3차원 (편파, 공간, 시간) 영역에서의 효율적인 적응 레이다 신호검출 알고리즘

An Efficient Adaptive Polarization-Space-Time Domain Radar Target Detection Algorithm

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요 약

본 논문에서는 잡음과 간섭신호가 존재하는 환경에서 편파영역에서의 레이다 신호처리 알고리즘과 공간 및 시간 영역에서의 레이다 신호처리 알고리즘을 효율적으로 결합하는 문제에 관해 고찰한다. 대부분의 직렬결합 방식들은 성능이 제한적이므로 3차원 즉, 편파, 공간, 및 시간영역에서 동시에 레이다 신호를 검출하는 적응 알고리즘에 초점을 맞춘다. 공간 및 시간 영역의 알고리즘을 3차원으로 단순 확장한 알고리즘과 달리, 본 논문에서 제안하는 알고리즘은 신호의 편파 파라미터에 대한 필터 뱅크가 불필요하다. 제안된 알고리즘에 대한 성능을 신호검출 확률과 오경보 확률에 의해 유도하고 신호의 편파정보를 이용하지 않거나 편파정보를 안다고 가정하는 알고리즘들과의 성능을 비교, 분석한다.

Abstract

This paper addresses the problem of combining adaptive polarization processing and space-time processing for further performance improvement of radar target detection in clutter and jammer environments. Since the most straightforward cascade combinations have quite limited performance improvement potentials, we focus on the development of adaptive processing in the joint polarization-space-time domain. Unlike a direct extension of some existing space-time processing algorithms to the joint domain, the processing algorithm developed in this paper does not need a potentially costly polarization filter bank to cover the unknown target polarization parameter. The performance of the new algorithm is derived and evaluated in terms of the probability of detection and the probability of false alarm, and it is compared with other algorithms that do not utilize the polarization information or assume that the target polarization is known.

I. Introduction

Radar systems design has often incorporated polarization diversity to improve detection performance, especially when a target and clutter are closely spaced in both the angle and Doppler domains. For these difficult cases, the polarization

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information provides a much needed discriminant for detection since algorithms utilizing only the angle and Doppler informations of radar returns may fail to detect the target from the clutter and jammer background. The effort on the development of polarization theory and measurement techniques may be traced back to the 1950s[4]. The polarization theory has been applied to the development of adaptive polarization filtering and mainly to the development of adaptive polarization canceller[4],[5].

There are many space-time processing algorithms developed to detect radar targets in clutter and jammer by utilizing the angle and Doppler information of radar returns. Among these algorithms, the Generalized Likelihood Ratio(GLR) detection algorithm by Kelly[2] is the most interesting due to its several attractive features such as CFAR (constant false alarm rate) and the robustness to non-Gaussian clutter. The space-time (ST) GLR algorithm, however, may be cumbersome when extended with a phased array radar with receiver polarization diversity. The extended algorithm will be called the SPST-GLR (specified polarization-space-time GLR) in this paper, since the steering vector of the SPST-GLR is specified in all three domains, i.e., polarization, angle, and Doppler domain. It can be shown that the SPST-GLR converges to the optimum processor in the joint polarization-space-time domain. However, its implementation requires the use of an additional filter bank to cover the unknown target polarization vector.

In this paper, we will develop a new GLR-based detection algorithm in the joint polarization-space-time domain. The algorithm is referred to as the PST-GLR (polarization-space-time domain GLR) algorithm. The steering vector of the PST-GLR algorithm is specified in the angle and Doppler domains but not specified in the polarization domain. The PST-GLR algorithm overcomes the implementation difficulty of the SPST-GLR algorithm and its performance is similar to that of the SPST-GLR algorithm, for which the polarization vector of the target return is assumed known.

Ⅱ. Problem Formulation

Consider now the data formulation of a phased array radar system with N_s dual channels. Each dual channel is assumed to consist of two orthogonally polarized antenna sensors, one horizontally polarized and the other vertically polarized. Each sensor in the array is assumed to receive N_t coherent pulses and produces an I and Q output pair. The I and Q output pairs are sampled to form range gate cells for each pulse. Thus the sampled outputs from each subarray may be represented by an $N_t \times N_s$ data matrix. The columns of the data matrix may be stacked to form an $N_t N_s \times 1$ column vector. Under hypothesis H_1, the target presence hypothesis, the primary data vector is assumed to contain the target component and may be written

$$\mathbf{x}_p = \begin{bmatrix} a_H\mathbf{s} + \mathbf{c}_H + \mathbf{n}_H \\ a_V\mathbf{s} + \mathbf{c}_V + \mathbf{n}_V \end{bmatrix}, \qquad (1)$$

where a_H and a_V are unknown complex constants representing the amplitude, phase, and polarization parameters of the target return. The $\mathbf{s}$ is a vector describing the target signal for both the H and V subarrays. The vectors $\mathbf{c}_H$ and $\mathbf{c}_V$ represent the clutter component and are assumed to be complex Gaussian random vectors with zero mean and covariance matrices $\mathbf{C}_{HH}$ and $\mathbf{C}_{VV}$. The $\mathbf{n}_H$ and $\mathbf{n}_V$ denote the receiver Gaussian noise vectors with zero mean and covariance $\sigma_n^2\mathbf{I}_N$, where $N = N_t N_s$. The noise vectors are assumed to be independent of each other and the clutter vectors. Under hypothesis H_1, the mean of the primary data vector $\mathbf{x}_p$ is

$$E\{\mathbf{x}_p\} = \mathbf{Sa}, \tag{2}$$

where

$$\mathbf{S} = \begin{bmatrix} \mathbf{s} & 0 \\ 0 & \mathbf{s} \end{bmatrix}, \ \mathbf{a} = \begin{bmatrix} a_H & a_V \end{bmatrix}^T. \tag{3}$$

Under hypothesis H_0, the target free hypothesis, the primary data vector $\mathbf{x}_p$ may be written

$$\mathbf{x}_p = \begin{bmatrix} \mathbf{c}_H + \mathbf{n}_H \\ \mathbf{c}_V + \mathbf{n}_V \end{bmatrix}. \tag{4}$$

Thus under H_0, the $\mathbf{x}_p$ is a Gaussian random vector with zero mean. The secondary data vectors may be written

$$\mathbf{x}_s(k) = \begin{bmatrix} \mathbf{c}_H(k) + \mathbf{n}_H(k) \\ \mathbf{c}_V(k) + \mathbf{n}_V(k) \end{bmatrix}, \ k = 1, 2, \ldots, K. \tag{5}$$

Each secondary data vector $\mathbf{x}_s(k)$, $k = 1,2,\ldots, K$, is assumed to have the same distribution as the primary data under hypothesis H_0.

Ⅲ. Polarization-Space-Time Domain GLR Test

Under hypothesis H_0, the primary data vector $\mathbf{x}_p$ and the secondary data vectors $\mathbf{x}_s(k)$, $k = 1, 2,\ldots, K$, have the same pdf. The joint pdf of $\mathbf{x}_p$ and $\mathbf{x}_s(k)$, $k = 1, 2,\ldots, K$, may be written

$$f_0(\cdot) = \left\{ \frac{1}{\pi^{2N} \|\mathbf{R}\|} \exp\left[-\mathrm{tr}(\mathbf{R}^{-1}\mathbf{T}_0) \right] \right\}^{K+1}, \tag{6}$$

where

$$\mathbf{T}_0 = \frac{1}{K+1} \left\{ \mathbf{x}_p \mathbf{x}_p^H + \sum_{k=1}^{K} \mathbf{x}_s(k) \mathbf{x}_s^H(k) \right\} \tag{7}$$

with $f_0(\cdot) = f_0\,[\mathbf{x}_p, \mathbf{x}_s(1), \ldots, \mathbf{x}_s(K)]$ and $(\cdot)^H$ denoting the complex conjugate transpose. Under hypothesis H_1, the joint pdf of $\mathbf{x}_p$ and $\mathbf{x}_s(k)$, $k = 1, 2,\ldots, K$, may be written

$$f_1(\cdot) = \left\{ \frac{1}{\pi^{2N} \|\mathbf{R}\|} \exp\left[-\mathrm{tr}(\mathbf{R}^{-1}\mathbf{T}_1) \right] \right\}^{K+1}, \tag{8}$$

where

$$\mathbf{T}_1 = \frac{1}{K+1} \left\{ (\mathbf{x}_p - \mathbf{Sa})(\mathbf{x}_p - \mathbf{Sa})^H + \sum_{k=1}^{K} \mathbf{x}_s(k) \mathbf{x}_s^H(k) \right\}, \tag{9}$$

with $f_1(\cdot) = f_1\,[\mathbf{x}_p, \mathbf{x}_s(1), \ldots, \mathbf{x}_s(K)]$. The steering vector of the PST-GLR algorithm is specified in the space and time domain but not specified in the polarization domain. Thus the PST -GLR is derived by maximizing Eq.(6) and Eq.(8) over the unknown covariance matrix $\mathbf{R}$ and target polarization vector $\mathbf{a}$. Maximizing $f_0(\cdot)$ and $f_1(\cdot)$ over $\mathbf{R}$ gives

$$\max_{\mathbf{R}} f_0(\cdot) = \left\{ \frac{1}{(e\pi)^{2N} \|\mathbf{T}_0\|} \right\}^{K+1}$$
$$\max_{\mathbf{R}} f_1(\cdot) = \left\{ \frac{1}{(e\pi)^{2N} \|\mathbf{T}_1\|} \right\}^{K+1}, \tag{10}$$

where $\mathbf{T}_0$ and $\mathbf{T}_1$ are the maximum likelihood estimates of $\mathbf{R}$ under hypotheses H_0 and H_1, respectively.

Let the likelihood ratio test be denoted by λ. Then we have

$$\lambda = \frac{\max_{(\mathbf{R},\mathbf{a})} f_1(\cdot)}{\max_{(\mathbf{R},\mathbf{a})} f_0(\cdot)} = \max_{\mathbf{a}} \left[\frac{\|\mathbf{T}_0\|}{\|\mathbf{T}_1\|}\right]^{K+1} = \frac{\|\mathbf{T}_0\|}{\min_a \|\mathbf{T}_1\|}. \quad (11)$$

We shall use the following determinant equality [7]

$$(K+1)^{2N}\|\mathbf{T}_0\| = \|\hat{\mathbf{R}}\|\left(1+\mathbf{x}_p^H \hat{\mathbf{R}}^{-1}\mathbf{x}_p\right)$$
$$(K+1)^{2N}\|\mathbf{T}_1\| = \|\hat{\mathbf{R}}\|\left\{1+(\mathbf{x}_p-\mathbf{Sa})^H \hat{\mathbf{R}}^{-1}(\mathbf{x}_p-\mathbf{Sa})\right\}, \quad (12)$$

where

$$\mathbf{R} = \sum_{k=1}^{K} \mathbf{x}_s(k)\mathbf{x}_s^H(k). \quad (13)$$

We see that minimizing $\|\mathbf{T}_1\|$ is equivalent to minimizing f given by

$$f = (\mathbf{x}_p-\mathbf{Sa})^H \hat{\mathbf{R}}^{-1}(\mathbf{x}_p-\mathbf{Sa}) = \left|\mathbf{C}(\mathbf{x}_p-\mathbf{Sa})\right|^2, \quad (14)$$

where $\hat{\mathbf{R}}^{-1} = \mathbf{C}^H\mathbf{C}$ and $|\cdot|$ denotes the Euclidean norm. Thus the least squares (LS) solution of $\mathbf{a}$ may be written

$$\hat{\mathbf{a}} = \left(\mathbf{S}^H\hat{\mathbf{R}}^{-1}\mathbf{S}\right)^{-1}\mathbf{S}^H\hat{\mathbf{R}}^{-1}\mathbf{x}_p. \quad (15)$$

Substituting this into Eq.(11) and using $\eta_o = (\lambda_o - 1)/\lambda_o$, we have the PST-GLR test

$$\eta = \frac{\mathbf{x}_p^H\hat{\mathbf{R}}^{-1}\mathbf{S}\left(\mathbf{S}^H\hat{\mathbf{R}}^{-1}\mathbf{S}\right)^{-1}\mathbf{S}^H\hat{\mathbf{R}}^{-1}\mathbf{x}_p}{1+\mathbf{x}_p^H\hat{\mathbf{R}}^{-1}\mathbf{x}_p} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_o. \quad (16)$$

Note that the above PST-GLR test may be reduced to the ST-GLR test by replacing the matrix $\mathbf{S}$ in Eq.(16) with the vector $\mathbf{s}$, the steering vector specified in the angle and Doppler domains.

The steering vector of the SPST-GLR (specified polarization-space-time domain GLR), a simple extension of the ST-GLR is specified in the polarization, space, and time domain and its test statistic is given by

$$\eta = \frac{\left|\mathbf{s}_1^H\hat{\mathbf{R}}^{-1}\mathbf{x}_p\right|^2}{\mathbf{s}_1^H\hat{\mathbf{R}}^{-1}\mathbf{s}_1\left(1+\mathbf{x}_p^H\hat{\mathbf{R}}^{-1}\mathbf{x}_p\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_o, \quad (17)$$

where $\mathbf{s}_1 = \mathbf{Sa}$ with $\mathbf{a}$ corresponding to true target polarization vector. When $\hat{\mathbf{a}}$ in Eq.(15) is used to replace $\mathbf{a}$ in the above expression, we obtain the PST-GLR. Thus the PST-GLR, which incorporates the estimation of the target polarization into its detector, is a generalization of the SPST -GLR.

Ⅳ. Detection Performance of the PST-GLR

The detection performance of the PST-GLR is derived and evaluated in terms of the probability of detection P_D and the probability of false alarm P_F by considering the statistical properties of the random variable η in Eq.(16).

4-1 Probability of false alarm

As in [2], to simplify our derivations, we shall first whiten the random vectors in η and then apply unitary transforms to the whitened vectors. Let

$$\mathbf{z}_p = \mathbf{R}^{-1/2}\mathbf{x}_p, \quad (18)$$

and

$$\mathbf{z}_s(k) = \mathbf{R}^{-1/2}\mathbf{x}_s(k),\ k = 1,2,\ldots,K. \tag{19}$$

Then $\mathbf{z}_p$ and $\mathbf{z}_s(k)$ are $2N\times 1$ complex Gaussian random vectors with zero mean and covariance matrix $\mathbf{I}_{2N}$. With this whitening transformation, we obtain

$$\mathbf{x}_p^H \hat{\mathbf{R}}^{-1}\mathbf{x}_p = \mathbf{z}_p^H \mathbf{Q}^{-1}\mathbf{z}_p, \tag{20}$$

where

$$\mathbf{Q} = \mathbf{R}^{-1/2}\hat{\mathbf{R}}\mathbf{R}^{-1/2} = \sum_{k=1}^{K}\mathbf{z}_s(k)\mathbf{z}_s^H(k). \tag{21}$$

Let $\mathbf{T} = \mathbf{R}^{-1/2}\mathbf{S}(\mathbf{S}^H\mathbf{R}^{-1}\mathbf{S})^{-1/2}$. Using $\mathbf{T}$, $\mathbf{Q}$, and $\mathbf{z}_p$, the PST-GLR test in Eq.(16) may be rewritten

$$\eta = \frac{\mathbf{z}_p^H\mathbf{Q}^{-1}\mathbf{T}\left(\mathbf{T}^H\mathbf{Q}^{-1}\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{Q}^{-1}\mathbf{z}_p}{1+\mathbf{z}_p^H\mathbf{Q}^{-1}\mathbf{z}_p} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_0 \tag{22}$$

Next, Let $\mathbf{U}$ be a unitary matrix such that

$$\mathbf{UT} = \mathbf{T}_1 = [\mathbf{I}_2\ \ 0]^T. \tag{23}$$

Appying the same unitary transform to $\mathbf{z}_p$ and $\mathbf{Q}$, we obtain

$$\eta = \frac{\mathbf{z}_p^H\mathbf{Q}^{-1}\mathbf{T}\left(\mathbf{T}^H\mathbf{Q}^{-1}\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{Q}^{-1}\mathbf{z}_p}{1+\mathbf{z}_p^H\mathbf{Q}^{-1}\mathbf{z}_p} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_o, \tag{24}$$

where we have used $\mathbf{T}$, $\mathbf{z}_p$ and $\mathbf{Q}$ again instead of $\mathbf{T}_1$, $\mathbf{z}_{p1}$, and $\mathbf{Q}_1$ only for convenience in notation.

Let

$$\mathbf{z}_p = \left[\mathbf{z}_A^T\ \mathbf{z}_B^T\right]^T, \tag{25}$$

and

$$\mathbf{Q} = \begin{bmatrix}\mathbf{Q}_{AA} & \mathbf{Q}_{AB}\\ \mathbf{Q}_{BA} & \mathbf{Q}_{BB}\end{bmatrix} \tag{26}$$

where $\mathbf{z}_A$ is a 2×1 vector and $\mathbf{Q}_{AA}$ is a 2×2 matrix. Also, let

$$\mathbf{Q}^{-1} = \begin{bmatrix}\mathbf{P}_{AA} & \mathbf{P}_{AB}\\ \mathbf{P}_{BA} & \mathbf{P}_{BB}\end{bmatrix}. \tag{27}$$

Then

$$\mathbf{T}^H\mathbf{Q}^{-1}\mathbf{T} = [\mathbf{I}_2\ \ 0]\begin{bmatrix}\mathbf{Q}_{AA} & \mathbf{Q}_{AB}\\ \mathbf{Q}_{BA} & \mathbf{Q}_{BB}\end{bmatrix}\begin{bmatrix}\mathbf{I}_2\\ 0\end{bmatrix} = \mathbf{P}_{AA}, \tag{28}$$

and

$$\mathbf{T}^H\mathbf{Q}^{-1}\mathbf{z}_p = \mathbf{P}_{AA}\mathbf{z}_A + \mathbf{P}_{AB}\mathbf{z}_B. \tag{29}$$

From the partitioned matrix inversion lemma, we have

$$\begin{aligned}\mathbf{P}_{AA} &= \left(\mathbf{Q}_{AA} - \mathbf{Q}_{AA}\mathbf{Q}_{BB}^{-1}\mathbf{Q}_{BA}\right)^{-1}\\ \mathbf{P}_{BA} &= -\mathbf{Q}_{BB}^{-1}\mathbf{Q}_{BA}\mathbf{P}_{AA}\\ \mathbf{P}_{AB} &= -\mathbf{P}_{AA}\mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}.\end{aligned} \tag{30}$$

Using the partitioned matrices and vectors yields

$$\mathbf{T}^H\mathbf{Q}^{-1}\mathbf{z}_p = \mathbf{P}_{AA}(\mathbf{z}_A - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{z}_B), \tag{31}$$

and

$$\begin{aligned}\mathbf{z}_p^H\mathbf{Q}^{-1}\mathbf{z}_p &= \mathbf{z}_A^H\mathbf{P}_{AA}\mathbf{z}_A + \mathbf{z}_B^H\mathbf{P}_{BA}\mathbf{z}_A\\ &\quad + \mathbf{z}_A^H\mathbf{P}_{AB}\mathbf{z}_B + \mathbf{z}_B^H\mathbf{P}_{BB}\mathbf{z}_B.\end{aligned} \tag{32}$$

Since $\mathbf{P}_{AA}$ is positive definite, we may define $\mathbf{P}_{AA} = A^H A$. Then

$$\mathbf{z}_p^H\mathbf{Q}^{-1}\mathbf{z}_p = \left|\mathbf{A}(\mathbf{z}_A - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{z}_B)\right|^2 + \mathbf{z}_B^H\mathbf{Q}_{BB}^{-1}\mathbf{z}_B. \tag{33}$$

Substituting the above expression into the test statistic in Eq.(24), we obtain

$$\eta = \frac{X}{1+X+\Sigma_B} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_0, \tag{34}$$

where $X = \left|\mathbf{A}(\mathbf{z}_A - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{z}_B)\right|^2$ and $\Sigma_B = \mathbf{z}_B^H \mathbf{Q}_{BB}^{-1}\mathbf{z}_B$.

Rearranging the above expression gives

$$X \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\eta_0}{1-\eta_0}(1+\Sigma_B). \tag{35}$$

The X above may be rewritten

$$X = (\mathbf{z}_A - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{z}_B)^H (\mathbf{Q}_{AA} - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{Q}_{BA})^{-1} (\mathbf{z}_A - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{z}_B), \tag{36}$$

where

$$\mathbf{Q}_{AA} - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{Q}_{BA} = \sum_{k=1}^{K} \left|\mathbf{z}_{sA}(k) - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{z}_{sB}(k)\right|^2. \tag{37}$$

Let

$$\mathbf{y}_p = \mathbf{z}_A - \mathbf{Q}_{AB}\mathbf{Q}_{BB}^{-1}\mathbf{z}_B.$$
$$\mathbf{y}_s(k) = \mathbf{z}_{sA}(k) - \sum_{i=1}^{K} \mathbf{z}_{sA}(i)\mathbf{z}_{sB}^H(i)\mathbf{Q}_{BB}^{-1}\mathbf{z}_{sB}(k). \tag{38}$$

Using $\mathbf{y}_p$ and $\mathbf{y}_s(k)$, the test statistic in Eq.(35) may be written

$$X = \mathbf{y}_p^H \left[\sum_{k=1}^{K} \mathbf{y}_s(k)\mathbf{y}_s^H(k)\right]^{-1} \mathbf{y}_p \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\eta_0}{1-\eta_0}(1+\Sigma_B). \tag{39}$$

Since $\mathbf{y}_p$ and $\mathbf{y}_s(k)$ are linearly transformed vectors of $\mathbf{x}_p$ and $\mathbf{x}_s(k)$, respectively, which are Gaussian vectors, they themselves are also Gaussian vectors. Let $g(k) = \mathbf{z}_{sB}^H(k)\mathbf{Q}_{BB}^{-1}\mathbf{z}_B$ and $G(i,k) = \mathbf{z}_{sB}^H(i)\mathbf{Q}_{BB}^{-1}\mathbf{z}'_{sB}(k)$, $i,k = 1,2,\ldots,K$. The $g(k)$ and $G(i, k)$ represent the elements of the $K\times 1$ vector $\mathbf{g}$ and the $K\times K$ matrix $\mathbf{G}$, respectively. It can be shown that

$$\mathbf{G}\mathbf{g} = \mathbf{g}$$
$$\mathbf{G}^2 = \mathbf{G}$$
$$\mathrm{Tr}(\mathbf{G}) = 2(N-1). \tag{40}$$

Using $g(k)$ and $G(i, k)$ we obtain

$$\mathbf{y}_p = \mathbf{z}_A - \sum_{k=1}^{K} \mathbf{z}_{sA}(k)g(k), \tag{41}$$

and

$$\mathbf{y}_s(k) = \mathbf{z}_{sA}(k) - \sum_{i=1}^{K} \mathbf{z}_{sA}(i)G(i,k). \tag{42}$$

The statistical properties of $\mathbf{y}_p$ and $\mathbf{y}_s(k)$ conditioned on the $\mathbf{B}$ components, are as follows

$$E\{\mathbf{y}_s(k) \mid \mathbf{B}\} = E\{\mathbf{y}_p \mid \mathbf{B}\} = 0 \tag{43}$$

$$E\{\mathbf{y}_p\mathbf{y}_s^H(k) \mid \mathbf{B}\} = 0 \tag{44}$$

$$E\{\mathbf{y}_p\mathbf{y}_p^H \mid \mathbf{B}\} = (1+\Sigma_B)\mathbf{I}_2, \tag{45}$$

and

$$E\{\mathbf{y}_s(k)\mathbf{y}_s^H(n) \mid \mathbf{B}\} = \left[\delta(n,k) - G(n,k)\right]\mathbf{I}_2, \tag{46}$$

where $\delta(n, k)$ is the Kronecker delta, i.e.,

$$\delta(n,k) = \begin{cases} 1; & n = k \\ 0; & n \neq k. \end{cases}$$

From the above expressions, we see that $\mathbf{y}_p$ and $\mathbf{y}_s(k)$ are 2×1 Gaussian random vectors with zero mean and are independent of each other. It should also be noted from Eq.(46) that $\mathbf{y}_s(k)$, $k = 1,2,\ldots, K$, have different covariance matrices.

Let $\mathbf{v}_p = \mathbf{y}_p/(1+\Sigma_B)^{1/2}$. Then $E\{\mathbf{v}_p \mid \mathbf{B}\} = 0$ and $E\{\mathbf{v}_p\mathbf{v}_p^H\} = \mathbf{I}_2$. By using $\mathbf{v}_p$, the test statistic in Eq.(35) may be written

$$X = \mathbf{v}_p^H \left[\sum_{k=1}^{K} \mathbf{y}_s(k)\mathbf{y}_s^H(k)\right]^{-1} \mathbf{v}_p \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\eta_0}{1-\eta_0}. \tag{47}$$

Note that there is no random variable in the X above that depends on the **B** components. Thus, the probability of false alarm to be derived from the above equation will no longer be conditioned and is the final result. In the above equation, $\mathbf{D}=\Sigma_{k=1}^{K}\mathbf{y}_s(k)\mathbf{y}_s^H(k)$ may be written

$$D=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} \Sigma_{k=1}^{K}|y_{s1}(k)|^2 & \Sigma_{k=1}^{K}y_{s1}(k)y_{s2}^*(k) \\ \Sigma_{k=1}^{K}y_{s1}^*(k)y_{s2}(k) & \Sigma_{k=1}^{K}|y_{s2}(k)|^2 \end{bmatrix}, \quad (48)$$

where we have used

$$\mathbf{y}_s(k)=[y_{s1}(k) \quad y_{s2}(k)]^T,\ k=1,2,\ldots,K. \quad (49)$$

Let $\mathbf{w}_{sj}=[y_{sj}(1),\ y_{sj}(2),\ldots,\ y_{sj}(k)]^T,\ \ j=1,2$. The $\mathbf{w}_{sj}$ is a Gaussian random vector with zero mean and covariance matrix $\mathbf{Y}=\mathbf{I}-\mathbf{G}$. Since $\mathbf{Y}$ is idem-potent, i.e., $\mathbf{Y}^2=\mathbf{Y}$, the eigenvalues of $\mathbf{Y}$ are either 0's or 1's. Also, since $\mathrm{Tr}(\mathbf{G})=2N-2$, $\mathrm{Tr}(\mathbf{Y})=K-2N+2$. Thus $\mathbf{Y}$ has $K-2N+2$ unit eigenvalues. Therefore, $\mathbf{Y}$ may be written

$$\mathbf{Y}=\sum_{k=1}^{K-2N+2}\mathbf{u}_k\mathbf{u}_k^H, \quad (50)$$

wherec $\mathbf{u}_k$ are the eigenvectors of $\mathbf{Y}$ corresponding to the unit eigenvalues. Let $\mathbf{w}_j=\mathbf{Y}^{-1/2}\mathbf{w}_{sj},\ j=1,2$. Then $E\{\mathbf{w}_j|\mathbf{B}\}=0$ and $E\{\mathbf{w}_j\mathbf{w}_j^H|\mathbf{B}\}=\mathbf{I}_K$. Thus

$$d_{11}=\mathbf{w}_{s1}^H\mathbf{w}_{s1} = \sum_{k=1}^{K-2N+2}|v_{s1}(k)|^2, \quad (51)$$

where $v_{s1}(k)=\mathbf{u}_k^H\mathbf{w}_1$. Note that $v_{s1}(k),\ k=1,2,\ldots,K-2N+2$, are i.i.d. Gaussian random variables with zero mean and unit variance. Using the same procedure, we have

$$\mathbf{D}=\sum_{k=1}^{K-2N+2}\mathbf{v}_s(k)\mathbf{v}_s^H(k), \quad (52)$$

where $\mathbf{v}_s(k)=[v_{s1}(k) \quad v_{s2}(k)]^T$. It is easy to show that $\mathbf{v}_s(k),\ k=1,2,\ldots,K-2N+2$, are i.i.d. Gaussian random vectors with zero mean and covariance matrix $\mathbf{I}_2$. Thus, the test statistic in Eq.(47) may now be written

$$X=\mathbf{v}_p^H\left[\sum_{k=1}^{K-2N+2}\mathbf{v}_s(k)\mathbf{v}_s^H(k,)\right]^{-1}\mathbf{v}_p \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\eta_0}{1-\eta_0}. \quad (53)$$

Similar to the derivations of the T^2-statistic [8], we can show that the above test statistic is equivalent to

$$t \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\eta_0}{1-\eta_0}\tau=(\lambda_0-1)\tau, \quad (54)$$

where $2t$ has the central χ^2 distribution with d.f. (degree of freedom) 4. The pdf of t is

$$f_t(t|H_0)=t\exp(-t). \quad (55)$$

The 2τ has the central χ^2 distribution with d.f. $2(K-2N+1)$. The pdf of τ is

$$f_\tau(\tau|H_0)=\frac{\tau^{K-2N}}{(K-2N)!}\exp(-\tau). \quad (56)$$

Therefore, the probability of false alarm is

$$P_F=P(t>(\lambda_0-1)\tau|H_0) = \frac{(1-\eta_0)^{K-2N+1}}{(K-2N)!}\sum_{j=1}^{2}(K-2N+2-j)!\eta_0^{2-j}. \quad (57)$$

Note from the above result that, like other GLR algorithms, the PST-GLR algorithm has the dedired CFAR feature.

4-2 Derivation of the Probability of Detection

Under H_1, the test statistic may also be written

$$X = \mathbf{v}_p^H \left[\sum_{k=1}^{K-2N+2} \mathbf{v}_s(k)\mathbf{v}_s^H(k) \right]^{-1} \mathbf{v}_p \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\eta_0}{1-\eta_0}. \quad (58)$$

The statistic of $\mathbf{v}_s(k)$, $k=1,2,\ldots, K-2N+2$, is the same as under H_0. The mean $\boldsymbol{v}$ of $\mathbf{v}_p$ however, is different and is

$$\mathbf{v} = \frac{1}{(1+\Sigma_B)^{1/2}} (\mathbf{S}^H \mathbf{R}^{-1} \mathbf{S})^{1/2} \mathbf{a}. \quad (59)$$

Again, similar to the derivation of the T^2-statistic in [8], we can show that the test statistic in Eq.(58) is equivalent to

$$t \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\eta_0}{1-\eta_0}\tau = (\lambda_0 - 1)\tau, \quad (60)$$

where $2t$ has the non-central χ^2 distribution with d.f. 4 and the non-centrality parameter μ with

$$\mu = \frac{1}{(1+\Sigma_B)} \left| \left(\mathbf{S}^H \mathbf{R}^{-1} \mathbf{S}\right)^{1/2} \mathbf{a} \right|^2. \quad (61)$$

The pdf of t is

$$f_t(t \mid \mathbf{B}, \mathbf{a}, H_1) = \sum_{m=0}^{\infty} \frac{\mu^m}{m!} \exp(-\mu) \frac{t^{m+1}}{(m+1)!} \exp(-t). \quad (62)$$

The 2τ has the same distribution as under H_0. Thus the probability of detection conditioned on the $\mathbf{B}$ components and $\mathbf{a}$ is given by

$$\begin{aligned} P_{D|\mathbf{B},\mathbf{a}} &= P(t > (\lambda_0 - 1)\tau \mid \mathbf{B}, \mathbf{a}, H_1) \\ &= 1 - \sum_{j=0}^{K-2N} \frac{\exp(-\mu)}{(K-2N-j)!} (1-\eta_0)^{K-2N-j} \\ &\times \sum_{m=0}^{\infty} \frac{(K-2N+1-j+m)!}{m!(m+1)!} \mu^m \eta_0^{m+2}. \end{aligned} \quad (63)$$

This equation may be expressed as a finite sum formula by using the result of Kelly [6], i.e.,

$$\begin{aligned} P_{D|\mathbf{a},\rho} &= 1 - \eta_0 (1-\eta_0)^{K-2N+1} \\ &\times \sum_{j=1}^{K-2N+1} \binom{K-2N+2}{j+1} \left(\frac{\eta_0}{1-\eta_0} \right)^j \\ &\times \exp[-\alpha\rho(1-\eta_0)] \sum_{m=0}^{j-1} \frac{1}{m!} [\alpha\rho(1-\eta_0)]^m. \end{aligned} \quad (64)$$

Note that the probability of detection above is conditioned on $\mathbf{B}$ through μ, where μ is a function of Σ_B.

Let $\alpha = \left|(\mathbf{S}\mathbf{R}^{-1}\mathbf{S})^{1/2}\mathbf{a}\right|^2$ and $\rho = 1/(1+\Sigma_B)$. Then

$$\mu = \alpha\rho, \quad (65)$$

where ρ represents a loss factor and has the beta distribution [2], [3]

$$f_\rho(\rho) = \frac{K!}{(2N-3)!(K-2N+2)!} (1-\rho)^{2N-3} \rho^{K-2N+2}. \quad (66)$$

Thus we obtain the probability of detection

$$P_{D|\mathbf{a}} = \int_0^1 P_{D|\mathbf{a},\rho} f_\rho(\rho) d\rho. \quad (67)$$

For a case of random target return, the randomness of $\mathbf{a}$ affects the probability of detection only through α in Eq.(64). The detection performance for this case can be obtained by taking a statistical average $\mathbf{P}_{D/\mathbf{a}}$ of over $\mathbf{a}$. The $\mathbf{a} = [a_H \ a_V]^T$ is herein assumed a Gaussian random vector with zero mean and covariance $\sigma_s^2 \mathbf{I}$, where σ_s^2 represents the signal power. Let

$$\begin{aligned} \alpha &= \left|(\mathbf{S}^H \mathbf{R}^{-1} \mathbf{S})^{1/2} \mathbf{a}\right|^2 \\ &= \mathbf{a}^H (\mathbf{S}^H \mathbf{R}^{-1} \mathbf{S}) \mathbf{a}. \end{aligned} \quad (68)$$

Since $\mathbf{S}^H \mathbf{R}^{-1} \mathbf{S}$ is positive definite, we have

$$\mathbf{S}^H \mathbf{R}^{-1} \mathbf{S} = \sum_{j=1}^{2} \lambda_j \mathbf{u}_j \mathbf{u}_j^H, \quad \text{with} \quad \lambda_1 > \lambda_2 > 0, \quad (69)$$

where λ_1 and $\mathbf{u}_j$ are the j-th eigenvalue and eigenvector of $\mathbf{S}^H \mathbf{R}^{-1}\mathbf{S}$, respectively.
Thus

$$\alpha = \sum_{j=1}^{2} \lambda_j |v_j|^2, \tag{70}$$

where $v_j = \mathbf{u}_j^H \mathbf{a}$ is a Gaussian random variable with zero mean and variance σ_s^2. The pdf of α is

$$f_\alpha(\alpha) = \sum_{j=1}^{2} \frac{b_j}{\lambda_j \sigma_s^2} \exp\left[-\frac{\alpha}{\lambda_j \sigma_s^2}\right], \tag{71}$$

where $b_1 = \lambda_1/(\lambda_1 - \lambda_2)$ and $b_2 = \lambda_2/(\lambda_2 - \lambda_1)$. For the random target amplitude case, $P_{D|\rho}$ is thus given by

$$\begin{aligned} P_{D|\rho} &= \int_0^\infty P_{D|\rho,\alpha} f_\alpha(\alpha) d\alpha \\ &= 1 - \frac{\eta_0}{\sigma_s^2(\lambda_1 - \lambda_2)}(1-\eta_0)^{K-2N+1} \\ &\times \sum_{j=1}^{K-2N+1} \binom{K-2N+2}{j+1} \left(\frac{\eta_0}{1-\eta_0}\right)^j \\ &\times \sum_{m=0}^{j-1} \left[\rho(1-\eta_0)\right]^m \left(\frac{1}{\gamma_1^{m+1}} - \frac{1}{\gamma_2^{m+1}}\right), \end{aligned} \tag{72}$$

where $\gamma_k = \rho(1-\eta_0) + 1/(\sigma_s^2 \lambda_k)$, $k = 1, 2$. Using the above results, we have

$$P_D = \int_0^1 P_{D|\rho} f_\rho(\rho) d\rho, \tag{73}$$

where $f_\rho(\rho)$ is given by Eq.(66).

V. Numerical Results and Discussion

Consider a radar system with a linear array consisting of N_s dual channels with equal spacing d. The total number of sensors in the array for the PST-GLR or the SPST-GLR is $2N_s$. Among the $2N_s$ sensors, both the H and V channel subarrays have N_s sensors. The array used for the ST-GLR is either the H channel subarray or the V channel subarray. In our numerical examples, we assumed $N_s = 4$ and $N_t = 5$ coherent pulses.

The clutter is assumed to have a two dimensional multi-peak Gaussian shaped power spectral density fuction in a stationary platform case, i.e.,

$$P_c(f_t, f_s) = \sum_{l=1}^{L} \frac{\sigma_{cl}^2}{2\pi\sigma_{ft}\sigma_{fs}} \exp\left[-\left[\frac{f_t^2}{2\sigma_{ft}^2} + \frac{(f_s - f_{csl})^2}{2\sigma_{fs}^2}\right]\right], \tag{74}$$

where L is the number of clutter peaks and σ_{cl}^2 is the power of the l-th clutter. Thus the total clutter power is $\sigma_c^2 = \sum_{l=1}^{L} \sigma_{cl}^2$. The σ_{fs} and σ_{ft} are parameters controlling the spread of the clutter spectrum in the angle and Doppler domains. The σ_{fs} and σ_{ft} are assumed to be same for each clutter peak in our simulation examples. The f_{csl} is the center frequency of the l-th clutter peak in the angle domain. The center frequencies of the clutter peaks in the Doppler domain are zeros in a stationary platform case. The spatial and temporal correlation functions may be obtained by inverse Fourier transform of the power spectral density function and specified by

$$r_s(n_s) = \sum_{l=1}^{L} \sigma_{cl}^2 \exp\left[-2(\pi\sigma_{cs}n_s)^2 + j2\pi n_s f_{csl}\right], \tag{75}$$

and

$$r_t(n_t) = \exp\left[-2(\pi\sigma_{ct}n_t)^2\right], \tag{76}$$

where n_s and n_t represent the numerical orders of the antenna element and pulse, respectively. In the case of moving platform, the clutter peaks are shifted in Doppler domain together with the spectral spread due to the angle-Doppler coupling effect. By considering platform motion, the correlation functions of the clutter may be expressed as [9]

$$r(\Delta n_t, \Delta n_s) = \sum_{l=1}^{L} \sigma_{cl}^2 \exp\left[-2\{\pi\sigma_{cs}(\Delta n_s + m_0 \Delta n_t)\}^2\right]$$
$$\times \exp\left[-j2\pi(\Delta n_s + m_0 \Delta n_t) f_{csl}\right]$$
$$\times \exp[-2(\pi\sigma_{ct}\Delta n_t)^2] \quad (77)$$

where $m_0 = 2v_a/(d \text{ PRF})$, $\Delta n_t = n_t - n_t^{'}$, and $\Delta n_s = n_s - n_s^{'}$. The v_a and PRF represent the speed of the platform and pulse repetition frequency of the radar system, respectively.

In our numerical examples, $L = 3$ and $m_0 = 1$ are chosen. The statistical average of the clutter polarization is assumed circulary polarized with the phase of the H channel clutter leading the phase of the V channel clutter by 90 degrees. The σ_{c2}^2 in Eq.(74) is determined such that $10\log(\sigma_{c2}^2/\sigma_n^2)$ =40 dB. The σ_{cl}^2, l=1,3, are 20 dB less than σ_{c2}^2. Thus the clutter-to-noise ratio (CNR), defined as CNR= σ_c^2/σ_n^2, is approximately 40 dB. We also assume that $\{(f_{cs1}, f_{ct1}), (f_{cs2}, f_{ct2}), (f_{cs3}, f_{ct3})\} = \{(-0.3, 0.0), (0.0, 0.0), (0.3, 0.0)\}$, $\sigma_{ft} = 0.01$, and $\sigma_{fs} = 0.025$. The clutter plus noise spectrum in moving platform case is depicted in Fig. 1. For the given array, the $(n_t\ n_s)$th component of the signal vector s may be written

$$\mathbf{s}_{n_t n_s} = \exp[j2\pi(n_t - 1) f_{st}]$$
$$\times \exp[j2\pi\{(n_s - 1) + m_0(n_t - 1)\} f_{ss}], \quad (78)$$

where $f_{st} = 2\nu(\lambda \text{ PRF})$ and $f_{ss} = d\sin\theta/\lambda$ are the normalized target Doppler and spatial frequencies, respectively, with ν denoting the radial velocity of the target and θ the direction of arrival of the target return with respect to the broadside of the array. In our numerical examples, the normalized Doppler and spatial frequencies of the target are chosen 0.1 and 0.0, i.e., f_{st} = 0.1 and f_{ss} = 0.0, respectively.

First, we evaluate the detection performances of

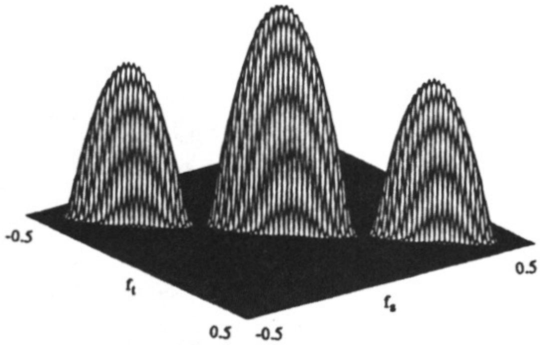

Fig. 1. Clutter-plus-noise spectrum.

the algorithms for the case of deterministic target return. For easy comparison with the ST-GLR, the power of the target return is assumed the same for both the H and V channels, i.e., $|a_H| = |a_V|$.

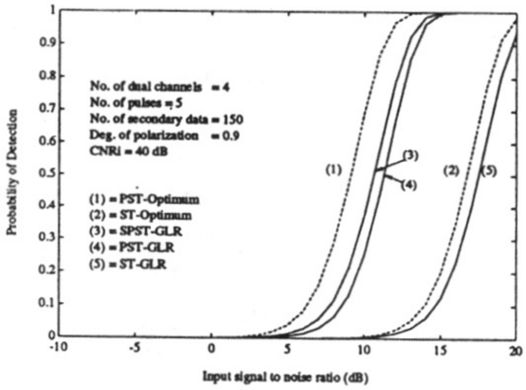

Fig. 2. Detection performance of the algorithms for SD = 180 degrees.

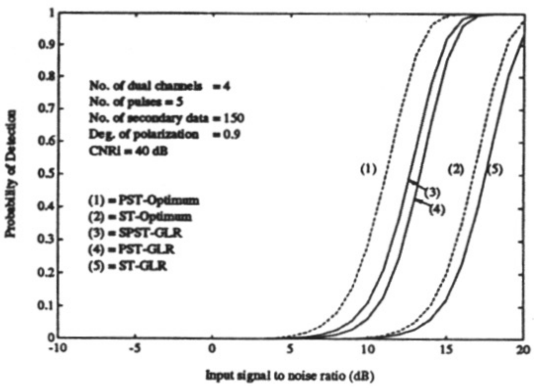

Fig. 3. Detection performance of the algorithms for SD = 90 degrees.

The following curves are obtained by setting the probability of false alarm $P_F = 10^{-5}$. Fig. 2~4 show the probability of detection P_D of the ST-GLR, PST-GLR, and SPST-GLR as a function of SNR. The target polarization is assumed known for the SPST-GLR. In Fig. 2, the polarization state of the target return is assumed to be circular but the phase of the V channel leads the phase of the H channel by 90 degrees such that the spherical distance (SD) between the target and the clutter is 180°. In Fig. 3, the polarization state of the target return is assumed linear, where the spherical distance is 90°. In Fig. 4, the polarization state of the target return is assumed to be separated from that of clutter by 5°. The figures show that the detection performance of the PST-GLR and SPST-GLR is significantly better than that of the ST-GLR when the target and clutter are sufficiently separated in polarization domain. Note also that the performance of each detector is close to that of the corresponding optimum detector. Also, note from Fig. 4 that when the target polarization is almost the same as the clutter polarization, the detection performance of the PST-GLR and SPST-GLR is almost same as that of the ST-GLR. For this case, the extra polarization cannot be used to improve the probability of detecting the target. Yet for this

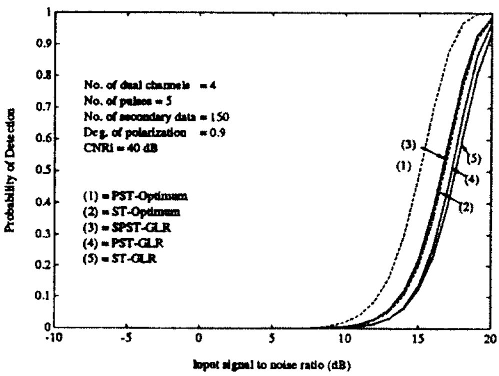

Fig. 4. Detection performance of the algorithms for SD = 5 degrees.

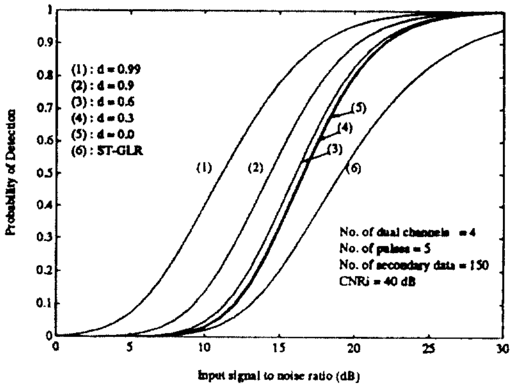

Fig. 5. Detection performance of the algorithms v.s degree of polarization for a random signal case.

case, the adaptive polarization canceller based detectors may fail completely to detect the target because the target power will be cancelled seriously together with the clutter power. It is also interesting to compare the performance of the PST-GLR with the SPST-GLR, which assumes that the target polarization is known. We note that the performance degradation of the PST-GLR due to estimating the target polarization is very small. This result also occurs in other clutter environments.

In Fig. 5, we compare the detection performances of the PST-GLR with that of the ST-GLR as a function of SNR with the d.p. of the clutter as a parameter. For this case, the target return is assumed random. The target amplitudes in both the H and V channels are modeled as Gaussian random variables with zero mean. Fig. 5 shows that for this random signal case, even when the d.p. of the clutter becomes zero, the detection performance of the PST-GLR is significantly superior to that of the ST-GLR. Note that when the d.p. of the clutter is low, the adaptive polarization canceller based detectors may have little performance improvement since the clutter cancellation will be insignificant.

We have selected a relatively large number of secondary data to avoid the severe performance degradation due to the requirement of estimating the covariance matrix **R**. However, under non-homogeneous clutter environments, which often arise for airborne radar systems, it is rarely possible to obtain such a large number of secondary data. It is also easily seen that the PST-GLR is affected more by the limited data constraints than the ST-GLR because the dimension of the covariance matrix estimated in the PST-GLR is twice the size of the one in the ST-GLR. This problem may be solved by using the localized processing concept. This idea will be discussed in detail in another paper.

Ⅵ. Conclusions

We have presented the PST-GLR algorithm for detecting radar targets. The algorithm is a generalized likelihood ratio detection algorithm, which is obtained by maximizing the likelihood ratio over the unknown covariance matrix and target polarization vector. The algorithm utilizes the radar return information in all three domains, i.e., the polarization, angle, and Doppler domain. We have also theoretically derived the probability of false alarm and the probability of detection for the algorithm. We have shown that the PST-GLR algorithm has the desired CFAR feature. Unlike in the Doppler domain, the unknown target polarization vector is estimated in the PST-GLR algorithm so there is no need to unrealistically implement a bank of filters in the polarization domain.

Our numerical results have shown that the performance degradation due to estimating the unknown polarization vector is quite small. Our numerical results have also shown that the performance of the PST-GLR algorithm may be significantly better than the angle and Doppler domain algorithms that do not exploit the information in the polarization domain, especially when the target and clutter polarization states differ significantly and when the degree of polarization of the clutter return is high. Even when the target and clutter polarization states coincide or when the degree of polarization of the clutter return is low, the PST-GLR still performs well, but the adaptive polarization canceller based detectors may fail completely or have little performance improvement.

Yet since the degrees of freedom of the PST-GLR algorithm are twice as many as that of the angle and Doppler domain algorithms, the performance of the PST-GLR degrades faster than that of the angle and Doppler domain algorithms for non-homogeneous environments. This limited secondary data problem may be overcome by using localized processing concepts and will be dealt with in detail in another paper.

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