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# Space Deformation of Parametric Surface Based on Extension Function

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**Abstract – In this paper, a new technique of space deformation for parametric surfaces with so-called extension function (EF) is presented. Firstly, a special extension function is introduced. Then an operator matrix is constructed on the basis of EF. Finally the deformation of a surface is achieved through multiplying the equation of the surface by an operator matrix or adding the multiplication of some vector and the operator matrix to the equation. Interactively modifying control parameters, ideal deformation effect can be got. The implementation shows that the method is simple, intuitive and easy to control. It can be used in such fields as geometric modeling and computer animation.**

*Keywords*: **parametric surface, space or free-form deformation, extension function**

## 1. Introduction

In the field of geometric modeling, the advent of non-uniform rational B-splines brought us a nearly perfect approach for mathematical description of free-form shape. However the interactive technique (changing weight factor or knot vector, moving control points) accompanying it for shape modification is limited. Therefore, to generate complex shape, people have to draw support from others high-level techniques for shape modification free-form or space deformation. So far, considerable achievements have been reached in research on those deformations and diversified methods of deformation have been playing an important role in practice. Some of them have become the core of certain commercial CAD/CAM softwares. Nevertheless finding new, effective and intuitive deformation approaches is still one increasingly significant research field in computer graphics.

Certainly we already have so many deformation methods. However, On the whole, those and other methods concerned still have room for improvement in such aspects as exact control of deformation region, quantitative control of modification extent and guarantee of continuity between deformed and undeformed region in local deformation or shape modification. Especially the existing techniques for shape modification among them direct only to NURBS [30]. So people can't help to ask whether there are some techniques that are more simple, easy to control and fit for deformation or shape modification of general parametric surfaces.

In answer to above question, we develop the technique of deformation based extension function for parametric surface. It can not only carry on shape modification but also yield relatively arbitrary shape. Unlike traditional methods, its main thoughts are acting on a surface's equation with the operator matrix constructed by so-called extension function to alter the shape of the surface. As we introduce control parameters with a different attribute, the method avoids oneness of the control means in traditional ways and increases the control precision to some extent. It is fit for any surfaces but those expressed by implicit form. Concrete manipulation is very simple and easy due to its application without any auxiliary tool.

The rest of the paper is structured in 5 sections. Secton 2 reviews existing methods. Section 3 provides the definition of EF. Section 4 describes the mathematical model and others key details of our method. Section 5 shows some performance examples. Section 6 concludes the paper with comparison and further research directions.

## 2. Previous Work

Global and local deformation [1] is the first modeling technique of deformation introduced into CAD/CAM field. This method and its improvement [2] can conduct regular deformation (for example, twisting, tapping, bending, rotating and scaling), but it is not easy to yield arbitrary shape with them. Free-form deformation (FFD) [3] overcomes the shortcoming of the above method. As is known to all, the central thoughts of geometric modeling are choosing regular shape information (such as point, line, plane etc.) and special weight factors, taking the weighted average of them and thus expressing a complex shape. In fact, FFD also make use of the ideas: firstly express a complex shape formed by infinite points through weighted average of relatively few control points and then move those control points to induce

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deformation of that complex shape while the local coordinates of object points are deemed to remain unchanged (the topological structure of deformation object is fixed) [5]. FFD is one of the most versatile and powerful deformation tools, but there yet are some unideal aspects in implementation. With this method it is not easy for deformation to reach anticipated effect exactly. Exact placement of object points is also hard to achieve. Literatures [6, 7, 8, 9, 10] improved FFD in shape of space embedded and basis function and used their results in human body animation and dynamic flexible deformation, but their basic principles accord with FFD. Those approaches increase control flexibility and meanwhile bring new troubles. Always they need to solve complex nonlinear equations with numerical method. Moreover the problem of continuity between adjoining deformation regions in different lattices arises. And extended free-form deformation (EFFD) is based on the same mathematic formula as FFD [4], but its auxiliary tool is a complex lattice formed by several lattices with arbitrary shape. In local deformation, however, it is not easy to determine the number of subdivision of the initial lattice and thus build the appropriate lattice. By introducing constraint points Hsu proposed direct manipulation of free-form deformation [14] that is an improvement of FFD. Though by it deformation can satisfy the layout of object points exactly, its implementation involves finding the best solution, in the least squares sense, to the system of complex equations. All those methods improve FFD in varying degrees, increase control flexibility and can achieve a variety of deformations. However, on the whole, as existing techniques of deformation adopt weight factors or points with parallel status as control elements they have some shortcomings that the properties of their control means are unitary. We think the unitary property is one of the reasons why their controllability is unideal and deformation effects are stiff. Recently another rather better improvement of FFD has been published [31]. Unlike above methods, axial deformation [11] takes a curve as control means. The method is good to generate deformation such as stretching, scaling, bending, twisting and so on. However, as its degree of freedom for control is limited, so is the effect of the deformation. It would be not easy to yield an arbitrary shaped bump by AxDf, for instance. Wires [12], a generalization of AxDf, can be used in creating wrinkled surface and stitching geometry together. Literature [13] presented an approach with the degree of freedom between FFD's and AxDf's of which the control tools are two parametric surfaces. Moreover, without any auxiliary tool, space deformation technique [15] uses specified points and displacements corresponding to them (called constraint) to control the deformation. And expectant shape can be achieved by choosing the solution satisfying the constraint. Whereas as the shape of deformation around a constraint point depends on so-called extrusion function, the last results of deformation don't closely correlate with the constraints. Besides the literature [16] provided a complementary method for deformation modeling, but it is short of intuitive and interactive control means. Due to introducing constraint points and radius the literature [17] presented a good technique of local deformation, which was improved so well by literature [18] that it can not only conduct deformation of points constraints but also lines, surfaces or volumes constraints. Léon *et al.* [21, 20, 19] linked the control polyhedron of a surface with the mechanical equilibrium of a bar network using the force density concept Surface deformations are achieved by adjusting mechanical parameters according to the criterion of equilibrium (i.e., all kind of parameters satisfy a linear equation). Though its effect of the deformation abounds in aesthetic felling, sophisticated results often needs solving high-order system of linear equations.

Above methods belong to the category of space deformation. Besides physics-based deformations [23, 22, 24] consider mechanical principles such as kinetics, elasticity, inelasticity and attributes such as mass, friction, internal force, so its effects of deformation are even more close to actual life. Although the techniques once were celebrated for a while, they generally need a large amount of computation and lack interactive control means, which limits their application in practice.

And there are several representative approaches of shape modification that fall into the deformation category. Piegl proposed a method modifying the shape of rational B-spline surface [27], in which the control points and their weight factors are recomputed directly from the definition of NURBS. It is intuitive and comprehensible in actual application. However, more often than not, it requires knot insertion even to achieve even simple effect. Using a perspective function transformation of arbitrary origin O Sánches-Reyes developed another way called a simple technique for NURBS shape modification [28]. User input for the modification amounts only to choosing origin O and displacing a control point along the radial direction through O. As the technique depends on surface equation, it is fit for carrying on global modification. The literatures [32, 29] also put forward two methods of shape modification.

## 3. Extension Function and Its Properties

**Definition** 1 Let C: $\varphi(u, v)=0$ be a simple closed curve in $(u, v)$ planethe function $\varphi(u, v)$ be continuous and have continuous partial derivatives with n-1-order over the curve. Once again let $U=\{(u, v) \mid \varphi(u, v) \leq 0\}$ represent a region enclosed by the curve. Then the composite function

$$E(u,v)=E(u,v,h,n)=\begin{cases} e^{h(\varphi(u,v))^n} & \varphi(u,v)\leq 0 \\ 1 & \varphi(u,v)>0 \end{cases}$$

is called extension function, where positive integer $n \geq 2$ real number $h \in R$. And curve $C$ is called bound curve, $n$ index and $U$ support region.

Extension function has following properties:

1) $E(u,v)|_C = 1.2 \left.\frac{\partial^i E(u,v)}{\partial u^k \partial v^l}\right|_C = 0,\ 0 \leq i = k+l, k, l \leq n-1$

3) $E(u,v)$ possesses extremal case similar to $\varphi(u,v)$ in the support region.

## 4. Mathematical Model of Deformation

### 4.1. The deformation using arbitrary point O' as its extension or contraction center

Let $p(u,v)=(x(u,v), y(u,v), z(u,v))^T$ be a $C^r$ surface defined on the domain $\Omega$, where $\Omega \subset R^2$; $E_{ij}(u,v) = E(u,v,h_{ij},n)$ extension functions whose the support regions belong to $\Omega$, where $n \leq r$, $i, j=1, 2, 3$; and $l_1$, $l_2$, $l_3$ unit vectors of linear independence.

$$\text{Write } D = \begin{pmatrix} E_{11} & \varepsilon_{12}(E_{12}-1) & \varepsilon_{13}(E_{13}-1) \\ \varepsilon_{21}(E_{21}-1) & E_{22} & \varepsilon_{23}(E_{23}-1) \\ \varepsilon_{31}(E_{31}-1) & \varepsilon_{32}(E_{32}-1) & E_{33} \end{pmatrix}$$

called operator matrix and take $\varepsilon_{ij}=\pm 1$, $i \neq j$, then after the deformation with O' as its center and $l_1$, $l_2$, $l_3$ as its extension or contraction directions, the deformed surface $p_d(u,v)$ and the original one $p(u,v)$ have following relation:

$$p_d(u,v) = F(u,v)(p(u,v)-O')+O',\quad (u,v) \in \Omega \qquad (1)$$

Where $F(u,v) = \frac{1}{[l_1 l_2 l_3]}(l_1 l_2 l_3) D((l_2 \times l_3)(l_3 \times l_1)(l_1 \times l_2))^T$,

$[l_1 l_2 l_3]$ denotes mixed product and $(l_1 l_2 l_3)$ is a matrix constructed by the vectors $l_1$, $l_2$, $l_3$ in column form.

### 4.2. The geometric meaning of the deformation technique

Set matrix $A=(a_{ij})_{3\times 3}$, column vector $X = x_1 l_1 + x_2 l_2 + x_3 l_3 = (l_1 l_2 l_3) X$,

where $X=(x_1 x_2 x_3)^T$.

Again let $= \frac{1}{[l_1 l_2 l_3]}(l_1 l_2 l_3) A((l_2 \times l_3)(l_3 \times l_1)(l_1 \times l_2))^T$, then

$$X = \frac{1}{[l_1 l_2 l_3]}(l_1 l_2 l_3)(a_{ij})_{3\times3}((l_2 \times l_3)(l_3 \times l_1)(l_1 \times l_2))^T (l_1 l_2 l_3) X$$

$$= \frac{1}{[l_1 l_2 l_3]}(l_1 l_2 l_3)(a_{ij})_{3\times3}[l_1 l_2 l_3] I X = (l_1 l_2 l_3)(a_{ij})_{3\times3}(x_1 x_2 x_3)^T$$

$$= \sum_{i,j=1}^{3} a_{ij} x_j l_i .$$

Especially,

1) when $A=I$, $a_{ij}=\delta_{ij}$, $i, j=1,2,3$. $X=\sum_{i=1}^{3} x_i l_i = X$, due to $X$'s arbitrariness, $=I$, i.e.,

(2) holds;

2) when $A=D(u,v)=(d_{ij})_{3\times 3}$, i.e., $=F(u,v)$. If we write

$P(u,v)-O' = p_1 l_1 + p_2 l_2 + p_3 l_3$, (2) can be expressed as

$$P_d(u,v) = \sum_{i,j=1}^{3} d_{ij} p_j l_i + O' = \sum_{i=1}^{3}\left(\sum_{j=1}^{3} d_{ij} p_j\right) l_i$$

$$= (l_1 l_2 l_3) D (p_1 p_2 p_3)^T + O'$$

From this formula we can easily find the geometric meanings of the deformation defined by (1) is conducting an affine transformation on the coordinates $(p_1 p_2 p_3)^T$ of the vector $P(u,v)$-$O'$ in affine coordinate system $[O', l_1, l_2, l_3]$, at every point $P(u,v)$ on original surface within support region $\Omega$. And transform matrix is the operator matrix $D(u,v)$

The displacement of the deformation satisfies $P(u,v) = P_d(u,v) - P(u,v)$

$$= [P_d(u,v) - O'] - [P(u,v) - O']$$

$$= (l_1 l_2 l_3) D (p_1 p_2 p_3)^T - (l_1 l_2 l_3) I (p_1 p_2 p_3)^T$$

$$= (l_1 l_2 l_3)(D-I)(p_1 p_2 p_3)^T$$

$$= (l_1 l_2 l_3)(D-I)\begin{pmatrix} p_2 \\ p_2 \\ p_3 \end{pmatrix}$$

### 4.3. The control means of the deformation model

Solving the problem of deformation not only lies in giving out deformation method itself, but what is more important is to bring forth some interactive control means accompanying the method. Generally speaking, it is hard for us to succeed only at one stroke, so we seldom get the anticipated effects through manipulating only once. In any case, shapes generated by computer system are rarely immediately acceptable and subsequent modifications are necessary, so when creating a method of deformation we bestow some degree of freedom on it, i.e., set up a number of parameters for shape control. Then we adjust these parameters so that the result of deformation approximates or reaches the anticipated effects at last. Let's review existing means of deformation and shape modification. Though they all have certain degree of freedom and can carry out interactive control, they can control deformation only to some extent quantitatively. Often we get these under the condition of losing those so that some side effects are yielded. Thereupon, it is not easy to control the deformation qualitatively. For example, in local deformation, using the methods such as literatures [2-15, 19-21] it is not easy to keep the region beyond the deformation remain unaffected. And method in literature [27, 28, 29, 30, 21, 20, 19] direct only to B-spline, NURBS surface. Even for these surface controlling its deformation region also requires degree elevation or knot insertion, which greatly increases the computational costs. Besides shape control for deformation in literature [3] needs moving the control points of parallelepiped lattice while traditional shape modification needs changing the vertex position of control polygon or its weight factors, but it is not clear that which points or its weight factors are best ones to be altered.

Our method can control the region over which deformation takes place exactly. In addition, with it we can adjust the shape qualitatively. Its control parameters consist of $h_{ij}$, $\varepsilon_{ij}$ $(i \neq j;\ i, j=1,2,3)$, $n$ and bound curve $C$. In addition, extension or contraction center $\boldsymbol{O}'$ and vectors $\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{l}_3$ also can be used to control the deformation. They all have obvious geometric meaning. Curve $C$ controls the deformation region, vectors $\boldsymbol{l}_1$, $\boldsymbol{l}_2$, $\boldsymbol{l}_3$ the principal directions, $h_{ij}$ the magnitude of deformation, $\varepsilon_{ij}(i \neq j)$ certain symmetric effects, $n$ the continuity of deformed surface on the bound curve and $\boldsymbol{O}'$ the center of extension or contraction.

In process of deformation, we control deformation through the following means:

1) Change $h_{ij}$. See Figure 4-12
2) Change $\varepsilon_{ij}$, which can create some symmetrical effect, .
3) Change $n$, which can alter the sooth degree of the deformed surface over bound curve. For example, increasing $n$ can make the deformed surface press close to the original surface near the bound curve. See Figure 2, 3 and 9.
4) Change bound curve, which can alter the region over that the deformation takes place. See Figure 3-4.
5) Change $\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{l}_3$. See Figure 15-16.
6) Change the position vector $\boldsymbol{O}'$. See Figure 10 and Figure 14.

**Remark** 4.1: Theoretically we can choose any curve like those described in the definition 1 as bound curve, But in actual application if we do so the infinite information of that kind of bound curve will make the interactive manipulation difficult. We must change infinite information into finite one. For example if we adopt circle as bound curve, we can easily control the deformation by adjusting only three parameters instead of whole curve.

## 4.4. The smooth degree of deformed surface on bound curve

**Lemma** If $\boldsymbol{l}_1, \boldsymbol{l}_2, \boldsymbol{l}_3$ are unit vectors of linear independence, $(\boldsymbol{l}_1\, \boldsymbol{l}_2\, \boldsymbol{l}_3)$ is a matrix constructed by the vectors $\boldsymbol{l}_1$, $\boldsymbol{l}_2$, $\boldsymbol{l}_3$ in their column form and $I$ is a 3-order unit matrixthen it follows that

$$\frac{1}{[\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3]}(\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3)((\boldsymbol{l}_2\times\boldsymbol{l}_3)(\boldsymbol{l}_3\times\boldsymbol{l}_1)(\boldsymbol{l}_1\times\boldsymbol{l}_2))^T = I \tag{2}$$

**proof** See section 3.4

**Theorem** Through the deformation (1), the deformed surface possesses n-1-order continuity on bound curve.

**proof** Write $D(u,v)=D$, $\{\partial^m D/\partial u^k \partial v^l\}=D_{kl}^{(m)}$, to which $\boldsymbol{p}_{kl}^{(m)}$ or $(\boldsymbol{p}_d)_{kl}^{(m)}$ is similar, then from the properties of extension function it follows that

$$D_{00}^{(0)}|_C = D|_C = I, D_{kl}^{(m)}|_C = O$$

$$\text{where } 0 \le k, l \le n-1, m=k+1, m=1,2,\Lambda,n-1 \tag{3}$$

Looking on the factors or the terms of (1) as matrix function and taking the mixed partial derivative of it with respect to $u$ and $v$ we get

$$(\boldsymbol{p}_d)_{kl}^{(k+l)} = \sum_{i=0}^{k}\sum_{j=0}^{l} C_l^j C_k^i \frac{1}{[\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3]}(\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3) D_{ij}^{(i+j)} ((\boldsymbol{l}_2\times\boldsymbol{l}_3)(\boldsymbol{l}_3\times\boldsymbol{l}_1)(\boldsymbol{l}_1\times\boldsymbol{l}_2))^T \boldsymbol{p}_{(k-i),(l-j)}^{(k+l-i-j)} \tag{4}$$

Evaluating the two sides of (4) on curve $C$ and using (3) and the lemma yields

$$(\boldsymbol{p}_d)_{kl}^{(k+l)}|_C = \boldsymbol{p}_{kl}^{(k+l)}|_C,$$

where $0 \le k, l \le n-1$ and $0 \le k+l \le n-1$. **QED.**

## 3.5. The continuous transition of deformation over the intersection of two support regions

Let $D_i$ be operator matrix corresponding to support regions $U_i$ enclosed by $C_i$, $n_i$ corresponding index of $D_i$, where $i=1,2$.

If $U_1 \text{ I } U_2 \neq \Phi$, then the equation of transition surface for deformation over it is

$$\tilde{\boldsymbol{p}}_d(u,v) = \frac{1}{[\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3]}(\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3) D_1 D_2 ((\boldsymbol{l}_2\times\boldsymbol{l}_3)(\boldsymbol{l}_3\times\boldsymbol{l}_1)(\boldsymbol{l}_1\times\boldsymbol{l}_2))^T (\boldsymbol{p}(u,v)-\boldsymbol{O}') + \boldsymbol{O}'(u,v) \in U_1 \text{I} U_2 \tag{5}$$

The deformed surface possesses $n_1$-1-order and $n_2$-1-order continuity respectively on $C_1$ and $C_2$ segment of $C_1 \text{Y} C_2$, where $C_1 \text{Y} C_2$ denotes $U_1 \text{I} U_2$'s boundary. See Figure 13, 22, 23.

In fact, taking the mixed partial derivative of (5) with respect to $u$ and $v$ we obtain

$$(\tilde{\boldsymbol{p}}_d)_{kl}^{(k+l)} = \sum_{i=0}^{k}\sum_{j=0}^{l} C_l^j C_k^i \frac{1}{[\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3]}(\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3)(D_1 D_2)_{ij}^{(i+j)} ((\boldsymbol{l}_2\times\boldsymbol{l}_3)(\boldsymbol{l}_3\times\boldsymbol{l}_1)(\boldsymbol{l}_1\times\boldsymbol{l}_2))^T \boldsymbol{p}_{(k-i),(l-j)}^{(k+l-i-j)}$$

$$= \sum_{i=0}^{k}\sum_{j=0}^{l}\sum_{p=0}^{i}\sum_{q=0}^{j} C_i^p C_j^q C_l^j C_k^i \frac{1}{[\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3]}(\boldsymbol{l}_1\boldsymbol{l}_2\boldsymbol{l}_3)(D_1)_{pq}^{(p+q)} (D_2)_{i-p,j-q}^{(i+j-p-q)} ((\boldsymbol{l}_2\times\boldsymbol{l}_3)(\boldsymbol{l}_3\times\boldsymbol{l}_1)(\boldsymbol{l}_1\times\boldsymbol{l}_2))^T \boldsymbol{p}_{(k-i),(l-j)}^{(k+l-i-j)} \tag{6}$$

Moreover, from the properties of extension function we get

$$(D_2)^{(0)}_{00}\big|_{(C_1 Y C_2) I C_2} = D_2\big|_{(C_1 Y C_2) I C_2} = I(D_2)^{(r)}_{st}\big|_{(C_1 Y C_2) I C_2}$$
$$= O, 1 \le r = s+t, s,t \le n_2 - 1$$

Substituting it into (6) yields

$$(\tilde{p}_d)^{(k+l)}_{kl}\big|_{(C_1 Y C_2) I C_2} = \sum_{i=0}^{k}\sum_{j=0}^{l} C_l^j C_k^i \frac{1}{[l_1 l_2 l_3]}(l_1 l_2 l_3)(D_1)^{(i+j)}_{ij}$$
$$((l_2\times l_3)(l_3\times l_1)(l_1\times l_2))^T p^{(k+l-i-j)}_{(k-i),(l-j)}\big|_{(C_1 Y C_2) I C_2}$$
$$=(p_d)^{(k+l)}_{kl}\big|_{(C_1 Y C_2) I C_2},\ 0\le k+l, k,l\le n_1-1$$

In a similar way we have

$$(\tilde{p}_d)^{(k+l)}_{kl}\big|_{(C_1 Y C_2) I C_1} = (p_d)^{(k+l)}_{kl}\big|_{(C_1 Y C_2) I C_1},\ 0\le k+1, k,l\le n_2-1$$

Generally, let $D_i$ be operator matrix corresponding to support regions $U_i$ enclosed by $C_i$, $n_i$ corresponding index of $D_i$, where $i=1,2,$ ; m. If $\mathop{\mathrm{I}}\limits_{i=1}^{m} U_i \ne \Phi$ then the equation of transition surface for deformation over it is

$$\tilde{p}_d(u,v) = \frac{1}{[l_1 l_2 l_3]}(l_1 l_2 l_3)\prod_{i=1}^{m} D_i((l_2\times l_3)(l_3\times l_1)(l_1\times l_2))^T$$
$$(p(u,v)-O')+O',(u,v)\in \mathop{\mathrm{I}}\limits_{i=1}^{m} U_i$$

which has $n_i-1$-order continuity respectively on $C_i$ segment of $\mathop{\mathrm{Y}}\limits_{i=1}^{m} C_i$, where $i=1,2,$ ; $m$ and $\mathop{\mathrm{Y}}\limits_{i=1}^{m} C_i$ denotes the boundary of $\mathop{\mathrm{I}}\limits_{i=1}^{m} U_i$. Then a uniform equation of deformation can be written as

$$\tilde{p}_d(u,v) = \frac{1}{[l_1 l_2 l_3]}(l_1 l_2 l_3)\prod_{i=1}^{m} D_i^{k_i}((l_2\times l_3)(l_3\times l_1)(l_1\times l_2))^T$$
$$(p(u,v)-O')+O',(u,v)\in \mathop{\mathrm{I}}\limits_{i=1}^{m} U_i^{k_i}$$

where $k_i=0$ or $1$, $D_i^0=I$, $D_i^1=D_i$, $U_i^0=\Omega\text{-}U_i$, $U_i^1=U_i$, and obviously $\mathop{\mathrm{Y}}\limits_{(k_1 k_2 \Lambda k_m)} (\mathop{\mathrm{I}}\limits_{i=1}^{m} U_i^{k_i}) = \Omega$.

**Remark** 4.2: If we want to adopt different principal directions over different support regions, the deformation should be carried out in turn. Here the deformation can be formulated in a recurrence form according to certain sequence.

### 4.6. The deformation of extension or contraction along radial direction with O' as center

$$p_d(u,v) = (p(u,v)-O')E+O',\ (u,v)\in\Omega$$

Generally, with $O_i$ as centers $E_i$ as extension function defined over the same or disjoint support regions the deformation equation is

$$p_d(u,v) = \sum_{i=1}^{n}(E_i-1)(p(u,v)-O_i)+p(u,v),\ (u,v)\in\Omega$$

### 4.7. The deformation of extension or contraction along a vector field

Let $p(u,v)=(x(u,v), y(u,v), z(u,v))^T$ be a $C^r$ surface defined on the domain $\Omega$, where $\Omega\subset R^2$; $E_{ij}(u,v) = E(u,v,h_{ij},n)$ extension functions whose the support regions belong to $\Omega$, where $n\le r$, $i$, $j=1,2,3$; and $s(u,v)$ a unit vectors field.

Set $D=\begin{pmatrix} E_{11}-1 & E_{12}-1 & E_{13}-1 \\ E_{21}-1 & E_{22}-1 & E_{23}-1 \\ E_{31}-1 & E_{32}-1 & E_{33}-1 \end{pmatrix}$, then the deformed surface $p_d(u,v)$ and the original one $p(u,v)$ have following relation

$$p_d(u,v) = p(u,v)+Ds(u,v)(u,v)\in\Omega$$

Similar to the section 4.1, now the control parameters involve index $n$, curve $C$ and $h_{ij}$. In order to achieve some special effect such as symmetry, we can still multiply the elements of the matrix $D$ by "−1" and increase the control flexibility.

In following special situations, we take $h_{11}=h_{22}=h_{33}$, $h_{ij}=0$, $i\ne j$.

1) When $s(u,v)$ is a constant vector, the deformation is extension or contraction along a fixed direction.
2) When $s(u,v)$ is a tangent vector field of a surface, the deformation is extension or contraction along a tangent line at every points.
3) When $s(u,v)$ is a normal vector field of a surface, the deformation is extension or contraction along a normal line at every points.

### 4.8. Major thought of the deformation

Motivated by the mould principle of foundry and manufacture industry, in this paper, we develop a new deformation model based on so-called extension function. The extension function and the operator matrix made of it correspond to mould. Adjusting every control parameter corresponds to changing the shape of the mould to achieve object expected. The operator matrix's acting on surface corresponds to extrusion or pouring. Its mathematical essentiality is that within certain range the coordinate space contracts or extends along certain directions with a certain point as its center while the magnitude of contraction and extension are variant depending on extension functions. If we adopt different extension functions in different direction, we can obtain rich deformation results. And coordinates beyond the range remain to be unchangeable. Compared with existing methods, the thought is simple can easily be understood by user without advanced mathematical foundation.

## 5. Experimental Results

For the sake of simplicity, we only apply our method to a biquadratic Bézier surface and a plane to demonstrate it mainly in orthogonal coordinate system. We also adapt circle, ellipse and so on as the bound curve of deformation and let $h_{ij}=0$ $(i \neq j)$. Fig. 1 shows an undeformed biquadratic Bézier surface with control points (-4, -4, 2), (0, -5, 2.5), (4, -4, 2), (-4, 0, 2.5), (0, 1, 4.5), (4, 0, 2.5), (-4, 4, 2), (0, 5, 2), (4, 4, 2). Fig. 2 is the deformed surface by taking a circle as bound curve, the point $O'=(0, 0, 2.2)$ as the center of contraction, 3 as the index and $h_{11}=h_{22}=h_{33}<0$. With 4 as the index and other control parameters similar to Fig. 2, Fig. 3 shows how the index influences the effect of the deformation. Figs. 4-12 reveal how the change of $h_{11}$, $h_{22}$ and $h_{33}$ affects the effect of the deformation, with $O'=(0, 0, 0)$ as the center of contraction. Fig. 13 displays a deformation with two intersecting support regions. Comparing with Fig. 10, Fig. 14 has the same control parameters with Fig. 10 but $O'=(0, 0, 1.5)$. Figs. 15-16 display the difference of deformation due to taking respectively $l_3=(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $l_3=(-1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$, mean

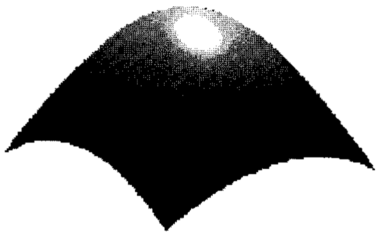

**Fig. 1.** The original surface.

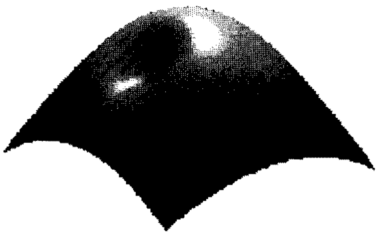

**Fig. 2.** The index $n=3$, $h_{11}=h_{22}=h_{33}<0$.

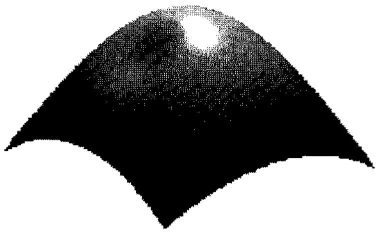

**Fig. 3.** The index $n=4$, $h_{11}=h_{22}=h_{33}<0$.

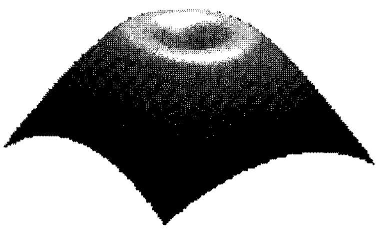

**Fig. 4.** The deformation with $h_{33}<0$, $h_{11}=h_{22}<0$.

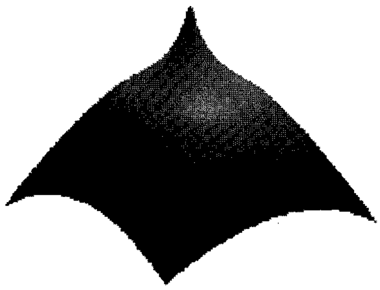

**Fig. 5.** The deformation with $h_{33}>0$, $h_{11}=h_{22}<0$.

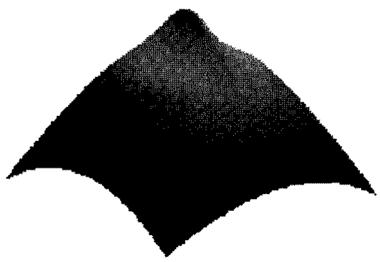

**Fig. 6.** The deformation with $h_{11}<0$, $h_{22}$, $h_{33}>0$.

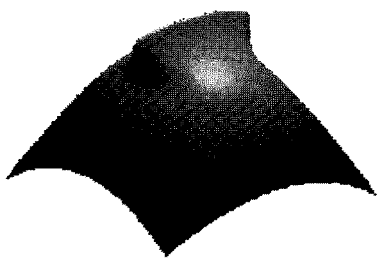

**Fig. 7.** The deformation with $h_{11}$, $h_{33}>0$, $h_{22}<0$.

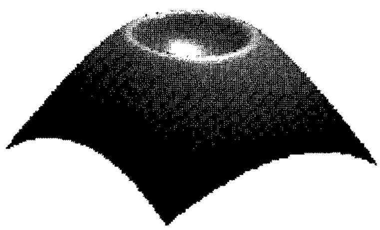

**Fig. 8.** The deformation with $h_{33}<0$, $h_{11}=h_{22}>0$.

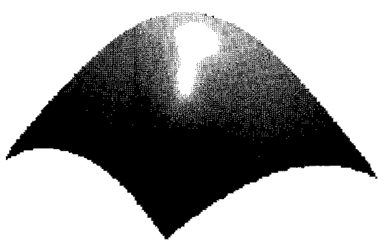

**Fig. 9.** The center of bound circle is (3/4, 3/4) with radius 1/18.

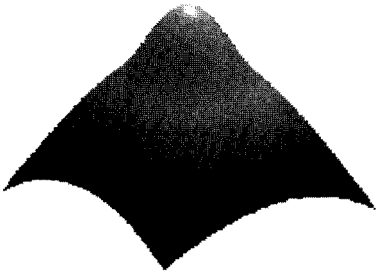

**Fig. 10.** The deformation with **O**'=(0, 0, 0).

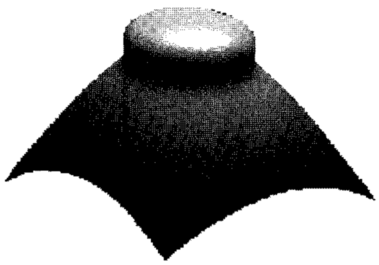

**Fig. 11.** The deformation with $h_{11}$, $h_{22}$, $h_{33}>0$, and $h_{33}<<h_{22}=h_{11}$.

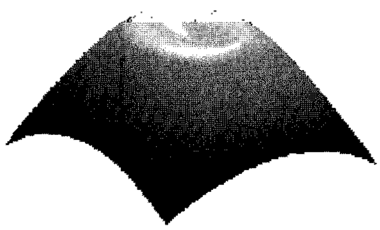

**Fig. 12.** The deformation with $h_{11}$, $h_{33}<0$, $h_{22}>0$.

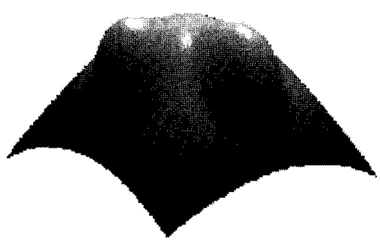

**Fig. 13.** The centers of two bound circles are respectively (3/4, 1/2), (1/2, 3/4).

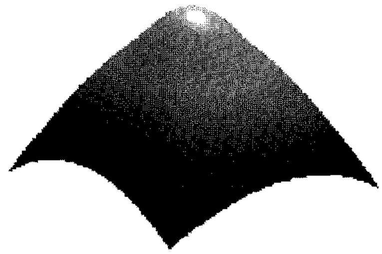

**Fig. 14.** The deformation with **O**'=(0, 0, 1.5).

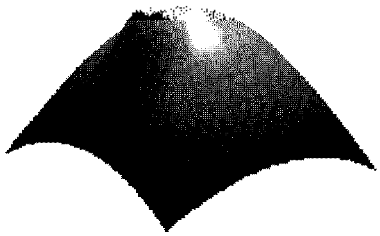

**Fig. 15.** The deformation with $l_3=(-1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$ $l_1=(1, 0, 0)$, $l_2=(0, 1, 0)$.

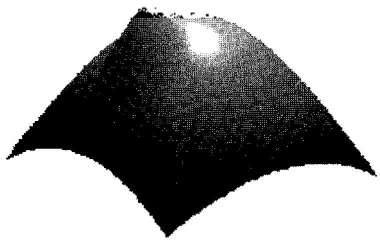

**Fig. 16.** The deformation with $l_3=(-1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$ $l_1=(1, 0, 0)$, $l_2=(0, 1, 0)$.

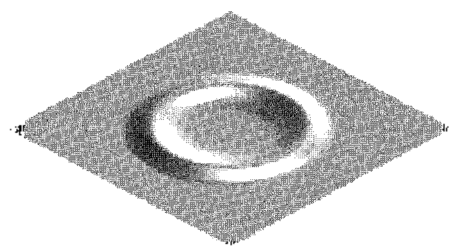

**Fig. 17.** The deformation with $h_{33}>0$, $h_{11}=h_{22}=0$, and bound curve $1\text{-}\cos(\pi(u^2+v^2)/4)=0$.

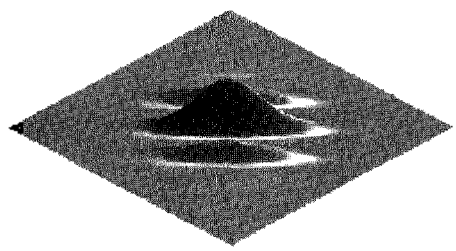

**Fig. 18.** The deformation with $h_{33}>0$, $h_{11}=h_{22}=0$, and bound curve $1+\cos(\pi(u\text{-}v))=0$.

while keeping $l_1=(1,0,0)$ and $l_2=(0,1,0)$. Fig. 17 shows that we can create a ring-like shape if we choose such bound curve as $1\text{-}\cos(\pi(u^2+v^2)/4)=0$. In actual application, we can choose unclosed curve as bound curve.

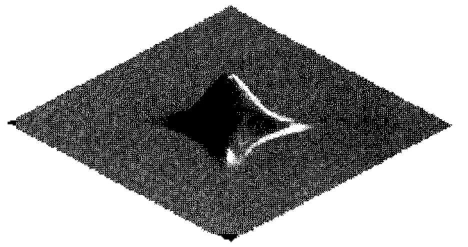

**Fig. 19.** The deformation with $h_{33}>0$, $h_{11}=h_{22}=0$, and bound curve 1-cos $(\pi(u^2-v^2)/2)=0$.

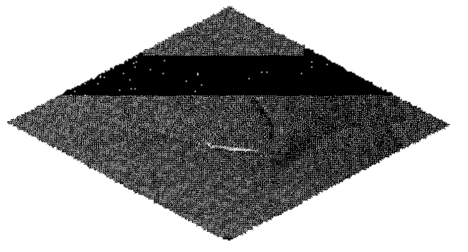

**Fig. 20.** The deformation with $h_{33}>0$, $h_{11}=h_{22}=0$, bound curve 1-cos $(2\pi(u^2+v))=0$ and a special support region.

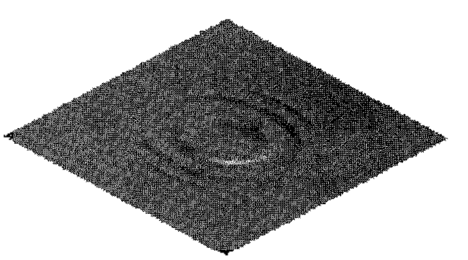

**Fig. 21.** The deformation with $h_{33}>0$, $h_{11}=h_{22}=0$ and bound curve 1-cos $(3\pi(u^2/3+v^2/5))=0$.

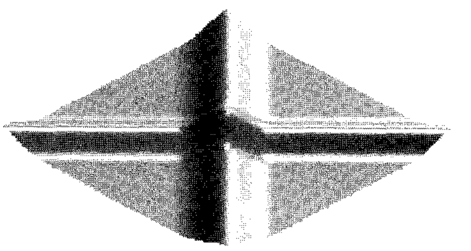

**Fig. 22.** The deformation with bound curve $1+\cos(\pi(u\pm v)/3)=0$.

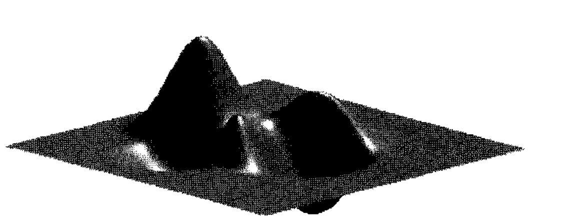

**Fig. 23.** The deformation with five intersecting support regions, three positive $h_{33}$ and two negative $h_{33}$, all $h_{11}$, $h_{22}$ being zero, and all indexes being 3 and five circles respectively being bound curves.

For example, Fig. 18 is the deformed plane through using $1+\cos(\pi(u-v))=0$ as bound curve and multiplying $h_{33}$ by $\exp((-u^2-v^2)/2))$. Figs. 19-20 displays that we can use unclosed curve $(1-\cos(2\pi(u^2+v))=0$, $1+\cos(\pi(u^2-v^2)/2)=0)$ as bound curve for some particular purpose. Fig. 21 tells that we can simulate ripple if we use the curve $1-\cos(3\pi(u^2/3+v^2/5))=0$ as bound curve and multiply $h_{33}$ by the factor $\exp(-u^2-v^2)$. Fig. 22 is achieved by taking $1+\cos(\pi(u\pm v)/3)=0$ as bound curve. Fig. 23 shows that our method can generates a complex shape as "multi-peak surface" and once again demonstrates the continuous transition of the deformation over the intersection of several support regions (the smooth degree on every bounding curve can be adjusted by changing its corresponding index).

**Remark** 5.1 In certain special case, self-intersection may take place. For example, in the deformation illustrated by Fig. 5 if we increase $h_{11}$, $h_{22}$ too much the deformed surface might intersects itself.

## 6. Comparison with Existing Methods

According to Barr's way [1] the deformation of surface is conducted mainly through following steps. First, convert the surface into a vector field by differentiating it. Then transform the vector field into another one according to a certain transformation rule for tangent vector. Finally integrate the new tangent vectors field to obtain the new position vectors equation of deformed surface. Obviously, by the method the deformation achieves figuration at one stroke. It is short of means for interactive control and not easy to generate arbitrary shape. The major reason for this lies in that we cant foreknow the relation between the transformation matrix and the shape of new surface. Moreover, in complex case quadrature itself is not easy. However, our method need not first differentiate the surface and then integrate. In addition, it has good controllability. As regards the techniques [3, 4], there are two troubled things. First, embed the object into lattices (convert the coordinates of object into the ones relative to the lattices) after the control points are determined according to the deformation region of the object. Then adjust the position of the control points concerned to deform the lattices such that the deformation of the lattices is passed to the inner object (i.e., compute the new global coordinates of object points in the deformed lattices corresponding to the same ones relative to the lattices). Thus an arbitrary shape is created. However, in those methods calculation concerned is completed with Bernstein or NURBS polynomial. Though the computation can be carried out through transforming the polynomial into the ones with power basis, the amount of computation is still very large. Moreover, in order to get an arbitrary shape, generally a lot of control points must be chosen, which induces new trouble. For example, people always can not make sure which control points should be moved. Even if they know to move which ones, the last effect of deformation is difficult to predicate. Especially in actual application overmany points result in screen clutter, which is disadvantageous to manipulation. Compared with FFD, RFFD [7] endows every control point with a weight factor so as to increase degree of freedom for deformation. The change of the effect of deformation induced by adjusting the weight factors is difficult to predicate too, which greatly limits the use of the technique by the common user not knowing spline theory. And the implementation of AXDf [11] must adopt a curve as the axis on which a local frame field (axial coordinate systems) is defined and convert their local coordinates into the ones of axial coordinate system. Then the shape of the axis is changed with traditional curve-editing techniques while the coordinates of object points relative to axial coordinate system keeps unchangeable. Lastly, the global coordinates of object points are computed. However the effect of deformation made by the method is dull. And it involves a large amount of computation in conversion between two kinds of coordinate. In contrast with above methods our one doesn't involve higher-degree polynomial and need no conversion between two kinds of coordinates in embedding or after deformation of auxiliary tool. So using it there is no too large computing cost. Though it has few control parameters, due to its very simple process of use, rich effect of deformation can be got by continuous implementations. Moreover it can quantitatively foresee or control the effect of deformation. What is more important is that the method need not draw support from any auxiliary tool for deformation.

As for existing techniques modifying the shape of surface, the one given by literature [27] is carried on by following way: knot insertion, moving control points, adjusting weight factors. Nevertheless, using it to modify a certain shape and facing too many degree of freedom, user always can not determine whether to move points or to change weight factors. Depending on geometric terms such as point, displacement, literature [28] introduced a perspective functional transformation of arbitrary center O with which the shape of surface modified is easy to expect. However it is still difficult to control the region of modification exactly or at will. In addition, another one introduced by the literature [28] can control position, 1-order or 2-order derivative through the control points of B-spline. However, for more constraint conditions it often need recur to knot insertion. When our method is used in shape modification of surface, its prominent advantage is that it possesses universality. It is fit for not only BézierB-SplineNURBS [30] surfaces but also any ones except those expressed by implicit function unlike the methods in literatures [27, 28] that directs only to BézierB-SplineNURBS surfaces. Furthermore, our method still has the following features:

1) It can control deformation region exactly and make sure the undeformed region remain unaffected.
2) Used it in local deformation, the smooth degree of surface on bound curve can be chosen artificially.
3) Due to its simple mathematical background, user without advanced mathematical knowledge can operate it.
4) It combines shape modification and deformation.
5) Applied over different regions continuously or simultaneously it can create rich effect of deformation.

We think further research should aims at constructing better extension function, locating the bound curve, building a database of extension function and analyzing the geometric information included in the deformed surface of the original surface. As for the situation of space curve, we discuss through other articles.

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