J. KSIAM Vol.6, No. 2, 109-120, 2002

Performance Analysis of A Modified Asymmetric Rectangular Fin

Hyung Suk Kang

ABSTRACT. The dimensionless heat loss from a modified asymmetric rectangular fin is investigated as a function of the fin top and tip Biot numbers using the two-dimensional separation of variables method. A rectangular fin is modified by attaching the wing on the top side of the fin. Fin effectiveness and efficiency with the variation of the location of the wing and the width of the wing are presented. The relationship between top surface Biot number and bottom surface Biot number as well as the relationship between the dimensionless wing height and the location of the wing for equal amount of heat loss is also discussed.

1. Introduction

The fin, or extended surface has been used to enhance the rate of heat transfer to a surrounding fluid in many thermal engineering applications such as the cooling of combustion engines, the air-conditioning, heat exchangers and so on. As a result, a great deal of attention has been directed to fin problems. Various shapes of fins have been studied. The most commonly studied fins are longitudinal rectangular, triangular, trapezoidal profile fins and the annular fin. For example, Bar-cohen [1] and Sen and Trinh [2] have discussed rectangular. Aziz and Nguyen [3] and Abrate and Newnham [4] were concerned with triangular while Kang and Look [5] and Kraus et al. [6] examined trapezoidal. Ullmann and Kalman [7] and Look [8] researched annular fins. Also studies on the pin fin and parabolic fin have been published. Gerencser and Razani [9] investigated the optimal pin fin array of variable cross section for a given fin material per unit base area while Kim and Kang [10] made a heat loss comparison between two parabolic fin models using two different numerical methods. Recently studies on the more unique shape of fin have been reported. Kunda and Das [11] present a numerical technique for the determination of the performance of eccentric annular fins with a variable base temperature. Bejan and Almogbel [12] report the geometric optimization of T-shaped fin assemblies, where the objective is to maximize the global thermal conductance of the assembly, subject to total volume and fin-material constraints. Usually most of the studies on the fin assume that the heat transfer coefficients for all surfaces of the fin are the same. But no literature seems to be available which presents a modified rectangular fin with unequal heat transfer coefficients. This paper presents an analysis of a thermally asymmetric

Key words: A modified rectangular fin, Wing, Heat loss, Effectiveness, Efficiency, Biot number

modified rectangular fin using the two-dimensional separation of variables method. A rectangular fin is modified by attaching the wing on the top side of the fin. In this study the top surface Biot number Bi_1 is different from the bottom surface Biot number Bi_2 and the fin tip Biot number Bi_3 varies from 0 to 1. Dimensionless heat losses are investigated as a function of the tip and top surface Biot numbers. Fin effectiveness and efficiency with the variation of the location of the wing and the width of the wing are presented. Further, for arbitrary fixed variables, the relationship between top surface Biot number and bottom surface Biot number as well as the relationship between the dimensionless wing height and the location of the wing for equal amount of heat loss are presented. For simplicity, the root temperature and the thermal conductivity of the fin's material are assumed constant as well as steady-state.

2. Two-Dimensional Analysis

For a modified asymmetric rectangular fin, illustrated in Fig. 1, the dimensionless governing differential equation is given by Eq. (1).

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0 \tag{1}$$

Three standard boundary conditions and one energy balance equation serve as the required problem formulation. They are

$$\theta = 1 \qquad \text{at } x = 0 \tag{2}$$

$$\frac{\partial\theta}{\partial y} - Bi_2 \cdot \theta = 0 \qquad \text{at } y = -1 \tag{3}$$

$$\frac{\partial\theta}{\partial x} + Bi_3 \cdot \theta = 0 \qquad \text{at } x = L \tag{4}$$

$$-\int_{-1}^{1} \frac{\partial\theta}{\partial x}\Big|_{x=0} dy = Bi_1\Big\{\int_0^a \theta\Big|_{y=1} dy + \int_1^H \theta\Big|_{x=a} dy + \int_a^b \theta\Big|_{y=H} dy + \int_1^H \theta\Big|_{x=b} dy + \int_b^L \theta\Big|_{x=b} dy\Big\} + Bi_2\int_0^L \theta\Big|_{y=-1} dy + Bi_3\int_{-1}^{1} \theta\Big|_{x=L} dy \tag{5}$$

By solving Eq. (1) with three boundary conditions listed as Eq. (2) through Eq. (4), the dimensionless temperature can be obtained by the separation of variables procedure. The result is

$$\theta(x, y) = \sum_{n=1}^{\infty} N_n \cdot f(x) \cdot f(y) \tag{6}$$

where,

$$f(x) = \cosh(\lambda_n x) - f_n \cdot \sinh(\lambda_n x) \tag{7}$$

$$f(y) = \cos(\lambda_n y) + g_n \cdot \sin(\lambda_n y) \tag{8}$$

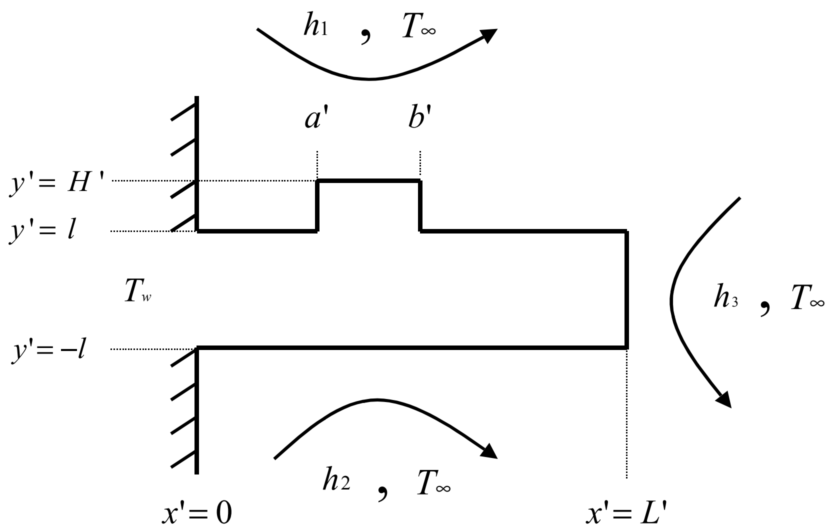

Fig. 1 Geometry of a modified asymmetric rectangular fin

$$N_n = \frac{4\sin(\lambda_n)}{[\{2\lambda_n + \sin(2\lambda_n)\} + g_n^2 \cdot \{2\lambda_n - \sin(2\lambda_n)\}]} \tag{9}$$

where, f_n and g_n are expressed by

$$f_n = \frac{\lambda_n \cdot \tanh(\lambda_n L) + Bi_3}{\lambda_n + Bi_3 \cdot \tanh(\lambda_n L)} \tag{10}$$

$$g_n = \frac{Bi_2 - \lambda_n \cdot \tan(\lambda_n)}{\lambda_n + Bi_2 \cdot \tan(\lambda_n)} \tag{11}$$

Eigenvalues λ_n are obtained by using the energy balance equation (Eq. (5)) and is listed as Eq. (12) to Eq. (21).

$$\begin{aligned} \{Bi_2 \cdot \sin(2\lambda_n) - \lambda_n \cdot AA_n\} \cdot [& Bi_1 \cdot \lambda_n \cdot \{\sinh(\lambda_n L) + BB_n\} \\ & + Bi_1 \cdot Bi_3 \cdot \{\cosh(\lambda_n L) + CC_n - 1\}] \\ & + \{DD_n - Bi_2 \cdot AA_n - \lambda_n \cdot \sin(2\lambda_n)\} \end{aligned}$$

$$\cdot (Bi_1 \cdot \lambda_n \cdot EE_n + Bi_1 \cdot Bi_3 \cdot FF_n) - GG_n \cdot (Bi_1 \cdot \lambda_n \cdot BB_n + Bi_1 \cdot Bi_3 \cdot CC_n) + \lambda_n \cdot HH_n \cdot II_n = 0 \tag{12}$$

where,

$$AA_n = \sin^2(\lambda_n) - \cos^2(\lambda_n) \tag{13}$$
$$BB_n = \sinh\{\lambda_n(L-b)\} - \sinh\{\lambda_n(L-a)\} \tag{14}$$
$$CC_n = \cosh\{\lambda_n(L-b)\} - \cosh\{\lambda_n(L-a)\} \tag{15}$$
$$DD_n = \lambda_n \cdot \sin\{\lambda_n(1+H)\} - Bi_2 \cdot \cos\{\lambda_n(1+H)\} \tag{16}$$
$$EE_n = \cosh\{\lambda_n(L-b)\} + \cosh\{\lambda_n(L-a)\} \tag{17}$$
$$FF_n = \sinh\{\lambda_n(L-b)\} + \sinh\{\lambda_n(L-a)\} \tag{18}$$
$$GG_n = Bi_2 \cdot \sin\{\lambda_n(1+H)\} + \lambda_n \cdot \cos\{\lambda_n(1+H)\} \tag{19}$$
$$HH_n = \lambda_n \cdot \sin(2\lambda_n) + 2Bi_2 \cdot \sin^2(\lambda_n) - Bi_2 \tag{20}$$
$$II_n = Bi_3 - Bi_3 \cdot \cosh(\lambda_n L) - \lambda_n \cdot \sinh(\lambda_n L) \tag{21}$$

The value of the heat loss from the modified asymmetric rectangular fin can be calculated with Eq. (22).

$$Q = 2k\theta_0 \sum_{n=1}^{\infty} N_n \cdot f_n \cdot \sin(\lambda_n) \tag{22}$$

Also equations of the fin effectiveness and the fin efficiency can be written by Eq. (23) and Eq. (24).

$$\varepsilon = \sum_{n=1}^{\infty} \frac{3N_n \cdot f_n \cdot \sin(\lambda_n)}{(Bi_1 + Bi_2 + Bi_3)} \tag{23}$$

$$\eta = \sum_{n=1}^{\infty} \frac{2N_n \cdot f_n \cdot \sin(\lambda_n)}{Bi_1 \cdot (L + 2H - 2) + Bi_2 \cdot L + 2Bi_3} \tag{24}$$

3. Results

Figure 2 presents the variation of the non-dimensional heat loss from a thermally asymmetric modified rectangular fin as the fin tip Biot number varies from 0.001 to 1.0 in the case of L=5, 10 and 20 for Bi_1=0.11, Bi_2=0.1, a=0.4L, b=0.6L and H=1.2. For short fin length (i.e. L=5), the effect of fin tip Biot number on the heat loss is important while this effect seems to be independent on the heat loss for the long fin length (i.e. L=20). It must be noted that the heat loss decreases as the fin length increases about beyond the value of Bi_3=0.25. This phenomenon can be explained by the fact that the fin tip Biot number is remarkably larger than fin top and bottom Biot numbers. Also it can be guessed that the maximum heat loss exists at a certain value of fin length when tip Biot number is less than about 0.25 from this figure and several dimensionless fin length for the maximum heat loss within this range of Bi_3 will be illustrated in Table 1.

Table 1 lists the dimensionless fin length for the maximum heat loss for two different thermally asymmetric conditions. The dimensionless fin length for the maximum heat loss decreases as tip Biot number increases. It also can be noted that the dimensionless fin length for the maximum heat loss when the wing is on the side with higher Biot number is slightly less than that when the wing is on

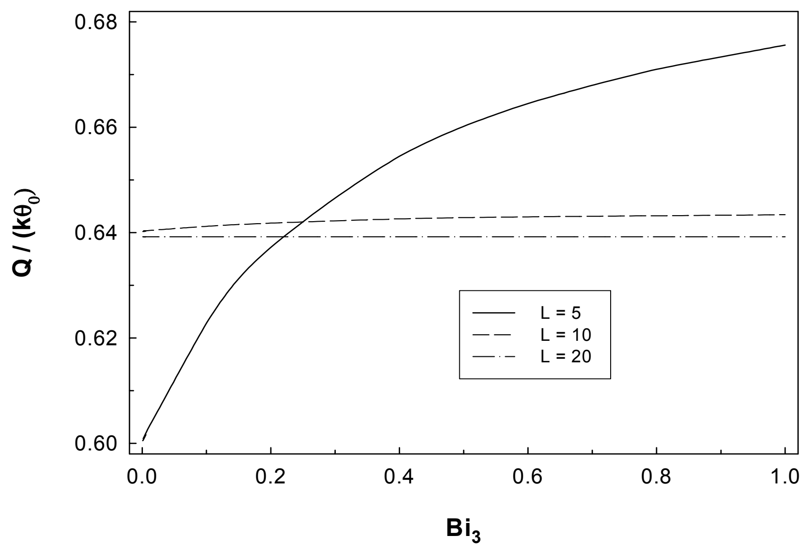

Fig. 2 The heat loss versus fin tip Biot number (Bi_1=0.11, Bi_2=0.1, a=0.4L, b=0.6L, H=1.2)

	L for the maximum heat loss		
	$Bi_3=0$	$Bi_3=0.1$	$Bi_3=0.2$
$Bi_1=0.11$ $Bi_2=0.1$	11.5	10.0	8.1
$Bi_1=0.1$ $Bi_2=0.11$	11.7	10.2	8.3

Table 1 Dimensionless fin length for the maximum heat loss in the case of a=0.4L, b=0.6L and H=1.2.

the side with lower Biot number.

The heat loss versus fin top Biot number for three different value of the dimensionless fin length in case of Bi_2=0.1−Bi_1, Bi_3=0.05, a=0.4L, b=0.6L and H=1.2

is presented by Fig. 3. For given circumstances, the heat loss increases continuously as fin top Biot number increases for L=5 while the effect of the fin top Biot number on the heat loss seems to be very small in case of L=10 and L=20.

Figure 4a presents the effect of the location of the wing with fixed height and width (i.e. H=1.2, b=a+1) on the fin effectiveness in case of Bi_3=0.01 and L=6 for three different combinations of top and bottom Biot numbers. As the wing approaches the fin root, fin effectiveness increases. It is also shown that fin effectiveness would be better when Biot number on the side with wing is larger than that on the side without wing if the average value of surrounding Biot numbers is the same.

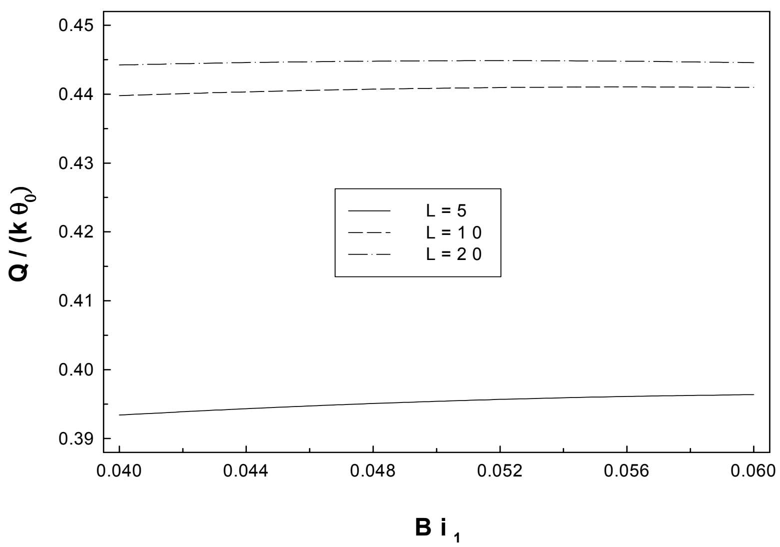

Fig. 3 The heat loss versus fin top Biot number (Bi_2=0.1-Bi_1, Bi_3=0.05, a=0.4L, b=0.6L, H=1.2)

Figure 4b shows the effect of the location of the wing on fin efficiency under the same condition as Fig. 4a. Fin efficiency also increases as the wing approaches the fin root. The range of fin efficiency variation is remarkable for $Bi_1>Bi_2$. Contrasted to fin effectiveness, fin efficiency is the best when bottom Biot number is larger than top Biot number.

The effect of width variation of the wing on fin effectiveness in case of Bi_3=0.01, a=1, L=6 and H=1.2 is shown in Fig. 5a. Comparing to Fig. 4a, it can be noted that the effect of width variation on fin effectiveness is less than the effect of the location of the wing on that. As b increases, which means the size of the wing is bigger, fin effectiveness decreases. This fact can be explained that even though b increases, the actual length of fin top line does not change for two-dimensional analysis. Figure 5b describes the effect of width variation of the wing on fin efficiency under the same condition as Fig. 5a. It also shows the trend of the effect of width variation on fin efficiency is similar to the effect of location change of the wing on that.

Figure 6 depicts the relationship between the height of the wing and the location of the wing to produce the equal amount of heat loss when the width of the wing is equal to 2 and L is equal to 10. It may be easily noted that the height of the wing increases as the wing moves from fin root direction to fin tip direction. Also this increasing rate of H becomes more remarkable as surrounding Biot numbers increase.

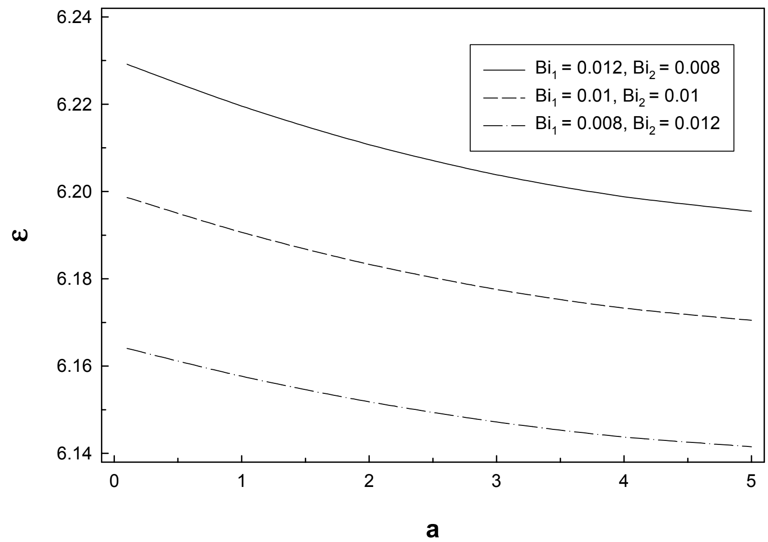

Fig. 4a Fin effectiveness versus beginning point of the wing (Bi_3=0.01, b=a+1, L=6, H=1.2)

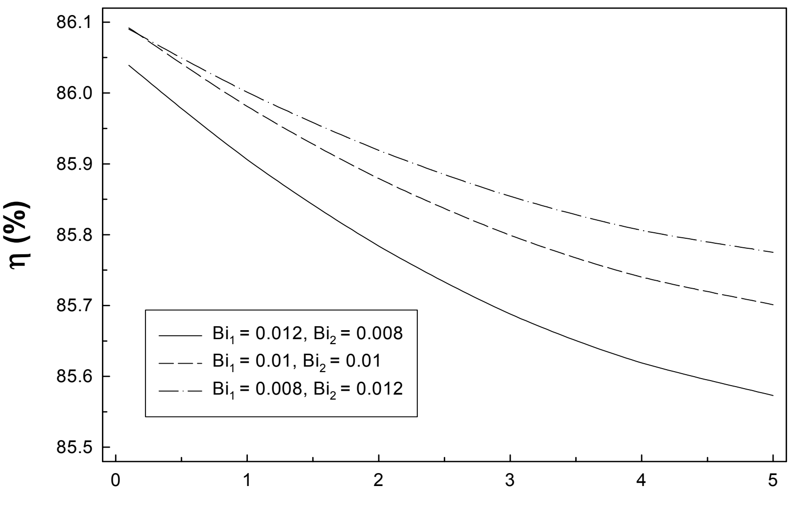

Fig. 4b Fin efficiency versus beginning point of the wing (Bi_3=0.01, b=a+1, L=6, H=1.2)

Figure 7 presents the relationship between Bi_1 and Bi_2 for an equal amount of heat loss in case of H=1.2 and 1.5 when other variables are fixed. As Bi_1 increases from 0.04 to 0.06, Bi_2 has to be decreased from 0.06 to 0.04 for an equal

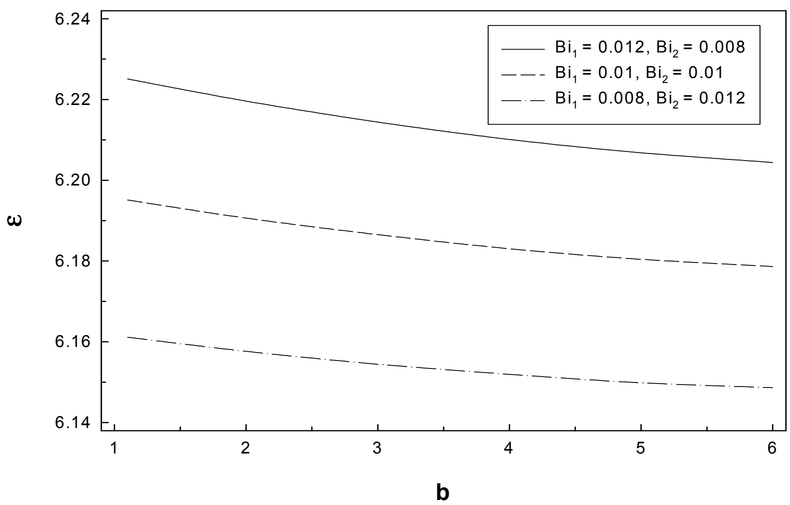

Fig. 5a Fin effectiveness versus ending point of the wing (Bi_3=0.01, a=1, L=6, H=1.2)

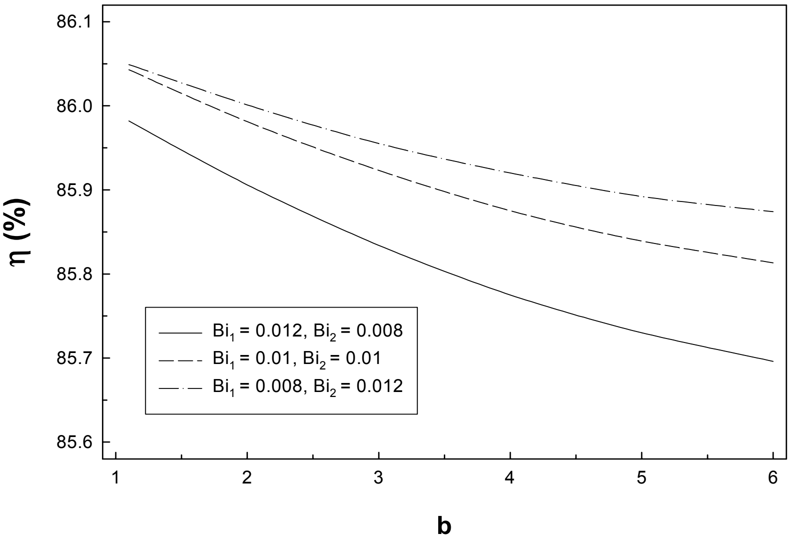

Fig. 5b Fin efficiency versus ending point of the wing (Bi_3=0.01, a=1, L=6, H=1.2)

amount of heat loss if the fin has no wing. But it is shown that as Bi_1 increases from 0.04 to 0.06, Bi_2 decreases from the value larger than 0.06 to the value less

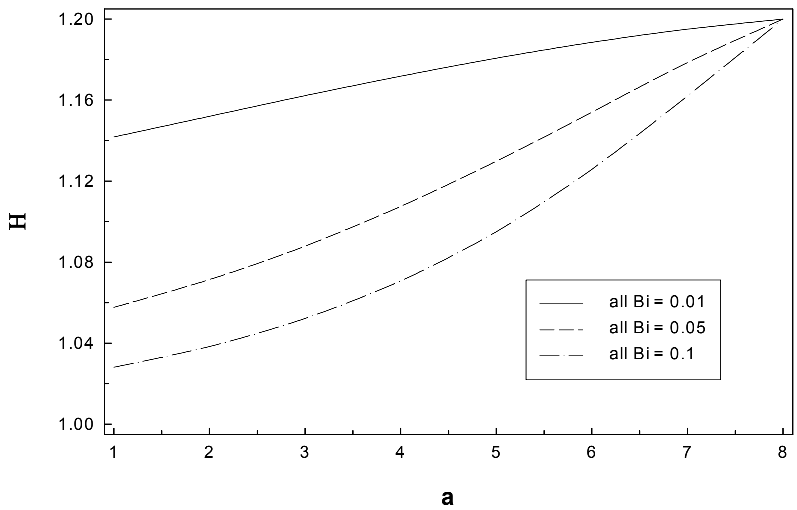

Fig. 6 Relationship between height and beginning point of the wing for an equal amount of heat loss (b=a+2, L=10)

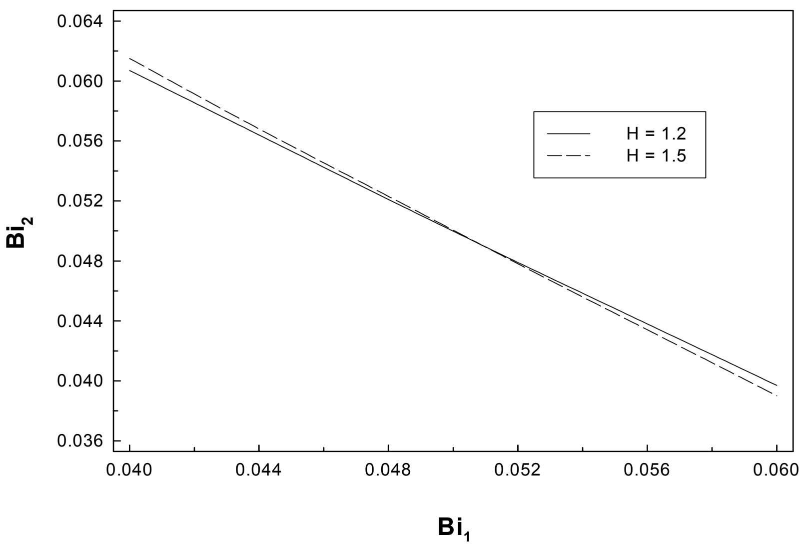

Fig. 7 Relationship between Bi_1 and Bi_2 for an equal amount of heat loss (a=0.4L, b=0.6L, Bi_3=0.05, L=6)

than 0.04 with the effect of the wing. The slope of the line for H=1.5 is greater than that for H=1.2 and this phenomenon means physically that the effect of the wing is more noticeable as the height of the wing increases.

4. Conclusion

From the two-dimensional analysis of an asymmetric modified rectangular fin presented here, the following conclusions can be drawn:
1) If the maximum heat loss exists at a certain fin length, fin length for the maximum heat loss decreases as tip Biot number increases.
2) When Biot number at the side with wing is larger than Biot number at the side without wing, it is better for fin effectiveness but not for fin efficiency.
3) For an equal amount of heat loss, the height of the wing increases as the wing with fixed width moves from fin root direction to fin tip direction and this trend is more noticeable as surrounding Biot numbers increase.

Nomenclature

a' : beginning point of the wing (m)
a : dimensionless beginning point of the wing
b' : ending point of the wing (m)
b : dimensionless ending point of the wing
Bi_i : Biot number of the ith fin surface (= $h_i l / k$)

h_i : heat transfer coefficient of the ith fin surface (W/m^2。C)
H' : the height of the wing (m)
H : dimensionless height of the wing
k : thermal conductivity (W/m。C)
l : one half fin root height (m)
L' : fin length (m)
L : dimensionless fin length (= L'/l)
Q : heat loss (W)
T : temperature (。C)
T_w : fin root temperature (。C)
T_∞ : ambient temperature (。C)
x' : coordinate along the fin length (m)
x : dimensionless coordinate along the fin length ($= x'/l$)
y' : coordinate along the fin height (m)
y : dimensionless coordinate along the fin height ($= y'/l$)

Greek Symbols
θ : dimensionless temperature ($T - T_\infty)/(T_w - T_\infty$)
θ_0 : adjusted fin root temperature ($= T_w - T_\infty$) (。C)
ε : fin effectiveness
η : fin efficiency
λ_n : eigenvalues (n = 1, 2, 3, · · ·)

Subscript
0 : root
1 : top side of a modified rectangular fin
2 : bottom side of a modified rectangular fin
3 : tip side of a modified rectangular fin
∞ : surrounding
w : wall

Acknowledgement

This work was funded by (Kisunghoi professor sending abroad) of Kangwon National University in 2001.

REFERENCES

[1] Bar-cohen, A., "Fin Thickness for an Optimized Natural Convection Array of Rectangular Fins", ASME J. of Heat Transfer, Vol. 101, 1979, pp. 564-566.

[2] Sen, A. K. and Trinh, S., "An Exact solution for the Rate of Heat Transfer

from a Rectangular Fin Governed by a Power Law-Type Temperature Dependence", ASME J. of Heat Transfer, Vol. 108, 1986, pp. 457-459.

[3] Aziz, A. and Nguyen, H., "Two-Dimensional Effects in a Triangular Convecting Fin", AIAA J. of Thermophysics and Heat Transfer, Vol. 6, No. 1, 1992, pp. 16 5~167.

[4] Abrate, S. and Newnham, P., "Finite Element Analysis of Triangular Fin Attached to a Thick Wall", Computer & Structures, Vol. 57, No. 6, 1995, pp. 945-957.

[5] Kang, H. S. and Look, D. C., "Two Dimensional Trapezoidal Fins Analysis", Computational Mechanics, Vol. 19, 1997, pp. 247~250.

[6] Kraus, A. D., Snider, A. D. and Doty, L. F., "An Efficient Algorithm for Evaluating Arrays of Extended Surface", ASME J. of Heat Transfer, Vol. 100, 1978, pp. 288-293.

[7] Ullmann, A. and Kalman, H., "Efficiency and Optimized Dimensions of Annular Fins of Different Cross-Section Shapes", Int. J. of Heat Mass Transfer, Vol. 32, 1989, pp. 1105~1110.

[8] Look, D. C., "Fin on a Pipe (Insulated Tip): Minimum Conditions for Fin to Be Beneficial", Heat Transfer Engineering, Vol. 16, No. 3, 1995, pp. 65-75.

[9] Gerencser, D. S. and Razani, A., "Optimization of Radiative-Convective Arrays of Pin Fins Including Mutual Irradiation between Fins", Int. J. of Heat Mass Transfer, Vol. 38, No. 5, 1995, pp. 899~907.

[10] Kim, K. T. and Kang, H. S., "A Heat Loss Comparison Between the Two Parabolic Fin Models Using Two Different Numerical Methods", Journal of KSIAM, Vol. 2, No. 2, 1998, pp. 97-109.

[11] Kundu, B. and Das, P. K., "Performance Analysis of Eccentric Annular Fins with a Variable Base Temperature", Numerical Heat Transfer, Part A, Vol. 36, 1999, pp. 751-766

[12] Bejan, A. and Almogel, M., "Constructal T-Shaped Fins", Int. J. of Heat Mass Transfer, Vol. 43, 2000, pp. 2101~2115.

Division of Mechanical and Mechatronics Engineering
College of Engineering
Kangwon National University
Chunchon, KOREA
hkang@kangwon.ac.kr