# 교량의 경험적 손상도 곡선

## Empirical Fragility Curves for Bridge


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**Abstract**

This paper presents a statistical analysis of empirical fragility curves for bridge. The empirical fragility curves are developed utilizing bridge damage data obtained from the 1995 Hyogoken Nanbu(Kobe) earthquake. Two-parameter lognormal distribution functions are used to represent the fragility curves with the parameters estimated by the maximum likelihood method. This paper also presents methods of testing the goodness of fit of the fragility curves and estimating the confidence intervals of the two parameters(median and log-standard deviation) of the distribution. An analytical interpretation of randomness and uncertainty associated with the median is provided.



**keywords** : fragility curves, goodness of fit, confidence interval


## 1. INTRODUCTION

Bridges are one of the most critical components of highway systems. Past earthquake, such as the 1971 San Fernando earthquake, and more recent ones, the 1994 Northridge earthquake and the 1995 Kobe earthquake, have demonstrated that bridges are vulnerable to earthquakes.[1)] Thus, it is important to evaluate the vulnerability of highway bridges in earthquake-prone regions. The development of vulnerability information in the form of fragility curves is a widely practiced approach when the information is to be developed accounting for a multitude of uncertain sources involved, for example, in estimation of seismic hazard, structural characteristics, soil-structure


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interaction, and site conditions.

In principle, the development of fragility curves will require synergistic use of the following methods ; (1) professional judgement, (2) quasi-static and design code consistent analysis, (3) utilization of damage data associated with past earthquakes, and (4) numerical simulation of seismic response of structures based on dynamic analysis.

This paper concentrates on the development of empirical fragility curves for bridges by utilizing the damage data associated with a past earthquake. At the same time, this paper introduces statistical procedures appropriate for the development of fragility curves under the assumption that they can be represented by two-parameter lognormal distribution functions. These procedures describe how the test of goodness of fit can be performed and confidence intervals of the two parameters estimated. The empirical fragility curves are developed utilizing bridge damage data obtained from the 1995 Hyogo-ken Nanbu(Kobe) earthquake.

Two-parameter lognormal distribution functions were traditionally used for fragility curve construction. This was motivated by its mathematical convenience in relating the actual structural strength capacity with the design strength primarily through a seismic factor of safety which can be factored into a number of multiplicative safety factors, each associated with a specific source of randomness and/or uncertainty. When the lognormal assumption is made for each of these factors, the overall seismic safety factor also distributes lognormally due to the multiplicative reproducibility of the lognormal variables. This indeed was the underpinning assumption that was made in the development of probabilistic risk assessment methodology for nuclear power plants in the 1970's and in the early 1980's.[8]

## 2. EMPIRICAL FRAGILITY CURVES

Empirical fragility curves for the Hanshin Expressway Public Corporation's(HEPC's) bridges (columns) are developed on the basis of the records of the damage resulting from the 1995 Kobe earthquake. It is assumed that the curves can be expressed in the form of two-parameter lognormal distribution functions, and the estimation of the two parameters(median and log-standard deviation) is performed with the aid of the maximum likelihood method. For this purpose, the PGA(Peak Ground Acceleration) is used to represent the intensity of the seismic ground motion.

The likelihood function for the present purpose is expressed as

$$L = \prod_{i=1}^{N} [F(a_i)]^{x_i} [1 - F(a_i)]^{1-x_i} \qquad (1)$$

where F(.) represents the fragility curve for a specific state of damage, $a_i$ is the PGA value to which bridge $i$ is subjected, $x_i = 1$ or 0 depending on whether or not the bridge sustains the state of damage under PGA $= a_i$, and $N$ is the total number of bridges inspected after the earthquake. Under the current lognormal assumption, $F(a)$ takes the following analytical form

$$F(a) = \Phi\left[\frac{\ln\left(\frac{a}{c}\right)}{\zeta}\right] \qquad (2)$$

in which "$a$" represents PGA and $\Phi[.]$ is the standardized normal distribution function.

The two parameter $c$ and $\zeta$ in Eq. (2) are computed as $c_e$ and $\zeta_e$ satisfying the following equations to maximize $\ln L$ and hence $L$ :

$$\frac{d\ln L}{dc} = \frac{d\ln L}{d\zeta} = 0 \quad (3)$$

This computation is performed by implementing a straightforward optimization algorithm.

Fragility curves are constructed[3] on the basis of a sample of 770 single-support reinforced concrete columns along two stretches of the viaduct, one in the HEPC's Kobe Route and the other in the Ikeda Route with total length of 40 km. The damage data reported by HEPC's engineers after the 1995 Kobe earthquake are utilized for this purpose. The state of damage is classified as collapse, major, moderate and minor. These bridge columns are of similar geometry and similarly reinforced. In this respect, the 770 columns under consideration here constitute approximately a homogeneous statistical sample. The PGA value at each column location under the Kobe earthquake is estimated by Nakamura et al(1998)[3] on the basis of the work by Nakamura et al(1996).[2]

Integrating the damage state information with that of the PGA and making use of the maximum likelihood method involving Eqs. (1)-(3), three fragility curves are constructed as shown in Fig. 1 together with values of the median $c_e$ and log-standard deviation $\zeta_e$. The curve with "minor" designation represents, at each PGA value "$a$", the probability that "at least minor" state of damage will be sustained by a bridge(arbitrarily chosen from the sample of bridges) when it is subjected to PGA "$a$". The same meaning applies to other curves with their respective damage state designations.

## 3. STATISTICAL ANALYSIS

The issues of hypothesis testing and confidence intervals relating to fragility curve development have not been addressed in the literature so far. It appears primarily because

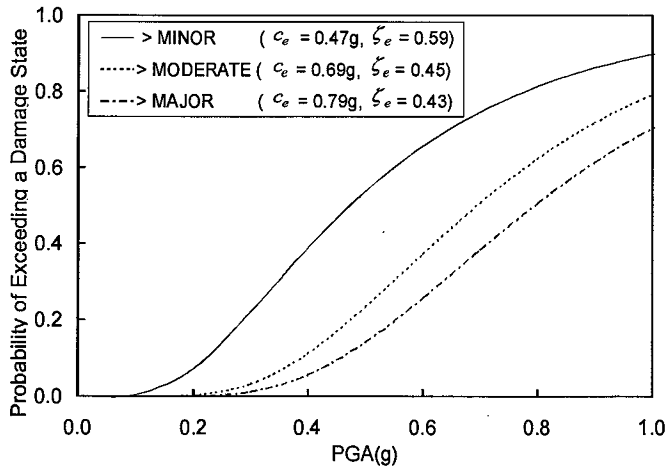


Fig. 1 Fragility Curves for HEPC's Bridges

the earthquake engineering community has never had the opportunity to collect damage data of a sufficiently large sample that can be used to develop fragility curves on the basis of legitimate statistical analysis. However, the 1994 Northridge and 1995 Kobe earthquakes, inflicting devastating damage upon many bridges, buildings, port facilities and other engineered structures, made it possible to consider statistical methods to analyze the probabilistic characteristics of damage in a more judicious fashion rather than relying on an ad hoc curve fitting exercise.

### 3.1 Test of Goodness of Fit

The fundamental probabilistic interpretation of a fragility curve $F(a)$ as a function of "$a$" suggests that a bridge will sustain a designated state of damage with probability $F(a)$ and will not sustain the damage state with probability $1-F(a)$ under the earthquake intensity represented by PGA equal to "$a$". This means that, under each PGA value, the probabilistic phenomena one deals with can be described by random variable $X_i$ following the Bernoulli distribution such that $X_i=1$ when the state of damage is reached under PGA$=a_i$, and $X_i=0$ otherwise. Then,

$$Y_i^2=(X_i-p_i)^2 \tag{4}$$

has mean and variance equal to

$$\mu_{Y_i^2}=p_i(1-p_i) \tag{5}$$

and

$$\sigma^2_{Y_i^2}=Var(Y_i^2)=p_i(1-p_i)(1-2p_i)^2 \tag{6}$$

respectively, where $p_i=F(a_i)$.

The sum of $Y_i^2$ shown below

$$Y^2=\sum_{i=1}^{N}(X_i-p_i)^2 \tag{7}$$

approaches asymptotically normal(Gaussian) as N becomes large due to the central limit theorem since each Bernoulli event under consideration is independent, where N is the sample size(the total number of the bridges considered) and in this analysis it is indeed a large value($\gg 1$).

Recalling that $X_i$ is independent of $X_j (i\neq j)$ and governed by the Bernoulli distribution, a straightforward analysis shows that the expected value $\mu_{Y^2}=E(Y^2)$ and the variance $\sigma^2_{Y^2}=Var(Y^2)$ can be written as

$$\mu_{Y^2}=E(Y^2)=\sum_{i=1}^{N}p_i(1-p_i) \tag{8}$$

and

$$\sigma^2_{Y^2}=Var(Y^2)=\sum_{i=1}^{N}p_i(1-p_i)(1-2p_i)^2 \tag{9}$$

On the other hand, if $x_i$ represents the realization(observation) of $X_i$ as defined in the likelihood function given by Eq. (1),

$$y^2=\sum_{i=1}^{N}(x_i-p_i)^2 \tag{10}$$

is the realization of $Y^2$.

Since $p_i$ depends on the values of $c_r$ and $\zeta_r$, the standard procedure of hypothesis testing suggests that if $\alpha$ is a significance level and

$$P_{\chi^2} = \Phi\left(\frac{\chi^2 - \mu_{\chi^2}}{\sigma_{\chi^2}}\right) \leq 1 - \alpha \qquad (11)$$

then, the hypothesis that $c_r$ and $\zeta_r$ are indeed the true values of $c$ and $\zeta$ cannot be rejected at the significance level $\alpha$ usually set equal to 0.05 or 0.10.

The $P_{\chi^2}$ values for the fragility curve developed for HPEC's bridges(Fig. 1) is given in Table 1. These values indicate that the hypotheses involved in all the cases can not be rejected at the significance level of 10%. It is noted here that this method can test the goodness of fit of the fragility curve only over the range of PGA where damage data sufficiently exist.

Table 1 $P_{\chi^2}$ Values for Goodness of Fit

| Damage State | HPEC's Bridges |
|---|---|
| Minor | 0.39 |
| Moderate | 0.86 |
| Major | 0.73 |

Fig. 2 shows the validity of the assumption of $Y^2$ in Eq. (7) being asymptotically normal by means of plotting the 100 realizations of $Y^2$. This requires simulation of $X_i$ at each $a_i$ using $p_i$ based on $c_r$ and $\zeta_r$ obtained from the empirically observed damage data. Upon simulating all $X_i$ for all $a_i$ and obtaining their realizations $\xi_i$, Eq. (10) is evaluated as

$$\eta^2 = \sum_{i=1}^{N} (\xi_i - p_i)^2 \qquad (12)$$

where $\eta$ and $\xi_i$ are written in place of $\chi$ and $x_i$ to distinguish the simulated data from the actual data. This process is repeated k times(k=100 here) to produce 100 realizations

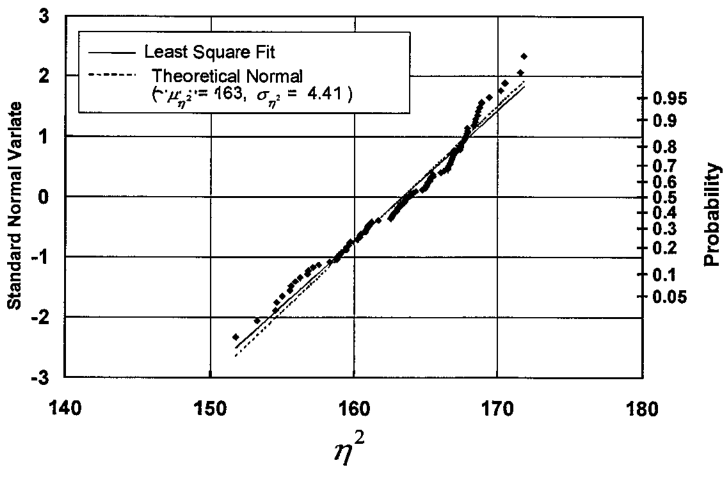


Fig. 2 Validation of $\eta^2$ Asymptotically Approaching a Normal Distribution

of $Y^2$ consisting of the same number of $\eta_i^2$, each representing one set of simulation of $\zeta_i (i=1,2,\ldots,N)$. This sample of $\eta_i^2$ is indeed plotted in Fig. 2 using the normal probability paper ; The dashed line represents the least square fit of the sample, while the solid line indicates the theoretical normal distribution with the mean and standard deviation given by Eqs. (8) and (9) respectively for the fragility curve for HEPC's bridges associated with the state of "at least minor" damage.

## 3.2 Estimation of Confidence Intervals

The estimators $\hat{c}$ and $\hat{\zeta}$ of $c$ and $\zeta$ cannot be explicitly given in terms of analytical forms as they represents optimal solutions obtained numerically by solving Eq. (3). From the uncertainty analysis point of view, however, it is most desirable to demonstrate the extent of the statistical variations of these estimators. To this end, Monte Carlo simulation techniques are used to generate realizations of $\hat{c}$ and $\hat{\zeta}$. These calls for the same simulation of $X_i$ at each $a_i$ using $p_i$ based on $c_r$ and $\zeta_r$ as exercised to demonstrate the asymptotic normal property of $Y^2$ in relation to the test of goodness of fit. Upon simulating $X_i$ for all $a_i$ and obtaining their realizations $\zeta_i (i=1,2,\ldots,N)$, Eq. (3) is solved for $c_r$ and $\zeta_r$ as a set of realizations of $\hat{c}$ and $\hat{\zeta}$.

Repeating this process a large number of times(500 times in this case), one obtains 500 sets of realizations of $\hat{c}$ and $\hat{\zeta}$ as plotted in Fig. 3. Assuming the marginal distribution of $\hat{c}$ being lognormal, and taking the 90% confidence interval between $\hat{c}=c_{0.05}$ and $\hat{c}=c_{0.95}$ associated with exceedance probabilities 95% and 5% of $\hat{c}$ one obtains $c_{0.05}=0.45g$ and $c_{0.95}=0.50g$ for the fragility curve of HEPC's bridges with the state of "at least minor" damage. Fig. 4 shows the three fragility curves

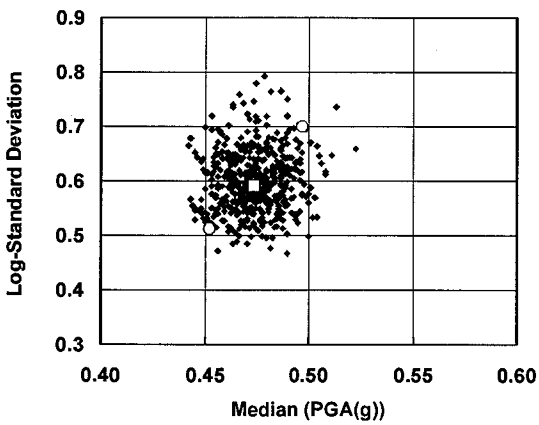


Fig. 3 Simulated Distribution of $\hat{c}$ and $\hat{\zeta}$

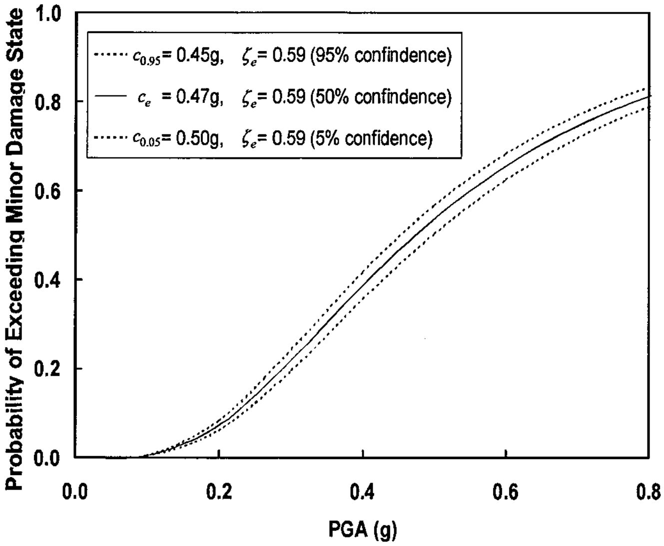


Fig. 4 95% and 5% Confidence Bands for a HEPC's Fragility Curve

with medians $c=c_{0.95}$, $c=c_e$ and $c_{0.05}$ with the identical log-standard deviation $\zeta_e=0.59$. The curves on the left and on the right respectively represent the fragility curves with 95% and 5% confidence. Following the tradition of the risk assessment procedure for the nuclear power plant[1], the log-standard deviations $\zeta=\zeta_e$ associated with $c=c_e$ is used for these three curves, although it is possible to combine $\zeta=\zeta_{0.95}$ and $\zeta=\zeta_{0.05}$, which are derivable from the simulated distribution of $\hat{\zeta}$, together with $c=c_{0.95}$ and $c=c_{0.05}$, respectively. This study contends as in PRA procedures Guide[1] that this is justifiable because the variation in $c$ has the first order effect on fragility values whereas that in $\zeta$ has the second order effect in general. Similar fragility curves with 95% and 5% confidence are obtained for other states of damage.

In the probabilistic risk assessment of nuclear power plants, an "average" fragility curve $F^*(a)$ is derived and utilized often. This "average" curve is obtained by unconditionalizing $F(a \mid c)=\Phi\left[\frac{\ln(a/c)}{\zeta}\right]$ under the assumption that $c$ is lognormally distributed :

$$F^*(a)=\int_{all\ z} F(a \mid z) f_c(z)\,dz \qquad (13)$$

where $f_c(z)$ is the lognormal density function of $c$ with median $\hat{c}$ and log-standard deviation $\zeta_c$. $F^*(a)$ is not lognormal. However, in approximation, it may be and indeed was used in practice as lognormal distribution in PRA Procedures Guide.[1]

$$f_c(z)=\Phi\left[\frac{\ln(z/\hat{c})}{\zeta_c}\right] \qquad (14)$$

## 4. CONCLUSINONS

This study developed empirical fragility curves for bridge associated with different states of damage of HEPC's bridges under the 1995 Kobe earthquake event. Two parameter lognormal distribution functions were used to represent the fragility curves with the two parameters estimated by the maximum likelihood method. Statistical procedures were presented to test the goodness of fit hypothesis for these fragility curves and to estimate the confidence intervals of the two parameters of the lognormal distribution.

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