J. KSIAM **Vol.6**, No.2, 63-73, 2002

SOLUTION OF THE BOUNDARY LAYER EQUATION FOR A MAGNETOHYDRODYNAMIC FLOW OF A PERFECTLY CONDUCTING FLUID

M. ZAKARIA

ABSTRACT. The influence of unsteady boundary layer magnetohydrodynamic flow with thermal relaxation of perfectly conducting fluid, past a semi-infinite plate, is considered. The governing non linear partial differential equations are solved using the method of successive approximations. This method is used to obtain the solution for the unsteady boundary layer magnetohydrodynamic flow in the special form when the free stream velocity exponentially depends on time. The effects of Alfven velocity α on the velocity is discussed, and illustrated graphically for the problem.

1. INTRODUCTION

The study of unsteady boundary layer magnetohydrodynamic flow problems in the presence of a magnetic field is motivated not only by its interest, but also by need for it in practical applications in chemical engineering technology.

The phenomenal growth of energy requirements in recent years has been attracting considerable attention all over the world. This has resulted in a continuous exploration for new ideas and avenues in hamessing various conventional energy sources like tidal waves, wind power, geo-thermal energy, etc. It is obvious that in oread to utilize the geo-thermal energy to the maximum, one should have a complete and precise knowledge of the amount of perturbations needed to generate convection currents in geo-thermal fluid. In addition, the knowledge of the quantity of perturbations essential to initiate convection currents in mineral fluids found in the earth's crust helps one to utilize minimal energy to extract the minerals. For example, in the recovery of hydro-carbons from underground petroleum deposit, the use of thermal processes is increasingly ganong importance as it enhances the recovery.Heat is being injected into the reservoir in the form of hot water or stream or heat can be generated by burning part of the crude in the reservoir. In all such thermal recovery processes fluid flow takes place through a conducting medium and the convection currents are detrimental.

Key words and phrases. Boundary layer., Magnetohydrodynamic., perfectly conducting.

The boundary layer flows for an electrically conducting fluid have been discussed by many authors[1]-[4], and historically Rossow[5] was the first to study the hydrodynamic behavior of the layer on a semi-infinite plate in the presence of a uniform transverse field.

Further extension of our research in the area has led to the study of the boundary layer equation for a unsteady hydrodynamic incompressible flow under the influence of a magnetic field. This paper presents the solution of unsteady hydrodynamic incompressible flow problem when the magnetic field is applied to the fluid. The problem involves two-dimensional boundary layer, where the free stream of flow exponentially depends on time.

2. Basic Equations

We consider the boundary layer equations for the two-dimensional unsteady flow of a viscous incompressible perfectly conducting fluid past a semi-infinite plate. We assume that the velocity at large distance from the body will depends on the time and x only. The x and y axes is taken along and perpendicular to the body, respectively.

Let a constant magnetic field of strength H_0acts in the direction of the y-axis. This produces an induced magnetic field **h** and induced electric field **E**. Which satisfy the linearized equations of electromagnetic, valid for slowly moving media of a perfect conductor[6].

$$curl\ h = J + \epsilon_0 \frac{\partial E}{\partial t}, \tag{1}$$

$$curl\ E = -\mu_0 \frac{\partial h}{\partial t}, \tag{2}$$

$$E = -\mu_0 (V \times H_0), \tag{3}$$

$$div\ h = 0, \tag{4}$$

where **J** is the electric current density, μ_0 and ϵ_0 are the magnetic and electric permeabilities, and **V** = (u, v, 0) is the velocity vector of the fluid that satisfies the equation of continuity

$$div\ V = 0, \tag{5}$$

the momentum equation

$$\rho \dot{V} = -\nabla p + \mu \nabla^2 V + \mu_0 (J \times H_0), \tag{6}$$

the momentum equation

$$\rho \left(\dot{T} + \tau_0 \frac{\partial \dot{T}}{\partial t} \right) = \frac{\lambda}{C_p} \nabla^2 T, \tag{7}$$

where ρ is the density of the fluid, p is the pressure, μ is the viscosity, T is the temperature of the fluid in the boundary layer, τ_0 is the relaxation time, C_p is the specific heat at constant pressure, λ is the thermal conductivity, and the over dot denotes the material derivative.

As mentioned above the applied magnetic field $\mathbf{H_0}$ has components $(0,\ H_0,\ 0)$. It can be easily seen from the above equations the induced magnetic field has the components $\mathbf{h} = (h_1,\ h_2,\ 0)$. The vector $\mathbf{E}$ and $\mathbf{J}$ will have non-vanishing components only in the z-direction, i.e.

$$\mathbf{E} = (\mathbf{0},\ \mathbf{0},\ \mathbf{E}),\ \mathbf{J} = (\mathbf{0},\ \mathbf{0},\ \mathbf{J})\ .$$

The governing equations within boundary layer approximation may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\ , \tag{8}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \\ &+ \frac{\alpha^2}{H_0}\left(\frac{\partial h_1}{\partial y} - \frac{\partial h_2}{\partial x} - \mu_0\epsilon_0 H_0\frac{\partial u}{\partial t}\right)\ , \end{aligned} \tag{9}$$

$$\frac{\partial h_1}{\partial t} = H_0\frac{\partial u}{\partial y}\ , \tag{10}$$

$$\frac{\partial h_2}{\partial t} = -\,H_0\frac{\partial u}{\partial x}\ , \tag{11}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\lambda}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \tau_0\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)\ , \tag{12}$$

where α is the Alfven velocity given by $\alpha^2 = \mu_0 H_0^2/\rho$ and ν is the kinematics viscosity, and the boundary conditions are given by

$$\begin{aligned} u &= 0\ ,\ v = 0\ ,\ T - T_\infty = T_0 U(x,t)\ ,\ at\ y = 0\ , \\ u &= U_\infty\ ,\ T - T_\infty = 0\ ,\ as\ y \to \infty\ , \end{aligned} \tag{13}$$

where T_0 is the temperature of the plate, T_∞ is the temperature of the fluid far away from the plate and U_∞ is the velocity of the fluid at large distance from the plate.

The pressure term in equation (9) shall be expressed in terms of the free stream velocity U_∞. At large distance from the plate the velocity component u is a function of x and t only. Equation (9) will then become a generalized Bernouill's equation,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial U_\infty}{\partial t} + U_\infty\frac{\partial U_\infty}{\partial x} + \frac{\alpha^2}{H_0}\frac{\partial h_{2\infty}}{\partial x} + \alpha^2\mu_0\epsilon_0\frac{\partial U_\infty}{\partial t}\ , \tag{14}$$

where $h_{2\infty}$(x,t) the component of induced magnetic field at large distance from the body. Inserting equation (14), equation (9)yields

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} &+ v\frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial t} + U_\infty\frac{\partial U_\infty}{\partial x} + v\frac{\partial^2 u}{\partial y^2} \\ &+ \frac{\alpha^2}{H_0}\left(\frac{\partial h_1}{\partial y} - \frac{\partial h_2}{\partial x} + \frac{\partial h_{2\infty}}{\partial x} - \mu_0\epsilon_0 H_0\left[\frac{\partial u}{\partial t} - \frac{\partial U_\infty}{\partial t}\right]\right)\ . \end{aligned} \tag{15}$$

Eliminating h_1 and h_2 between equations (10), (11), and (15), and taking into account the boundary layer approximations. Equation (15) yields

$$\begin{aligned}\frac{\partial^2 u}{\partial t^2} + u\frac{\partial^2 u}{\partial t\partial x} + \frac{\partial u}{\partial t}\frac{\partial u}{\partial x} &+ v\frac{\partial^2 u}{\partial t\partial y} + \frac{\partial v}{\partial t}\frac{\partial u}{\partial y} \\ &= \frac{\partial^2 U_\infty}{\partial t^2} + U_\infty\frac{\partial^2 U_\infty}{\partial t\partial x} + \frac{\partial U_\infty}{\partial t}\frac{\partial U_\infty}{\partial x} \\ &+ v\frac{\partial^3 u}{\partial t\partial^2 y} + \alpha^2\left(\frac{\partial^2 u}{\partial y^2} - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}(u - U_\infty)\right) .\end{aligned} \tag{16}$$

We introduce the following non-dimensional quantities

$$\begin{aligned} x^* &= \frac{U_0 x}{\nu} , \ y^* = \frac{U_0 y}{\nu} , \ t^* = \frac{U_0^2 t}{\nu} , \ h_1^* = \frac{h_1}{H_0} , \ h_2^* = \frac{h_2}{H_0} , \\ u^* &= \frac{u}{U_0} , \ v^* = \frac{v}{U_0} , \ P = \frac{\rho C_p \nu}{\lambda} , \ T^* = \frac{T - T_\infty}{T_0} , \\ \tau_0^* &= \frac{U_0^2\tau_0}{\nu} , \ \alpha^* = \frac{\alpha}{U_0} , \ U_\infty^* = \frac{U_\infty}{U_0} , \end{aligned} \tag{17}$$

where P is the Prandtl number.

By taking into account $U_\infty = U_0$ U(x,t). Invoking the non-dimensional quantities above, equations (8), (12), and (16) are reduced to the non-dimensional equations, dropping the asterisks for convenience,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{18}$$

$$\begin{aligned} a\frac{\partial^2 u}{\partial t^2} + u\frac{\partial^2 u}{\partial t\partial x} &+ \frac{\partial u}{\partial t}\frac{\partial u}{\partial x} + v\frac{\partial^2 u}{\partial t\partial y} + \frac{\partial v}{\partial t}\frac{\partial u}{\partial y} \\ &= a\frac{\partial^2 U}{\partial t^2} + U\frac{\partial^2 U}{\partial t\partial x} + \frac{\partial U}{\partial t}\frac{\partial U}{\partial x} \\ &+ \frac{\partial^3 u}{\partial t\partial^2 y} + \alpha^2\frac{\partial^2 u}{\partial y^2} , \end{aligned} \tag{19}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{P}\frac{\partial^2 T}{\partial y^2} - \tau_0\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) , \tag{20}$$

where $a = 1 + \alpha^2/c^2$ and c is the speed of light given by $c^2 = 1/\epsilon_0\mu_0$.

From equation (13) the reduced boundary conditions are

$$\begin{aligned} u &= 0 , \ v = 0 , \ T = U(x,t) , \ at \ y = 0 , \\ u &= U(x,t) , \ T = 0 , \ as \ y \to \infty . \end{aligned} \tag{21}$$

3. The Method of Successive Approximations

The unsteady boundary layer equations (18), (19), and (20) will be integrated by a process of successive approximations[7]. Selecting a system of coordinates which is at rest with respect to the plate and that the magnetohydrodynamic flow of a perfectly conducting fluid moves with respect to the plane surface, we can make the assumption that the velocities u, v and the temperature possess a series solutions of the form

$$u(x,y,t) = \sum_{i=0}^{\infty} u_i(x,y,t), \ v(x,y,t) = \sum_{i=0}^{\infty} v_i(x,y,t), \ T(x,y,t) = \sum_{i=0}^{\infty} T_i(x,y,t), \tag{22}$$

where $u_i = 0(\epsilon^i)$ i-integer and ϵ is a small number.

Each term in the series (22) must satisfy the continuity equation (18)

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0 \ , \ (i = 0,1,2...) \ . \tag{23}$$

Substituting the series (22) into equations (19), and (20) and setting equal to zero terms of the same order, one obtains for finding components of the series (22)

$$a\frac{\partial^2 u_0}{\partial t^2} - \frac{\partial^3 u_0}{\partial t \partial^2 y} - \alpha^2 \frac{\partial^2 u_0}{\partial y^2} = a\frac{\partial^2 U}{\partial t^2} \ , \tag{24}$$

$$\begin{aligned} a\frac{\partial^2 u_1}{\partial t^2} - \frac{\partial^3 u_1}{\partial t \partial y^2} - \alpha \frac{\partial^2 u_1}{\partial y^2} &= U\frac{\partial^2 U}{\partial t \partial x} + \frac{\partial U}{\partial t}\frac{\partial U}{\partial x} - u_0 \frac{\partial^2 u_0}{\partial t \partial x} \\ &- v_0 \frac{\partial^2 u_0}{\partial t \partial y} - \frac{\partial u_0}{\partial t}\frac{\partial u_0}{\partial x} - \frac{\partial v_0}{\partial t}\frac{\partial u_0}{\partial y} \ , \end{aligned} \tag{25}$$

$$\frac{\partial T_0}{\partial t} + \tau_0 \frac{\partial^2 T_0}{\partial t^2} - \frac{1}{P}\frac{\partial^2 T_0}{\partial y^2} = 0 \ , \tag{26}$$

$$\begin{aligned} \frac{\partial T_1}{\partial t} + \tau_0 \frac{\partial T_1}{\partial t^2} - \frac{1}{P}\frac{\partial^2 T_1}{\partial y^2} &= -u_0 \frac{\partial}{\partial x}\left(T_0 + \tau_0 \frac{\partial T_0}{\partial t}\right) \\ &- v_0 \frac{\partial}{\partial y}\left(T_0 + \tau_0 \frac{\partial T_0}{\partial t}\right) \ . \end{aligned} \tag{27}$$

Combining equation (22) and equation (21) we have the corresponding boundary conditions

$$\begin{aligned} u_i &= 0 \ , \ i = 0,1,2... \ for \ y = 0 \ , \\ v_i &= 0 \ , \ i = 0,1,2... \ for \ y = 0 \ , \\ T_0 &= U(x,t) \ , \ T_i = 0 \ , \ i = 1,2... \ for \ y = 0 \ , \\ u_0 &= U(x,t) \ , \ u_i = 0 \ , \ i = 1,2... \ for \ y \to \infty \ , \\ T_i &= 0 \ , \ i = 0,1,2... \ for \ y \to \infty \ . \end{aligned} \tag{28}$$

In the following analysis, the first two terms in the series solution (22) will be retained. It is known fact[7] that solution is satisfactory in the phases of the non-periodic motion after it has been started from rest (till the moment when the first separation of boundary layer occurs) and in the case of periodic motion when the amplitude of oscillation is small. Higher-order approximations u_2, can be obtained easy in principle. However, the complexity of the method of successive approximations increase rapidly as higher approximations are considered. It is also known that third and higher terms series solutions give small changes in the results compared with the two terms series solutions.

4. Unsteady Magnetohydrodynamic Flow of a Perfectly Conducting Fluid when Free Stream Exponentially Depends on Time

Let us suppose that the free stream flow at large distance from the surface has the form

$$U(x,t) \ = \ e^{\omega t} V(x) \ , \tag{29}$$

where $\omega \ > \ 0$. Suppose that the solution of the differential equation (24) is of the form

$$u_0(x,y,t) \ = \ e^{\omega t} V(x) f_1'(y) \ , \tag{30}$$

using equation (23)

$$v_0(x,y,t) \ = \ - \ e^{\omega t} \frac{dV(x)}{dx} f_1(y) \ ,$$

(primes indicate derivatives with respect to y) from equations (24) and (30) and using equation (29) one obtains the differential equation of the unknown function $f_1'(y)$ and the corresponding boundary conditions

$$f_1''' \ - \ k_1^2 f_1' \ = \ - \ k_1^2 \ , \tag{31}$$

$$\begin{aligned} f_1 \ &= \ 0 \ , \ f_1' \ = \ 0 \ , \ y \ = \ 0 \ , \\ f_1' \ &= \ 1 \ , \ y \ \to \ \infty \ , \end{aligned} \tag{32}$$

where $k_1^2 \ = \ \frac{a\omega^2}{\omega \ + \ \alpha^2}$.

The solution of the system (31) and (32) is given by

$$f_1(y) \ = \ y \ + \ \frac{1}{k_1} \left(e^{-k_1 y} \ - \ 1 \right) \ . \tag{33}$$

Assuming the solution of the differential equation (25) is of the form

$$u_1(x,y,t) \ = \ e^{2\omega t} V \frac{dV}{dx} f_2'(y) \ , \tag{34}$$

and using equations (29), (30), and (34), one obtains from equations (25) and (28) the differential equation for $f_2(y)$ and the corresponding boundary conditions

$$f_2''' \ - \ k_2^2 f_2' \ = \ \omega_1 \left[\ - \ 1 \ + \ f_1'^2 \ - \ f_1 f_1'' \right] \ , \tag{35}$$

$$\begin{aligned} f_2 \ &= \ 0 \ , \ f_2' \ = \ 0 \ , \ y \ = \ 0 \ , \\ f_2' \ &= \ 0 \ , \ y \ \to \ \infty \ , \end{aligned} \tag{36}$$

where

$$\omega_1 \ = \ \frac{2\omega}{2\omega \ + \ \alpha^2} \ , \ k_2^2 \ = \ 2a\omega\omega_1 \ .$$

Using equation (33) one obtains the solution of the system (35)and (36)

$$f_2(y) \ = \ C_1 \left(C_2 \ + \ y \right) e^{-k_1 y} \ + \ C_3 e^{-k_2 y} \ + \ C_4 \ , \tag{37}$$

where

$$C_1 = \frac{\omega_1}{k_1^2 - k_2^2}, \; C_2 = \frac{2\left(2k_1^2 - k_2^2\right)}{k_1^2 - k_2^2},$$

$$C_3 = \frac{C_1}{k_2}\left[1 - k_1 C_2\right], \; C_4 = \frac{C_1}{k_2}\left[C_2\left(k_1 - k_2\right) - 1\right].$$

Suppose that the solution of the differential equation (26) is of the form

$$T_0 = e^{\omega t} V(x) \Phi_1(y), \tag{38}$$

from equation (26) and (38) one obtains the differential equation of the unknown function $\Phi_1(y)$ and the corresponding boundary conditions

$$\Phi_1'' - P\omega\left(1 + \omega\tau_0\right)\Phi_1 = 0, \tag{39}$$

$$\Phi_1 = 1, \; for \; y = 0, \; \Phi_1 = 0, \; for \; y \to \infty. \tag{40}$$

The solution of the system (39) and (40) is given by

$$\Phi_1(y) = e^{-k_3 y}, \tag{41}$$

where $k_3^2 = P\omega\left(1 + \omega\tau_0\right)$.

Assuming the solution of the differential equation (27) is of the form

$$T_1(x, y, t) = e^{2\omega t} V \frac{dV}{dx} \Phi_2(y), \tag{42}$$

and using equations (38), (42), one obtains from equations (27) and (28) the differential equation for $\Phi_2(y)$ and the corresponding boundary conditions

$$\Phi_2'' - 2\omega P\left(1 + 2\omega\tau_0\right)\Phi_2 = \frac{k_3^2}{\omega}\left(\Phi_1 f_1' - \Phi_1' f_1\right), \tag{43}$$

$$\Phi_2 = 0, \; for \; y = 0, \; \Phi_2 = 0, \; for \; y \to \infty. \tag{44}$$

The solution of the system (43) and (44) is given by

$$\Phi_2(y) = A_1 e^{-k_4 y} + A_2 e^{-(k_1 + k_3) y} + A_3\left(A_4 - y\right) e^{-k_3 y}, \tag{45}$$

where

$$A_1 = -A_2 - A_3 A_4,$$

$$k_4^2 = 2\omega P\left(1 + 2\omega\tau_0\right), \; A_2 = \frac{k_3^2\left(k_3 - k_1\right)}{\omega k_1\left[\left(k_1 + k_3\right)^2 - k_4^2\right]},$$

$$A_3 = \frac{k_3^2}{\omega\left(k_3^2 - k_4^2\right)}, \; A_4 = \frac{k_3^2}{\omega\left(k_3^2 - k_4^2\right)}\left[\frac{k_1 - k_3}{k_1 k_3} + \frac{2k_3^2}{k_3^2 - k_4^2}\right].$$

From the velocity field, we can study the wall shear stress τ, as given by

$$\tau(x, t) = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{46}$$

From equations (17), (22), (30), and (34) we obtain

$$
\begin{aligned}
\tau &= \rho U_0^2 \left(\frac{\partial u}{\partial y}\right)_{y=0} \\
&= \rho U_0^2 V(x) e^{\omega t} \left(k_1 + \epsilon \left\{e^{\omega t} \frac{dV}{dx} \left[k_1 C_1 \left(k_1 C_2 - 2\right) + k_2^2 C_3\right]\right\}\right) .
\end{aligned} \tag{47}
$$

The local friction coefficient C_f is then given by

$$C_f = \frac{\tau}{\frac{1}{2}\rho U_0^2} ,$$

It follows from equation (47)

$$C_f = 2V(x) e^{\omega t} \left(k_1 + \epsilon \left\{e^{\omega t} \frac{dV}{dx} \left[k_1 C_1 \left(k_1 C_2 - 2\right) + k_2^2 C_3\right]\right\}\right) . \tag{48}$$

The local heat flux q may be written by Fourier's law as

$$q = - k \left(\frac{\partial T}{\partial y}\right)_{y=0} . \tag{49}$$

From equations (17), (22), (38), and (42) we obtain

(50)

$$
\begin{aligned}
q(x,t) &= - k \frac{U_0}{\nu} \left(T_0 - T_\infty\right) \left(\frac{\partial T}{\partial y}\right)_{y=0} , \\
&= k V(x) e^{\omega t} \frac{U_0}{\nu} \left(T_0 - T_\infty\right) \\
&\times \left[k_3 + \epsilon \left\{e^{\omega t} \frac{dV}{dx} \left(k_4 A_1 + \left(k_1 + k_3\right) A_2 + A_3 \left(A_4 k_3 + 1\right)\right)\right\}\right] .
\end{aligned}
$$

The local heat transfer coefficient is given by

(51)

$$
\begin{aligned}
h(x,t) &= \frac{q(x,t)}{\left(T_0 - T_\infty\right)} , \\
&= k V(x) e^{\omega t} \frac{U_0}{\nu} \\
&\times \left[k_3 + \epsilon \left\{e^{\omega t} \frac{dV}{dx} \left(k_4 A_1 + \left(k_1 + k_3\right) A_2 + A_3 \left(A_4 k_3 + 1\right)\right)\right\}\right] .
\end{aligned}
$$

5. Conclusions

In order to illustrate the above results graphically the stream velocity was taken in the form

$$U_\infty(x,y,t) = C e^{\omega t} x^n ,$$

where C and n are fixed constants. The investigation of the effects of magnetic fields on the flow characteristic of a thermal relaxation of perfectly conducting fluid, past a semi-infinite plate has been carried out in the preceding section.

This enabled us to conclude the following points:

(i) The velocity is plotted against y in Figs 1 and 2 for n=0,1 ,ω=1, α=0.1, 0.2, 0.3 and for n=0 in this case V(x) = constant, and from equation (34) $u_1 = 0$. Hence, u_0 as given by equation (30) constitutes the exact solution of equation (16) i.e it is the exact solution of this problem. In these Figs. The dotes lines represent the solution of this flow when t = 0.5, and the solid lines represent the solution of its flow obtained when, t = 1.0. It can be seen from these figures that the velocity field decreases with increasing values of the Alfven velocity parameter α and an increases in the value of t leads to an increase in the velocity. Also the velocity field is found ti increase when n = 0 as compared to n = 1.

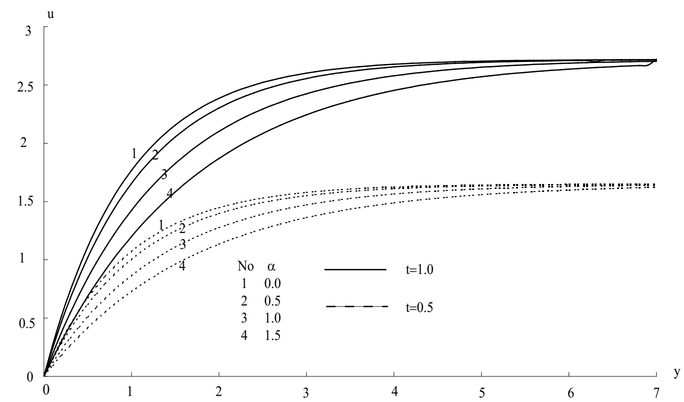

Fig. 1: Effect of Alfven velocity on velocity field, m=0

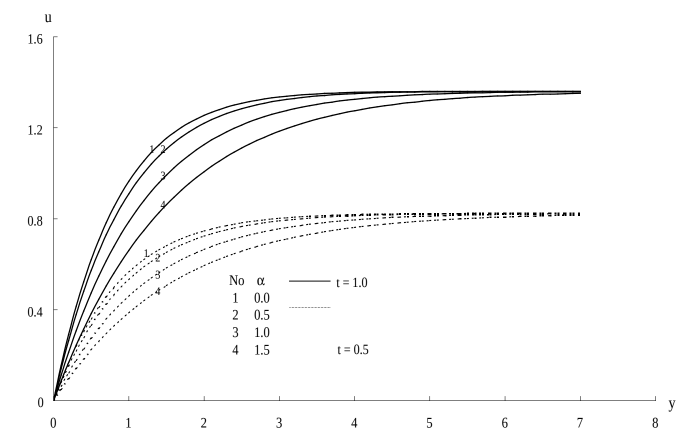

Fig. 2: Effect of Alfven velocity on velocity f

(ii) In Figs. 3 and 4 the temperature field is plotted for different value of P and two value of τ_0. The dotes lines represent the solution of this flow when τ_0 - 0.0, and the solid lines represent the solution of thus flow obtained when $\tau_0 = 0.02$. It is observed that an increases in the value of P leads to a decrease in the temperature field and an increases in the value of τ_0 leads to increase in the temperature field. It also noticed that the temperature field is found to increase when n = 0 as compared to n = 1.

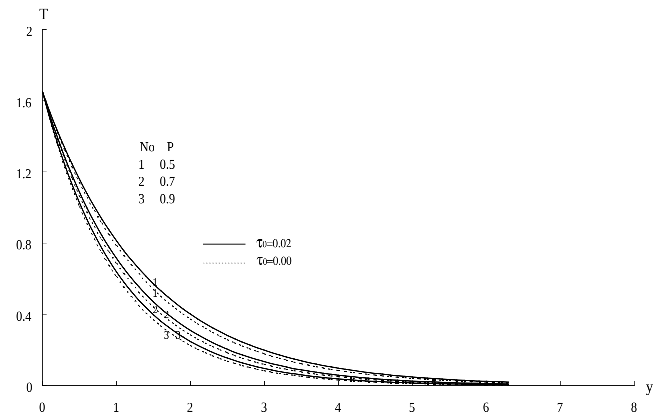

Fig. 3: Effect of Prandtl number on temperatu

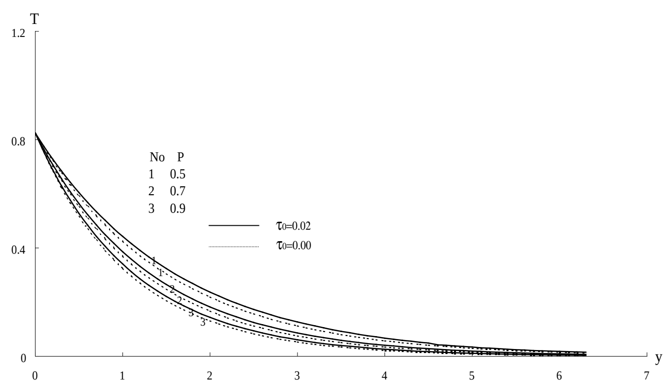

Fig. 4: Effect of Prandtl number on temperature fi

(iii) The skin friction coefficient C_f is plotted against x in Fig. 5 for different value of α, and two value of t. The effects of Alfven velocity α is observed from Fig. 5 An increase in the value of α leads to a decrease in the skin friction coefficient

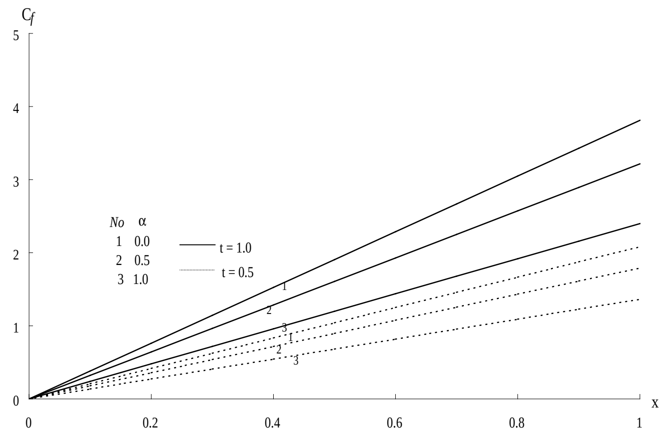

Fig. 5: Local friction factor versus

References

[1] J. N. Kapur, R. C. Srivastava :*similar solution of the boundary equation for power law fluid,* ZAMP 14, 383-389(1963).

[2] M. Kumari, G. Nath :*Conjugate MHD flow past a flat plate,* Acta Mech. 106, 215-220(1994).

[3] M. Zakaria :*Hydromagnetic fluctuating flow of a couple stress fluid through q porous medium,* Appl. Math. Computing 10, 175-191(2002).

[4] M. Zakaria :*An electromagnetic free convection flow of a micropolar fluid with relaxation time*, Korean J. comput & Appl. Math. 8, 447-458(2001).

[5] V. J. Rossow :*on flow of electrically conducting fluids over a flat plate in the presence of a transverse magnetic field,* NACA TN 3971(1957).

[6] W. Nowacki :*Dynamic problem of thermoelasticity,* Norrdhoff International, Leyden, The Netherlands(1975).

[7] H. Schlichting :*Boundary layer theory,* 6^{th}ed. McGraw. Hill. (1968).

Department of Mathematics,
Faculty of Education, El-Shatby 21526
Alexandria University,
Alexandria, Egypt.
E-mail: zakariandm@Yahoo.Com