J. KSIAM **Vol.6**, No.1, 81-89, 2002

EXISTENCE OF SOLUTIONS OF FUZZY DELAY DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITION

K.BALACHANDRAN AND P.PRAKASH

ABSTRACT. In this paper we prove the existence of solutions of fuzzy delay differential equations with nonlocal condition. The results are obtained by using the fixed point principles.

1. INTRODUCTION

The theory of fuzzy differential equations has been studied by many authors [2-5,9,10] by using the H-differentiability for the fuzzy valued mappings of a real variable whose values are normal, convex, upper semicontinuous and compactly supported fuzzy sets in R^n. Seikkala [8] defined the fuzzy derivative which is generalization of the Hukuhara derivative in [6]. The local existence theorems are given in [9], and the existence theorems under compactness-type conditions are investigated in [10], for the Cauchy problem $x' = f(t, x),\ x(t_0) = x_0$ when the fuzzy valued mapping f satisfies the generalized Lipschitz condition. Park et al [5] studied the fuzzy differential equation with nonlocal condition. Nieto [4] proved an existence theorem for fuzzy differential equations on the metric space (E^n, D).

In this paper we prove the existence of solutions of fuzzy delay differential equations with nonlocal condition of the form

$$\begin{aligned} x'(t) &= f(t, x(\sigma_1(t)), x(\sigma_2(t)), \cdots, x(\sigma_n(t))), \quad t \in J = [0, a] \qquad (1) \\ x(0) &- g(t_1, t_2, \cdots, t_p, x(\cdot)) = x_0, \end{aligned}$$

where $\sigma_i : J \to J,\ i = 1, 2, \cdots, n$ are continuous functions and $f : J \times E^{n^2} \to E^n$ is levelwise continuous function and $\sigma_i(t) \le t$ for all $t \in J,\ g : J^p \times E^n \to E^n$ satisfies the Lipschitz condition. The symbol $g(t_1, t_2, \cdots t_p, x(\cdot))$ is used in the sense that in the place of $'\cdot'$, we can substitute only elements of the set $\{t_1, t_2, \cdots, t_p\}$. For example, $g(t_1, t_2, \cdots, t_p, x(\cdot))$ can be defined by the formula

$$g(t_1, t_2, \cdots, t_p, x(\cdot)) = c_1 x(t_1) + c_2 x(t_2) + \cdots + c_p x(t_p),$$

2000 AMS Subject Classification : 45G10
Key words: Fuzzy delay differential equations, Nonlocal condition

where $c_i (i = 1, 2, \cdots, p)$ are given constants.

2. Preliminaries

Let $P_K(R^n)$ denote the family of all nonempty, compact, convex subsets of R^n. Addition and scalar multiplication in $P_K(R^n)$ are defined as usual. Let A and B be two nonempty bounded subsets of R^n. The distance between A and B is defined by the Hausdorff metric

$$d(A, B) = \max \left\{ \sup_{a \in A} \inf_{b \in B} ||a - b||, \sup_{b \in B} \inf_{a \in A} ||a - b|| \right\},$$

where $|| \cdot ||$ denote the usual Euclidean norm in R^n. Then it is clear that $(P_K(R^n), d)$ becomes a metric space. Let $I = [t_0, t_0 + a] \subset R$ $(a > 0)$ be a compact interval and let E^n be the set of all $u : R^n \to [0, 1]$ such that u satisfies the following conditions:

: (i) u is normal, that is, there exists an $x_0 \in R^n$ such that $u(x_0) = 1$,
: (ii) u is fuzzy convex, that is, $u(\lambda x + (1 - \lambda)y) \geq \min\{u(x), u(y)\}$, for any $x, y \in R^n$ and $0 \leq \lambda \leq 1$,
: (iii) u is upper semicontinuous,
: (iv) $[u]^0 = \text{cl}\{x \in R^n : u(x) > 0\}$ is compact.

If $u \in E^n$, then u is called a fuzzy number, and E^n is said to be a fuzzy number space. For $0 < \alpha \leq 1$, denote $[u]^\alpha = \{x \in R^n : u(x) \geq 0\}$. Then from (i)-(iv), it follows that the α-level set $[u]^\alpha \in P_K(R^n)$ for all $0 \leq \alpha \leq 1$.

If $g : R^n \times R^n \to R^n$ is a function, then using Zadeh's extension principle we can extend g to $E^n \times E^n \to E^n$ by the equation

$$\tilde{g}(u, v)(z) = \sup_{z = g(x,y)} \min\{u(x), v(y)\}.$$

It is well known that $[\tilde{g}(u, v)]^\alpha = g([u]^\alpha, [v]^\alpha)$ for all $u, v \in E^n$, $0 \leq \alpha \leq 1$ and continuous function g. Further, we have $[u + v]^\alpha = [u]^\alpha + [v]^\alpha, [ku]^\alpha = k[u]^\alpha$, where $k \in R$. Define $D : E^n \times E^n \to [0, \infty)$ by the relation $D(u, v) = \sup_{0 \leq \alpha \leq 1} d([u]^\alpha, [v]^\alpha)$, where d is the Hausdorff metric defined in $P_K(R^n)$. Then D is a metric in E^n.

Further we know that [7]

: (i) (E^n, D) is a complete metric space,
: (ii) $D(u + w, v + w) = D(u, v)$ for all $u, v, w \in E^n$,
: (iii) $D(\lambda u, \lambda v) = |\lambda| D(u, v)$ for all $u, v \in E^n$ and $\lambda \in R$.

It can be proved that $D(u + v, w + z) \leq D(u, w) + D(v, z)$ for u, v, w and $z \in E^n$

Definition 2.1.[2] A mapping $F : I \to E^n$ is strongly measurable if for all $\alpha \in [0, 1]$ the set-valued map $F_\alpha : I \to P_K(R^n)$ defined by $F_\alpha(t) = [F(t)]^\alpha$ is Lebesgue measurable when $P_K(R^n)$ has the topology induced by the Hausdorff metric d.

Definition 2.2.[2] A mapping $F : I \to E^n$ is said to be integrably bounded if there is an integrable function $h(t)$ such that $\|x(t)\| \le h(t)$ for every $x(t) \in F_0(t)$.

Definition 2.3. The integral of a fuzzy mapping $F : I \to E^n$ is defined levelwise by $[\int_I F(t)dt]^\alpha = \int_I F_\alpha(t)dt =$ The set of all $\int_I f(t)dt$ such that $f : I \to R^n$ is a measurable selection for F_α for all $\alpha \in [0, 1]$.

Definition 2.4.[1] A strongly measurable and integrably bounded mapping $F : I \to E^n$ is said to be integrable over I if $\int_I F(t)dt \in E^n$.

Note that if $F : I \to E^n$ is strongly measurable and integrably bounded, then F is integrable. Further if $F : I \to E^n$ is continuous, then it is integrable.

Proposition 2.1. Let $F, G : I \to E^n$ be integrable and $c \in I, \lambda \in R$. Then

: (i) $\int_{t_0}^{t_0+a} F(t)dt = \int_{t_0}^{c} F(t)dt + \int_{c}^{t_0+a} F(t)dt;$

: (ii) $\int_I (F(t) + G(t))dt = \int_I F(t)dt + \int_I G(t)dt,$

: (iii) $\int_I \lambda F(t)dt = \lambda \int_I F(t)dt,$

: (iv) $D(F, G)$ is integrable,

: (v) $D\left(\int_I F(t)dt, \int_I G(t)dt\right) \le \int_I D(F(t), G(t))dt.$

Definition 2.5 A mapping $F : I \to E^n$ is Hukuhara differentiable at $t_0 \in I$ if for some $h_0 > 0$ the Hukuhara differences

$$F(t_0 + \Delta t) -_h F(t_0), \quad F(t_0) -_h F(t_0 - \Delta t)$$

exist in E^n for all $0 < \Delta t < h_0$ and there exists an $F'(t_0) \in E^n$ such that

$$\lim_{\Delta t \to 0+} D((F(t_0 + \Delta t) -_h F(t_0))/\Delta t, F'(t_0)) = 0$$

and

$$\lim_{\Delta t \to 0+} D((F(t_0) -_h F(t_0 - \Delta t)/\Delta t, F'(t_0)) = 0.$$

Here $F'(t)$ is called the Hukuhara derivative of F at t_0.

Definition 2.6. A mapping $F : I \to E^n$ is called differentiable at a $t_0 \in I$ if, for any $\alpha \in [0, 1]$, the set-valued mapping $F_\alpha(t) = [F(t)]^\alpha$ is Hukuhara differentiable at point t_0 with $DF_\alpha(t_0)$ and the family $\{DF_\alpha(t_0) : \alpha \in [0, 1]\}$ define a fuzzy number $F(t_0) \in E^n$.

If $F : I \to E^n$ is differentiable at $t_0 \in I$, then we say that $F'(t_0)$ is the fuzzy derivative of $F(t)$ at the point t_0.

Theorem 2.1. Let $F : I \to E^n$ be differentiable. Denote $F_\alpha(t) = [f_\alpha(t), g_\alpha(t)]$. Then f_α and g_α are differentiable and $[F'(t)]^\alpha = [f'_\alpha(t), g'_\alpha(t)]$.

Theorem 2.2. Let $F : I \to E^n$ be differentiable and assume that the derivative F' is integrable over I. Then, for each $s \in I$, we have

$$F(s) = F(a) + \int_a^s F'(t)dt.$$

Definition 2.7. A mapping $f : I \times E^n \to E^n$ is called levelwise continuous at a point $(t_0, x_0) \in I \times E^n$ provided, for any fixed $\alpha \in [0, 1]$ and arbitrary $\epsilon > 0$, there exists a $\delta(\epsilon, \alpha) > 0$ such that

$$d([f(t, x)]^\alpha, [f(t_0, x_0)]^\alpha) < \epsilon$$

whenever $|t - t_0| < \delta(\epsilon, \alpha)$ and $d([x]^\alpha, [x_0]^\alpha) < \delta(\epsilon, \alpha)$ for all $t \in I, x \in E^n$.

Corollary 2.1 [2] Suppose that $F : I \to E^n$ is continuous. Then the function

$$G(t) = \int_a^t F(s)ds, \quad t \in I$$

is differentiable and $G'(t) = F(t)$.
Now, if F is continuously differentiable on I, then we have the following mean value theorem

$$D(F(b), F(a)) \leq (b - a) \cdot \sup\{D(F'(t), \hat{0}), t \in I\}.$$

As a consequence, we have that

$$D(G(b), G(a)) \leq (b - a) \cdot \sup\{D(F(t), \hat{0}), t \in I\}.$$

Theorem 2.3. Let X be a compact metric space and Y any metric space. A subset Ω of the space $C(X, Y)$ of continuous mappings of X into Y is totally bounded in the metric of uniform convergence if and only if Ω is equicontinuous on X, and $\Omega(x) = \{\phi(x) : \phi \in \Omega\}$ is a totally bounded subset of Y for each $x \in X$.

3. Main Results

Definition 3.1. A mapping $x : J \to E^n$ is a solution to the problem (1) if and only if it is levelwise continuous and satisfies the integral equation

$$x(t) \ = \ x_0 + g(t_1, t_2, \cdots, t_p, x(\cdot)) + \int_0^t f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds \quad (2)$$

for all $t \in J$.

Let $Y = \{\xi \in E^n : H(\xi, x_0) \leq b\}$ be the space of continuous functions with $H(\xi, \psi) = \sup_{0 \leq t \leq \gamma} D(\xi(t), \psi(t))$ and b is a positive number.

Theorem 3.1. Assume that:

: (i) The mapping $f : J \times Y \to E^n$ is levelwise continuous in t on J and there exists a constant G_0 such that

$$D(f(t,x_1,x_2,\cdots,x_n), f(t,y_1,y_2,\cdots,y_n)) \le G_0 \sum_{i=1}^{n} D(x_i,y_i)$$

: (ii) There exists a constant G_1 such that for all $x, y \in Y$ and $\sigma_i : J \to J,\quad i = 1,2,\cdots,n$

$$D(x(\sigma_i(t)), y(\sigma_i(t))) \le G_1 D(x(t), y(t))$$

: (iii) $g : J^p \times Y \to E^n$ is a function and there exists a constant $G_2 > 0$ such that

$$D(g(t_1,t_2,\cdots,t_p,x(\cdot)), g(t_1,t_2,\cdots,t_p,y(\cdot))) \le G_2 D(x,y).$$

Then there exists a unique solution $x(t)$ of (1) defined on the interval $[0,\gamma]$ where

$$\begin{aligned}
\gamma &= \min\{a, (b-N)/M, (1-G_2)/G_0G_1\},\\
M &= \max D(f(t, x(\sigma_1(t)), x(\sigma_2(t)), \cdots, x(\sigma_n(t))), \hat{0})) \quad \text{and}\\
N &= D(g(t_1,t_2,\cdots,t_p,x(\cdot)), \hat{0}), \hat{0} \in E^n.
\end{aligned}$$

Proof: Define an operator $\Phi : Y \to Y$ by

$$\Phi x(t) \;=\; x_0 + g(t_1,t_2,\cdots,t_p,x(\cdot)) + \int_0^t f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds. \quad (3)$$

First, we show that $\Phi : Y \to Y$ is continuous whenever $\xi \in Y$ and that $H(\Phi\xi, x_0) \le b$. Since f is levelwise continuous and σ is continuous, we take

$$M = \max D(f(t, x(\sigma_1(t)), x(\sigma_2(t)), \cdots, x(\sigma_n(t))), \hat{0})$$

$$\begin{aligned}
&D(\Phi\xi(t+h), \Phi\xi(t))\\
&= D\Bigg(x_0 + g(t_1,t_2,\cdots,t_p,\xi(\cdot)) + \int_0^{t+h} f(s,\xi(\sigma_1(s)),\xi(\sigma_2(s)),\cdots,\xi(\sigma_n(s)))ds,\\
&\qquad x_0 + g(t_1,t_2,\cdots,t_p,\xi(\cdot)) + \int_0^{t} f(s,\xi(\sigma_1(s)),\xi(\sigma_2(s)),\cdots,\xi(\sigma_n(s)))ds\Bigg)\\
&\le D\Bigg(\int_0^{t+h} f(s,\xi(\sigma_1(s)),\xi(\sigma_2(s)),\cdots,\xi(\sigma_n(s)))ds,\\
&\qquad \int_0^{t} f(s,\xi(\sigma_1(s)),\xi(\sigma_2(s)),\cdots,\xi(\sigma_n(s)))ds\Bigg)\\
&\le \int_t^{t+h} D(f(s,\xi(\sigma_1(s)),\xi(\sigma_2(s)),\cdots,\xi(\sigma_n(s))),\hat{0})ds\\
&= hM \to 0 \text{ as } h \to 0.
\end{aligned}$$

That is, the map Φ is continuous. Now

$$\begin{aligned}
&D(\Phi\xi(t), x_0)\\
&= D\left(x_0 + g(t_1, t_2, \cdots, t_p, \xi(\cdot)) + \int_0^t f(s, \xi(\sigma_1(s)), \xi(\sigma_2(s)), \cdots, \xi(\sigma_n(s)))ds, x_0\right)\\
&\le D(g(t_1, t_2, \cdots, t_p, \xi(\cdot)), \hat{0}) + \int_0^t D(f(s, \xi(\sigma_1(s)), \xi(\sigma_2(s)), \cdots, \xi(\sigma_n(s))), \hat{0})ds)\\
&= N + Mt
\end{aligned}$$

and so

$$H(\Phi\xi, x_0) = \sup_{0\le t\le\gamma} D(\Phi\xi(t), x_0) \le N + M\gamma \le b.$$

Thus Φ is a mapping from Y into Y. Since $C([0,\gamma], E^n)$ is a complete metric space with the metric H, we only show that Y is a closed subset of $C([0,\gamma], E^n)$. Let $\{\psi_n\}$ be a sequence in Y such that $\psi_n \to \psi \in C([0,\gamma], E^n)$ as $n \to \infty$. Then

$$D(\psi(t), x_0) \le D(\psi(t), \psi_n(t)) + D(\psi_n(t), x_0),$$

that is,

$$\begin{aligned}
H(\psi, x_0) &= \sup_{0\le t\le\gamma} D(\psi(t), x_0) \le H(\psi, \psi_n) + H(\psi_n, x_0)\\
&\le \epsilon + b
\end{aligned}$$

for sufficiently large n and arbitrary $\epsilon > 0$. So $\psi \in Y$. This implies that Y is closed subset of $C([0,\gamma], E^n)$. Therefore Y is a complete metric space.

By using Proposition 2.1 and assumptions (i),(ii) and (iii), we will show that Φ is a contraction mapping. For $\xi, \psi \in Y$,

$$\begin{aligned}
&D(\Phi\xi(t), \Phi\psi(t))\\
&= D\Big(x_0 + g(t_1, t_2, \cdots, t_p, \xi(\cdot)) + \int_0^t f(s, \xi(\sigma_1(s)), \xi(\sigma_2(s)), \cdots, \xi(\sigma_n(s)))ds,\\
&\qquad x_0 + g(t_1, t_2, \cdots, t_p, \psi(\cdot)) + \int_0^t f(s, \psi(\sigma_1(s)), \psi(\sigma_2(s)), \cdots, \psi(\sigma_n(s)))ds\Big)\\
&\le D(g(t_1, t_2, \cdots, t_p, \xi(\cdot)), g(t_1, t_2, \cdots, t_p, \psi(\cdot)))\\
&\qquad + \int_0^t D(f(s, \xi(\sigma_1(s)), \xi(\sigma_2(s)), \cdots, \xi(\sigma_n(s))),\\
&\qquad\quad f(s, \psi(\sigma_1(s)), \psi(\sigma_2(s)), \cdots, \psi(\sigma_n(s))))ds\\
&\le G_2 D(\xi(\cdot), \psi(\cdot)) + \int_0^t G_0 G_1 D(\xi(s), \psi(s))ds
\end{aligned}$$

Then we obtain

$$\begin{aligned} H(\Phi\xi, \Phi\psi) &\le \sup_{t\in\gamma}\left\{G_2 D(\xi(\cdot),\psi(\cdot)) + \int_0^t G_0 G_1 D(\xi(s),\psi(s))ds\right\} \\ &\le G_2 D(\xi(\cdot),\psi(\cdot)) + \gamma G_0 G_1 D(\xi(t),\psi(t)) \\ &\le (G_2 + G_0 G_1 \gamma) H(\xi,\psi). \end{aligned}$$

Since $\gamma G_0 G_1 + G_2 < 1, \Phi$ is a contraction map. Therefore Φ has a unique fixed point $x \in C([0,\gamma], E^n)$ such that $\Phi x = x$, that is,

$$x(t) = x_0 + g(t_1, t_2, \cdots, t_p, x(\cdot)) + \int_0^t f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds.$$

Theorem 3.2. Let f, σ and g be as in Theorem 3.1. Denote by $x(t, x_0), y(t, y_0)$ the solutions of equation (1) corresponding to x_0, y_0, respectively. Then there exists constant $q > 0$ such that

$$H(x(\cdot, x_0), y(\cdot, y_0)) \le q D(x_0, y_0)$$

for any $x_0, y_0 \in E^n$ and $q = 1/(1 - G_2 - \gamma G_0 G_1)$.

Proof: Let $x(t, x_0), y(t, y_0)$ be solutions of equations (1) corresponding to x_0, y_0, respectively. Then

$$\begin{aligned} &D(x(t,x_0), y(t,y_0)) \\ &= D\Big(x_0 + g(t_1,t_2,\cdots,t_p,x(\cdot)) + \int_0^t f(s,x(\sigma_1(s)),x(\sigma_2(s)),\cdots,x(\sigma_n(s)))ds, \\ &\qquad y_0 + g(t_1,t_2,\cdots,t_p,y(\cdot)) + \int_0^t f(s,y(\sigma_1(s)),y(\sigma_2(s)),\cdots,y(\sigma_n(s)))ds\Big) \\ &\le D(x_0,y_0) + D(g(t_1,t_2,\cdots,t_p,x(\cdot)), g(t_1,t_2,\cdots,t_p,y(\cdot))) \\ &\qquad + \int_0^t D(f(s,x(\sigma_1(s)),x(\sigma_2(s)),\cdots,x(\sigma_n(s))), \\ &\qquad\qquad f(s,y(\sigma_1(s)),y(\sigma_2(s)),\cdots,y(\sigma_n(s))))ds \\ &\le D(x_0,y_0) + G_2 D(x(\cdot),y(\cdot)) + \int_0^t G_0 G_1 D(x(s),y(s))ds \end{aligned}$$

Thus, $H(x(\cdot, x_0), y(\cdot, y_0)) \le D(x_0, y_0) + (G_2 + \gamma G_0 G_1) H(x(\cdot, x_0), y(\cdot, y_0))$,

that is, $H(x(\cdot, x_0), y(\cdot, y_0)) \le 1/(1 - G_2 - \gamma G_0 G_1) D(x_0, y_0)$.

This completes the proof of the theorem.

Next we generalize the above theorem for the fuzzy delay differential equation (1) with nonlocal condition.

Theorem 3.3. Suppose that $f : J \times E^{n^2} \to E^n$ is level wise continuous and bounded, $\sigma_i : J \to J$ $(i = 1 \cdots n)$ are continuous and $g : J^p \times E^n \to E^n$ is continuous. Then the initial value problem (1) possesses at least one solution on the interval J.

Proof: Since f is continuous and bounded and g is a continuous function there exists $r \geq 0$ such that

$$D(f(t, x(\sigma_1(t)), x(\sigma_2(t)), \cdots, x(\sigma_n(t))), \hat{0}) \leq r, \ t \in J, x \in E^n.$$

Let B be a bounded set in $C(J, E^n)$. The set $\Phi B = \{\Phi x : x \in B\}$ is totally bounded if and only if it is equicontinuous and for every $t \in J$, the set $\Phi B(t) = \{\Phi x(t) : t \in J\}$ is a totally bounded subset of E^n. For $t_0, t_1 \in J$ with $t_0 \leq t_1$, and $x \in B$ we have that

$$\begin{aligned}
&D(\Phi x(t_0), \Phi x(t_1)) = \\
&\quad D\Big(x_0 + g(t_1, t_2, \cdots, t_p, x(\cdot)) + \int_0^{t_0} f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds, \\
&\quad x_0 + g(t_1, t_2, \cdots, t_p, x(\cdot)) + \int_0^{t_1} f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds\Big) \\
&\leq D\Big(\int_0^{t_0} f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds, \\
&\quad \int_0^{t_1} f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds\Big) \\
&\leq \int_{t_0}^{t_1} D(f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s))), \hat{0})ds \\
&\leq |t_1 - t_0| \cdot \sup\{D(f(t, x(\sigma_1(t)), x(\sigma_2(t)), \cdots, x(\sigma_n(t))), \hat{0}) \ t \in J, \} \\
&\leq |t_1 - t_0| \cdot r.
\end{aligned}$$

This shows that ΦB is equicontinuous. Now, for $t \in J$ fixed. we have

$$D(\Phi x(t), \Phi x(t')) \ \leq \ |t - t'| \cdot r, \quad \text{for every} \ \ t' \in J, \ x \in B.$$

Consequently, the set $\{\Phi x(t) : x \in B\}$ is totally bounded in E^n. By Ascoli's theorem we conclude that ΦB is a relatively compact subset of $C(J, E^n)$. Then Φ is compact, that is, Φ transforms bounded sets into relatively compact sets.

We know that $x \in C(J, E^n)$ is a solution of (1) if and only if x is a fixed point of the operator Φ defined by (3).

Now, in the metric space $(C(J, E^n), H)$, consider the ball

$$B = \{\xi \in C(J, E^n), H(\xi, \hat{0}) \leq m\}, \ m = a \cdot r.$$

Thus, $\Phi B \subset B$. Indeed, for $x \in C(J, E^n)$,

$$\begin{aligned} D(\Phi x(t), \Phi x(0)) &= D\Big(x_0 + g(t_1, t_2, \cdots, t_p, x(\cdot)) \\ &\quad + \int_0^t f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s)))ds, \\ &\qquad x_0 + g(t_1, t_2, \cdots, t_p, x(\cdot))\Big) \\ &\leq \int_0^t D(f(s, x(\sigma_1(s)), x(\sigma_2(s)), \cdots, x(\sigma_n(s))), \hat{0})ds \\ &\leq |t| \cdot r \leq a \cdot r. \end{aligned}$$

Therefore, defining $\hat{0} : J \to E^n,\ \hat{0}(t) = \hat{0},\ t \in J$ we have

$$H(\Phi x, \Phi\hat{0}) = \sup\{D(\Phi x(t), \Phi\hat{0}(t)) : t \in J\}.$$

Therefore Φ is compact and, in consequence, it has a fixed point $x \in B$. This fixed point is a solution of the initial value problem (1).

References

[1] R.J.Aumann, Integrals of set-valued functions, *Journal of Mathematical Analysis and Applications*, 12 (1965), 1-12.
[2] O.Kaleva, Fuzzy differential equations, *Fuzzy Sets and Systems*, 24 (1987), 301-317.
[3] P.E.Kloeden, Remarks on Peano-like theorems for fuzzy differential equations, *Fuzzy Sets and Systems*, 44 (1991), 161-163.
[4] J.J.Nieto, The Cauchy problem for continuous fuzzy differential equations, *Fuzzy Sets and Systems*, 102 (1999), 259-262.
[5] J.Y.Park, H.K.Han and K.H.Son, Fuzzy differential equation with nonlocal condition, *Fuzzy Sets and Systems*, 115 (2000), 365-369.
[6] M.L.Puri and D.A.Ralescu, Differentials of fuzzy functions, *Journal of Mathematical Analysis and Applications*, 91 (1983), 552-558.
[7] M.L.Puri and D.A.Ralescu, Fuzzy random variables, *Journal of Mathematical Analysis and Applications*, 114 (1986), 409-422.
[8] S.Seikkala, On the fuzzy initial value problem, *Fuzzy Sets and Systems*, 24 (1987), 319-330.
[9] C.Wu, S.J.Song and E.Lee, Approximate solutions, existence and uniqueness of the Cauchy problem of fuzzy differential equations, *Journal of Mathematical Analysis and Applications*, 202 (1996), 629-644.
[10] C.Wu and S.J.Song, Existence theorem to the Cauchy problem of fuzzy differential equations under compactness type conditions, *Journal of Informations Sciences*, 108 (1998), 123-134.

Department of Mathematics
Bharathiar University
Coimbatore - 641 046
INDIA
balachandran_k@lycos.com