hitchcock said, if individuals have one-half propensity to *A* and one-half propensity to *C*, then the probability of *Y* in the presence of *AvC* is greater than the probability of *Y* in the presence of *B*. As far as *the experiment* is concerned, it is natural and intuitive that that *B* has negative causal significance for *Y* when the probability of *Y* given *B* is compared with the probability of *Y* given *AvC*. If our intuition must insist that *B* always have positive causal significance for *Y* in the experiment, then the intuition seems odd. Also, recall that Eells' solution is intended to show that

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6) I am indebted to Sungsu Kim for this example of the imaginary world.

$B$  has neither unanimously positive nor negative nor neutral significance for  $Y$  but is mixed for  $Y$ . And the mixed causal significance of  $X$  for  $Y$  depends on  $r(A)$  and  $r(C)$  that refer to propensities with which individuals would distribute themselves in  $A$  and  $C$  if not  $B$  (had they not been  $B$ ). Then, it is not tenable that  $B$  always must be positively causally relevant to  $Y$  in the comparison between the probability of  $Y$  in the presence of  $B$  and the probability of  $Y$  in the absence of  $B$ .

Let us turn now to the second problem. Hitchcock claims that, according to Eells' solution, the causal significance of  $B$  for  $C$  is sensitive to the "mechanism by which the subjects were assigned to the three treatments groups" (Hitchcock 1993, p.346). Notice the following passage of Eells concerning  $r(A)$  and  $r(C)$ .

I say the probabilities determined are causally significant because of the way they are determined by different kinds of individuals' propensities (the  $r$ 's and  $s$ 's) to be different kinds of  $X$ 's and non- $X$ 's were they different, with respect to  $X$  and  $-X$  from the way they are; the values determined are not simply averages depending on how individuals happen to be actually distributed among the different kinds of  $X$ 's and non- $X$ 's (Eells 1988, p.207).

In the above experiment,  $r(A)$  and  $r(C)$  represent the different *propensities* individuals (patients) in a population (the experimental group)  $P$  would have to distribute themselves between  $A$  and  $C$  if they were not  $B$ . This point is based on the core idea of the unanimity theory such that a factor is a causal factor for another factor *relative to a population of a certain kind or type*. Considering this feature of the unanimity theory, it seems to me that the mechanism is not a problem with Eells' solution. Rather, it clearly reveals a desirable feature of the unanimity theory. In the unanimity theory, causal significance of a factor  $X$  for another factor  $Y$  depends on which population type  $Q$  a population  $P$ , within which  $X$  may be a causal factor for  $Y$ , is considered as exemplifying. As Hitchcock (1993, p.338) also says, a population (strictly speaking, a *kind* or *type* a population is considered as exemplifying) is a chance set-up. A (hypothetical relative) frequency of  $Y$  given  $X$  reflects propensity of  $X$  for  $Y$  that comes from a population as a chance-setup. The

unanimity theory explicates the propensity in terms of probabilistic relevance (based on the relative frequency). So Eells' solution, which is sensitive to the random mechanism, exactly represents various propensities that come from the study population of the patients as a chance set-up.

In the experiment, a population  $P$  is considered as exemplifying a population type  $Q_1$ , the study group of patients for the experiment. This population  $P$  of  $Q_1$  is divided into three mutually exclusive and exhaustive treatment groups. In the experiment, the mechanism of the subjects being assigned to the three different treatment groups depends on *propensities* relative to the study group population of patients  $P$  of  $Q_1$ . If the individuals in the experiment distribute, with the propensity of equal likelihood, among the three groups, then the propensity reflects the mechanism which the experiments choose. It reflects the propensity in a population of a population type  $Q_1$  (i.e., the initial conditions, or random experiment setting). On the other hand, let us consider a population  $P$  as exemplifying a population type  $Q_2$  (i.e., a set of different initial conditions). Then, it is not necessary that individuals in this population  $P$  of  $Q_2$  have the same propensity (i.e., propensity equally likely to distribute between  $r(A)$  and  $r(C)$ ) as they have in the population of  $Q_1$ . Individuals in the population  $P$  of  $Q_2$  may have propensities  $r^2(A)$  and  $r^2(C)$  different from  $r^1(A)$  and  $r^1(C)$  individuals in the population  $P$  of  $Q_1$  has. More strictly speaking, if the mechanism the experiment chooses in  $Q_1$  is random, then it assigns the patients to  $A, B, C$  with the ratio of 1 to 1 to 1. On the other hand, the mechanism the experiment chooses in  $Q_2$  may assign the patients to  $A, B, C$  with the ration of 10 to 1 to 1. Likewise, if we consider the other population types  $Q_3, \dots, Q_n$ , then individuals in a population  $P$  of each of the different population types will have different propensities to distribute themselves between  $A$  and  $C$ , e.g.,  $\langle r^3(A), r^3(C) \rangle$ ,  $\langle r^4(A), r^4(C) \rangle$ , ... , and  $\langle r^p(A), r^p(C) \rangle$ . Thus, in the experiment, individuals in a population may have different propensities to distribute themselves between  $A$  and  $C$ , depending on which population type they are relative to. So if we grant that  $r(A)$  and  $r(C)$  are intended to represent propensities in the population  $P$  of  $Q_1$ , then the equal probability distribution between  $A$  and  $C$  in the study group population of

patients does not really pose a problem for Eells' solution anymore.

Let us also notice that  $A$  and  $C$  are the only alternatives to  $B$  in the population  $P$  of  $Q_1$ . But in the population of  $P$  of  $Q_2$ , there may be many other alternatives (e.g.,  $D$ ,  $E$ , ...) in addition to  $A$  and  $C$ . If so, then we can expect individuals in the population to distribute, with many different probabilities (propensities), themselves among these alternatives if not  $B$ . For example, say that not just  $A$  and  $C$  but  $D$  and  $E$  are alternatives to  $B$  in the population of  $P$  of  $Q_2$ . Then, individuals in the population of  $P$  of  $Q_2$  will distribute themselves with different propensities among  $A$ ,  $C$ ,  $D$ , and  $E$ , if not in  $B$ . In this case, the individuals may have propensity equally likely to distribute among  $A$ ,  $C$ ,  $D$ , and  $E$ . That is,  $r(A)$ ,  $r(C)$ ,  $r(D)$ , and  $r(E)$  have the same value. But it is natural that the individuals may also have different propensities to distribute among  $A$ ,  $C$ ,  $D$ , and  $E$ . That is,  $r(A)$ ,  $r(C)$ ,  $r(D)$ , and  $r(E)$  may have different values. So the result in the population  $P$  of  $Q_1$  does not have significance in the population  $P$  of  $Q_2$  anymore. The point is that the unanimity theory is not a binary theory but *a type of three-place theory* such that a factor is a causal factor for another factor relative to a population of a certain type or kind. So the unanimity theory is, as it should be, sensitive to the mechanism by which the subjects were assigned to the three treatments groups. This sensitivity is not a problem with the unanimity theory but clearly reveals the desirable feature of the unanimity theory.<sup>7)</sup>

#### 4. Conclusions

I discussed how the theory of probabilistic causation meets the problem of disjunctive causal factors and the other two problems due to it. I conclude that Hitchcock should notice that the unanimity theory is not a binary theory but a type of three-place, or ternary theory. The three problems rather reveal this

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7) We can consider a population having individuals of many kinds corresponding to  $Q_1$ ,  $Q_2$ , ... etc. Then, we have interactive factors  $Q_1$ ,  $Q_2$ , ..., etc.

desirable feature of the unanimity theory such that a factor is a causal factor for another factor relative to a population of a certain type or kind.



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