Performance Analysis of Blind Channel Estimation for Precoded Multiuser Systems

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Abstract: **Precoder has been shown to be able to provide source diversity and design flexibility. In this paper we employ precoding techniques for block transmission based on a multirate filterbank structure. To accommodate multiuser communication with variable data rates, different precoders with corresponding coefficients and up/down sampling rates are used. However, due to unknown multipath distortion, different interferences may exist in the received data, such as multiuser interference, intersymbol interference and interblock interference. To estimate channel parameters for a desired user, we employ all structured signature waveforms associated with different symbols of that user and apply subspace techniques. Therefore better performance of channel estimator can be achieved than the conventional subspace method based only on the signature of the current symbol. The delay for that user can also be jointly estimated. Channel identifiability conditions and asymptotic channel estimation error are investigated in detail. Numerical examples are provided to justify the proposed method.**

Index Terms: **Channel estimation, subspace decomposition, perturbation analysis.**

I. INTRODUCTION

There is increasing interest in designing new wireless communication networks which can provide multimedia services. The new networks will support not only traditional voice communication, but also image, video, and data transmissions at a much higher rate. Different multiuser communication schemes have been deployed such as time division multiple access (TDMA), frequency division multiple access (FDMA), code division multiple access (CDMA) or hybrid schemes [1]. These multiple access schemes allow many users to share a finite amount of radio spectrum. However, CDMA allows different users with different spreading codes to simultaneously transmit, thus increasing the system capacity. For this reason, CDMA technology has become prevalent in the new wireless networks.

One typical characteristic of CDMA systems is the exploitation of spreading sequences which differentiate users at the receiver end. It can be easily observed that direct sequence (DS) spreading before transmission imposes significant redundancy for each transmitted symbol. This highly redundant scheme offers the low-rate system a unique capability to combat multipath effects and mitigate inter-symbol interference (ISI). However, new wireless networks require accommodation of multirate sources from different users. Most current proposals suggest either multicode (MC) or multiple processing gain (MPG) mechanism [2], while requiring data rates to be integral multiples of some basic low-rate. In order to support variable rate transmission however, a comprehensive scheme needs to be investigated.

From signal processing perspective, multirate requirement can be easily satisfied by designing an appropriate multirate filterbank at the transmitter with adjustable upsampling rate P and downsampling rate M (assume $M \leq P$) [3]. In fact, to provide multirate service is a key objective in designing a digital communication system. For different purposes, the precoder usually deals with block transmission [4], [5]. In high data rate transmissions, the communication channel may introduce severe ISI. Channel equalization becomes necessary in symbol recovery [6]. Various solutions have been proposed based on different criteria such as maximum likelihood (ML), minimum mean square error (MMSE), or constant modulus (CM). A precoder offers its unique capability of suppressing ISI from channel distortion [5], [7]. With precoded transmission, equalization of frequency selective channels becomes more efficient. Extensive studies have been performed on either design of precoders or joint optimization of precoders and equalizers [8], [9].

Under the filterbank framework, recently proposed methods [5], [7] can unify multiple modulation schemes. With a moderate amount of leading/trailing zeros in each block, the limited interblock interference (IBI) can be mitigated and perfect equalization of any FIR channel by a FIR equalizer becomes possible in the absence of noise. The precoder can be optimized jointly with the receiver based on maximizing the output SNR or minimizing the mean square error (MSE) under different constraints. The precoded transmission method has been shown to be directly applicable to a downlink multiuser communication system. No blocking is suggested at the transmitter. However, this strategy restricts flexible design of the communication system, especially for a variable rate system. Moreover, in high speed communications, the channel could span several block periods resulting in significant IBI due to the precoder. The level of IBI might be many orders higher than the ISI. Furthermore, the multiuser communication requirement complicates the equalization procedure, since multiuser interference (MUI) impedes reliable symbol detection.

In this work, we propose a more general scheme to gain much flexibility in system design and unify various multiple access methods [10]. Different from [7], we explicitly consider in the received data IBI in addition to MUI and ISI. For such a system, blind multiuser detection can be performed based on channel estimates. We observe that various signatures of symbols from a desired user exhibit intriguing structures, which are convolutions of the precoder coefficients with common unknown multi-

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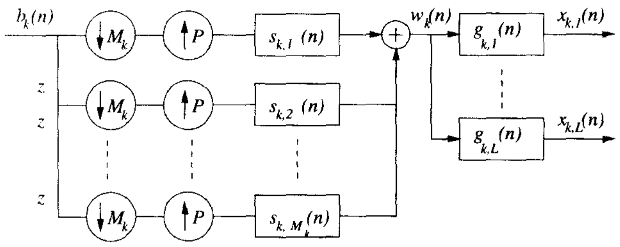

Fig. 1. Precoded multirate multiuser system with receiver antenna diversity.

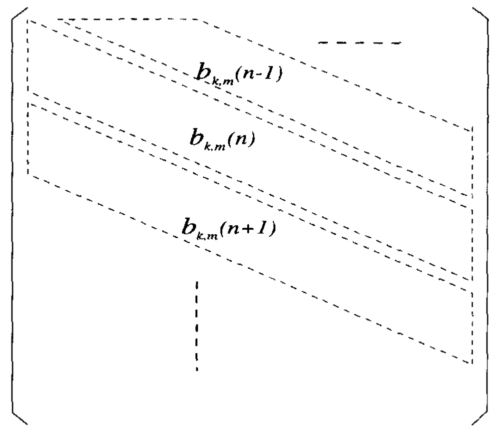

Fig. 2. Structure of matrix $T_{k,m}(n)$.

path channel parameters. Thus we minimize total projections of these signatures onto the estimated noise subspace to obtain better channel estimate and the delay. Similar idea has been used in [11] in a different scenario based on a subspace intersection technique. The structured signatures have also been employed to estimate downlink CDMA channels in [12]. We attempt to generalize the technique to variable data rate communication systems. The identifiability conditions are also investigated. Due to finite data sample effect in estimating the noise subspace, the mean square error of the channel estimate is derived in a closed-form as a function of the number of data samples and system parameters based on perturbation theory. Simulation results show that significant improvement is achieved by employing a complete set of signature waveforms. However, optimization of the precoder coefficients is beyond the scope of the current work. Many interesting results on precoder design/optimization have appeared in the literature [8], [13], and [14].

This paper is organized as follows. A multirate precoding framework for a multiuser communication system with antenna diversity is described in Section II. Blind channel estimation technique is proposed and its identifiability conditions are discussed in Section III. Then the performance of the estimation method is analyzed in Section IV. Connections with existing approaches are demonstrated in Section V. Numerical examples are provided in Section VI. Finally some conclusions are drawn in the last section.

II. MULTIRATE PRECODING FRAMEWORK IN MULTIUSER COMMUNICATIONS

Let us assume K users are simultaneously provided service in a wireless communication system. User k has i.i.d. symbol stream $b_k(n)$ to transmit (see Fig. 1). This stream is partitioned into consecutive blocks with each block M_k symbols and converted into M_k parallel sub-streams. In the n-th block, let $b_{k,m_k}(n)$ denote the m_k-th symbol $b_k(nM_k + m_k)$. The m_k-th branch is followed by a downsampler by factor M_k and an upsampler by factor P with filter coefficients $s_{k,m_k}(n)$. By design, the precoders are assumed to be FIR with length P for all users and linear time-invariant (LTI). We adopt common upsampling rate P for all users but different downsampling rates ($M_k < P$). Therefore variable information rates are converted into a fixed transmission rate to maximize the channel's capacity. For simplicity of description, we still term signals after precoders by "chips" as in CDMA communications. If we denote the symbol period for user k by T_{b_k} and common chip period by T_c, then within M_k symbol periods, there are P chips to be transmitted. By simple calculation, we can find that $M_k = \frac{T_c}{T_{b_k}}P$, or equivalently $M_k = \frac{R_{b_k}}{R_c}P$. By adjusting M_k for each user, its information rate can be easily satisfied.

The transmission structure is a typical multirate filterbank [3]. Its output is a superposition of signals from M_k branches (e.g., [7])

$$w_k(n) = \sum_{m=1}^{M_k} w_k^{(m)}(n),\ w_k^{(m)}(n) = \sum_{i=-\infty}^{\infty} b_{k,m}(i)s_{k,m}(n-iP) \tag{1}$$

These coded chips are then sent to the destination through wireless channels. The communication channel is assumed to be FIR and LTI. Assume L antennas are employed at the receiver. The composite channel impulse response for the l-th sub-channel including physical channel and the antenna response can be combined and denoted by $g_{k,l}(n)$. We factorize its order q_k by P: $q_k = Q_kP + \eta_k$ where Q_k and η_k are both integers with $0 \leq \eta_k < P$. Hence the noise-free data from user k of the l-th sub-channel and m-th branch can be expressed by

$$x_{k,l}^{(m)}(n) = \sum_{q=0}^{Q_kP+\eta_k} g_{k,l}(q) \sum_{i=-\infty}^{\infty} b_{k,m}(i)s_{k,m}(n-q-iP). \tag{2}$$

To explicitly reveal the channel input/output (I/O) relationship, we will employ matrix representations. Let us collect ν chip samples of $x_{k,l}^{(m)}(n)$ in a vector $\boldsymbol{x}_{k,l}^{(m)}(n)$ from $x_{k,l}^{(m)}(nP)$ to $x_{k,l}^{(m)}(nP+\nu-1)$. Since the channel spans at most Q_kP+P-1 chip periods, ν is chosen as $\nu = QP$ which satisfies $\nu \geq \max(P+Q_kP+P) = \max(Q_k+2)P$. We also arrange channel coefficients and precoder coefficients in different vectors $\boldsymbol{g}_{k,l} = [g_{k,l}(0), \cdots, g_{k,l}(q_k)]^T$, $\boldsymbol{s}_{k,m} = [s_{k,m}(0), \cdots, s_{k,m}(P-1)]^T$. After careful arrangement, the convolution in (2) can be nicely written in a matrix form as $\boldsymbol{x}_{k,l}^{(m)}(n) = \boldsymbol{T}_{k,m}(n)\boldsymbol{g}_{k,l}$ where $\boldsymbol{T}_{k,m}(n)$ has the structure in Fig. 2 with subblocks (in dashed dotted box) associated with different symbols. Each subblock is obtained by shifting a $\nu \times (q_k+1)$ filtering matrix $\boldsymbol{S}_{k,m}$ up or down by multiple of P rows

$$S_{k,m} = \begin{bmatrix} s_{k,m}(0) & & 0 \\ \vdots & \ddots & s_{k,m}(0) \\ s_{k,m}(P-1) & & \vdots \\ 0 & \ddots & s_{k,m}(P-1) \\ \vdots & 0 & 0 \end{bmatrix}, \quad (3)$$

and scaling it by different symbols. The shift operation can be made by a $\nu \times \nu$ Jordan matrix J with all $1's$ in the first diagonal below the main diagonal. For convenience, we will use the symbol J^{-1} to denote J^T although J is singular ([5]): $J^{-1} \triangleq J^T$ and define J^0 as an identity matrix. Assume the k-th user arrives at the receiver in a quasi-synchronous way with delay τ_k in chip periods. After collecting L data vectors of $x_{k,l}^{(m)}(n)$ successively in a big vector and considering M_k branches, it can be easily shown that the received data vector has the following form

$$y(n) = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \sum_{j=-(Q_k+1)}^{Q-1} A_{k,m,j} g_k b_{k,m}(n+j) + v(n), \quad (4)$$

where g_k is a channel vector with L sub-vectors $g_{k,l}$ as its successive entries, $A_{k,m,j}$ is a code filtering matrix

$$A_{k,m,j} = I_L \otimes (J^{Pj+\tau_k} S_{k,m}), \quad (5)$$

where I_L is an identity matrix of dimension L, "$\otimes$" represents the Kronecker product, and $v(n)$ is AWGN. The I/O model (4) shows that $y(n)$ includes not only ISI within a block (indexed by m), but also IBI (indexed by j) from neighboring blocks. However, all signatures $A_{k,m,j} g_k$ of different symbols from user k contain information about the channel vector g_k. Based on this observation, we will derive a subspace based cost function to blindly estimate channel vector g_k and delay τ_k next. Without loss of generality, user 1 is assumed to be the desired user.

III. BLIND CHANNEL ESTIMATION AND ITS IDENTIFIABILITY

It is well known that subspace technique is a powerful tool for blind channel estimation. It was originally proposed by [15], and has been successfully applied to single rate CDMA systems [16]–[19]. The method has been improved based on a subspace intersection idea [11]. It has been shown that it is also applicable to block equalization [5] in a precoded system with limited IBI and fixed data rate. We will extend this idea to channel and delay estimation in a variable rate multiuser communication scenario.

Let us first examine the data vector $y(n)$ in (4). The signature vector corresponding to $b_{k,m}(n+j)$ is $A_{k,m,j} g_k$ and denoted by $h_{k,m,j}$. If we collect these vectors for user k in a matrix H_k column by column

$$H_k = [h_{k,m,j}], \; j = -(Q_k+1), \cdots, Q-1; \; m = 1, \cdots, M_k, \quad (6)$$

and also put $b_{k,m}(n+j)$ for all j and m at time n in a vector $b_k(n)$, then (4) is simplified as

$$y(n) = \sum_{k=1}^{K} H_k b_k(n) + v(n) = Hb(n) + v(n), \quad (7)$$

where H is obtained by stacking H_k column by column, and $b(n)$ is a long vector with $b_k(n)$ as its entries. Without loss of generality, we assume signature waveforms $\{H_k\}_{k=1}^{K}$ are linearly independent. The data correlation matrix R is computed from (7) as

$$R = E\{y(n)y^H(n)\} = \sum_{k=1}^{K} \alpha_k H_k H_k^H + \sigma_v^2 I, \quad (8)$$

where superscript "H" denotes complex conjugate transpose, α_k is the signal power of user k, σ_v^2 is the noise power. To obtain the noise subspace, eigenvalue decomposition of R yields

$$R = U_s \Lambda_s U_s^H + U_n \Lambda_n U_n^H, \; \Lambda_s = diag(\lambda_i^2 + \sigma_v^2), \; \Lambda_n = \sigma_v^2 I \quad (9)$$

where U_s spans the signal subspace which is also a range space of H and U_n spans the noise subspace. Since signature vectors $h_{1,m,j}$ of the desired user lie in the signal subspace, they are orthogonal to U_n

$$U_n^H h_{1,m,j} = U_n^H A_{1,m,j} g_1 = 0, \quad (10)$$

for all possible j and m. Hence our cost function to estimate g_1 based on the estimated noise subspace follows [10], [15], and [19]

$$\xi(g) = g^H [\sum_{m=1}^{M_1} \sum_{j=-(Q_1+1)}^{Q-1} A_{1,m,j}^H \hat{U}_n \hat{U}_n^H A_{1,m,j}] g. \quad (11)$$

To guarantee a unique solution, it is required that $\hat{U}_n^H A_{1,m,j}$ have full column rank. This matrix has $\mu \triangleq LQP - \sum_{k=1}^{K} M_k(Q+Q_k+1)$ rows and $L(q_1+1)$ columns. Therefore

$$L(QP - q_1 - 1) \geq \sum_{k=1}^{K} M_k(Q + Q_k + 1). \quad (12)$$

(12) provides flexible choices for parameters based on the trade-off between design complexity and system capacity. If some users have low rate information sources, then the system can accommodate more users. If there are more antennas (large L), then short observation window is sufficient (small Q).

It might be argued that (11) offers little novelty. However, structured signatures together with multiple contributions from ISI and IBI to the channel estimator may affect identifiability conditions. Similar structured signatures have been employed in [12] to estimate downlink CDMA channels. Detailed identifiability results are also provided therein. For uplink channels, we currently restrict our attention to the case where the channel span is less than the symbol interval and establish the following lemma.

Lemma 1: Let S be a code matrix for the desired user constructed by stacking $A_{1,m,j}$ for all m, j column by column, and

H_{int} be an interference matrix whose columns consist of signatures from other users

$$S = [A_{1,m,j}], \quad H_{int} = [H_2, \cdots, H_K].$$

If $[S\ H_{int}]$ has full column rank and (12) holds, then g_1 is a unique minimum solution of (11) within a scalar ambiguity.

Proof: For simplicity of presentation and without loss of generality, we may assume $L = 1$, $M_1 = 1$. The proof can be easily extended to a general scenario. Then we can denote $A_{1,j,m}$ by A_j and $S_{1,m}$ by S for notational convenience. The proof follows an extension of [18] and can proceed by contradiction [5], [20]. The condition on the channel span within one symbol interval is embedded in the rank condition on $[S\ H_{int}]$. If it is violated, then it can be easily verified that $[S\ H_{int}]$ becomes rank deficient. Matrix S has contributions corresponding to the desired symbol, ISI, and IBI.

Suppose there is another vector $\widetilde{g}_1$ which makes $A_j\widetilde{g}_1 \in U_s$ for $\forall j$. Since span$\{U_s\}$ = span$\{H\}$, $A_j\widetilde{g}_1$ can be expressed by different linear combinations of column vectors in H for different j. By stacking vectors $A_j\widetilde{g}_1$ for all j in a matrix, we can conclude that there exists matrices D_1 and D such that

$$S\ diag(\widetilde{g}_1) = S\ diag(g_1)\ D_1 + H_{int}D, \qquad (13)$$

where $H_1 = S\ diag(g_1)$ has been used and $diag(\cdot)$ represents a block diagonal matrix with diagonal entry to be the inner quantity. Since $[S\ H_{int}]$ has full column rank, from (13) we have

$$diag(\widetilde{g}_1) = diag(g_1)\ D_1. \qquad (14)$$

By equating each sub-block of both sides of (14) we conclude our proof. □

Different from existing approaches in channel estimation for CDMA systems, our method employs all available information associated with the unknown channel. However, the delay τ_1 for user 1 is not known a priori and thus needs to be estimated. Taking into account (11), joint estimation of the delay and channel vector for the desired user can be formulated as follows

$$(\widehat{\tau}_1, \widehat{g}_1) = \arg\min_{\tau, g} g^H [\sum_{m=1}^{M_1} \sum_{j=-(Q_1+1)}^{Q-1} A_{1,m,j}^H \widehat{U}_n \widehat{U}_n^H A_{1,m,j}] g, \qquad (15)$$

where $A_{1,m,j}$ is a function of the delay and given by (5). The unit norm constraint on g should be imposed to avoid trivial solutions. Since chip synchronization is assumed, it only requires checking all possible integers in $[0, P-1]$ for the delay. For each possible value, the normalized channel vector is the eigenvector of the matrix inside the brackets corresponding to its minimum eigenvalue within a phase ambiguity. Due to the unknown multipath distortion, the arbitrarily long channel span exceeding one symbol interval will require new identifiability conditions and thus constitutes an open research topic. Some novel ideas from [12] on identifying CDMA downlink channels will be definitely beneficial to such an investigation.

Once the channel is estimated, signature vector of each symbol $b_{1,m}(n+j)$ from the desired user can be constructed as $A_{1,m,j}\widehat{g}_1$. They can serve for symbol detection purposes. Since at each time n, contributions to the data vector are made by M_1 symbols from the current block (block n) and also various symbols from neighboring blocks (block $n+j$ for $j \neq 0$), attention can be restricted to directly detect M_1 symbols from the current block ($j = 0$) only at each time. Other options also exist by detecting interfering symbols from the neighboring blocks first and then subtracting their contributions from the data vector to improve the detection performance.

IV. PERFORMANCE ANALYSIS OF THE CHANNEL ESTIMATION METHOD

The detection scheme will depend on the accuracy of channel estimate. Therefore, channel estimation error needs to be quantified. Since the estimated noise subspace is obtained from eigen decomposition of sample covariance matrix from N data sample vectors

$$\widehat{R} = \frac{1}{N} \sum_{n=1}^{N} y(n) y^H(n), \qquad (16)$$

the number of available data samples will affect the accuracy of the subspace estimate, thus the performance of the estimator. In this section, we first study the perturbation of the noise subspace if R is estimated by (16). Then we investigate the covariance and MSE of our channel estimator as functions of N. The analyses will be based on the perturbation technique [21]–[23]. However, we do not assume a particular distribution of the source [24] such as a Gaussian process assumed in most array signal processing work and used in obtaining MSE in our previous paper [10]. Since channel parameters are jointly estimated with time delay, the performance is also determined by the delay estimation error. This joint problem will be simplified by assuming perfect timing in the following derivation (for example, [25]).

A. Perturbation in the Noise Subspace

Our method employs the noise subspace of estimated correlation matrix instead of the data matrix as used in [17], [25], therefore the results therein are not directly applicable. However, similar procedures can be followed. We will denote a perturbation by preceding the corresponding quantity by δ. Then an estimation error occurs in estimating R due to finite data samples [26]: $\delta R = \widehat{R} - R$. δR is Hermitian and is a small perturbation when N is large enough [23]. This perturbation will cause an error in the estimate of noise subspace and finally transfer an error to the channel estimate. We will evaluate this error analytically by first deriving the perturbation of the noise subspace of R and then the perturbation in the channel estimate.

For notational convenience, let Z be the noise-free data correlation matrix

$$Z = R - \sigma_v^2 I = U_s \Omega U_s^H, \quad \Omega = diag\{\lambda_i^2\}, \qquad (17)$$

and X be the objective matrix in the cost function without perturbation

$$X = \sum_{m=1}^{M_1} \sum_{j=-(Q_1+1)}^{Q-1} A_{1,m,j}^H U_n U_n^H A_{1,m,j}. \qquad (18)$$

First, the perturbation in the noise subspace is given in the following lemma as proved in APPENDIX A.

Lemma 2: If $\boldsymbol{R}$ is perturbed to be $\boldsymbol{R} + \delta\boldsymbol{R}$, the first order perturbation of the noise subspace has the following form

$$\delta \boldsymbol{U}_n \approx -\boldsymbol{Z}^\dagger \delta \boldsymbol{R} \boldsymbol{U}_n, \tag{19}$$

where $(\cdot)^\dagger$ denotes pseudo-inverse.

This lemma shows that the first order perturbation in the noise subspace of a matrix is characterized by the perturbation in that matrix. Such result is not oriented to a particular channel estimation method. However, as a general result, it is definitely applicable to analysis of the performance of our channel estimator. Notice that our result in (19) has a form very similar to that in [22], although the corresponding terms have different meanings.

B. Channel Estimation Error

The bias of channel estimate pertaining to the proposed method is dependent on $\delta\boldsymbol{U}_n$ thus $\delta\boldsymbol{R}$. The following lemma provides an analytical result for channel estimate when $\boldsymbol{R}$ is not accurately estimated.

Lemma 3: Due to perturbation $\delta\boldsymbol{R}$, the first order perturbation of channel estimate is given by

$$\delta \boldsymbol{g}_1 \approx \sum_{m=1}^{M_1} \sum_{j=-(Q_1+1)}^{Q-1} \boldsymbol{X}^\dagger \boldsymbol{A}_{1,m,j}^H \boldsymbol{U}_n \boldsymbol{U}_n^H \delta \boldsymbol{R} \boldsymbol{Z}^\dagger \boldsymbol{A}_{1,m,j} \boldsymbol{g}_1. \tag{20}$$

Proof: In the absence of subspace perturbation, $\boldsymbol{g}_1$ (which has been assumed to be unitary) is an eigenvector of the $(q_1 + 1) \times (q_1 + 1)$ matrix $\boldsymbol{X}$ associated to its unique null eigenvalue. According to (18), the perturbation of $\boldsymbol{X}$ due to $\delta\boldsymbol{U}_n$ has the form

$$\delta \boldsymbol{X} \approx \sum_m \sum_j \boldsymbol{A}_{1,m,j}^H [\delta \boldsymbol{U}_n \boldsymbol{U}_n^H + \boldsymbol{U}_n \delta \boldsymbol{U}_n^H + \delta \boldsymbol{U}_n \delta \boldsymbol{U}_n^H] \boldsymbol{A}_{1,m,j}.$$

After substituting (19), $\delta\boldsymbol{X}$ is related to $\delta\boldsymbol{R}$ by

$$\delta \boldsymbol{X} \approx -\sum_m \sum_j \boldsymbol{A}_{1,m,j}^H [\boldsymbol{Z}^\dagger \delta \boldsymbol{R} \boldsymbol{U}_n \boldsymbol{U}_n^H + \boldsymbol{U}_n \boldsymbol{U}_n^H \delta \boldsymbol{R} \boldsymbol{Z}^\dagger] \boldsymbol{A}_{1,m,j}. \tag{21}$$

Then $\delta\boldsymbol{g}_1$ has the following form [25], [26], and [27]

$$\delta \boldsymbol{g}_1 \approx -\boldsymbol{X}^\dagger \delta \boldsymbol{X} \boldsymbol{g}_1. \tag{22}$$

After substituting (21) in (22) and noticing that $\boldsymbol{U}_n^H \boldsymbol{A}_{1,m,j} \boldsymbol{g}_1 = \boldsymbol{0}$, we obtain (20). □

It is observed that Lemma 3 provides a direct link between the bias of channel estimate for a precoded multirate multiuser system and perturbation in correlation estimation. It is fundamental to the performance of the proposed channel estimator. According to (20), $\delta\boldsymbol{g}_1$ is a random vector due to the randomness of $\delta\boldsymbol{R}$. Since $\delta\boldsymbol{R}$ depends on the second-order statistics of the received data, the channel mean-square-error (MSE) which is defined as $E\{||\delta\boldsymbol{g}_1||^2\}$ depends on the fourth-order statistics of the input and the noise. For notational convenience, denote $\boldsymbol{y}(n)$ in (7) by $\boldsymbol{y}_n$ and $\boldsymbol{v}(n)$ by $\boldsymbol{v}_n$

$$\boldsymbol{y}_n = \boldsymbol{H}\boldsymbol{b}_n + \boldsymbol{v}_n, \tag{23}$$

where inputs from all users are collected in a vector $\boldsymbol{b}_n$ whose variance is denoted by $E\{\boldsymbol{b}_n\boldsymbol{b}_n^H\} = \boldsymbol{\Gamma}$. Assume all inputs have zero odd moments, equal variance $E\{|b_{k,m}(n)|^2\} = \sigma_b^2$. They are also independent of each other, rendering $\boldsymbol{\Gamma}$ a diagonal matrix $\sigma_b^2\boldsymbol{I}$. In the case when users have different transmission power, one can properly scale the corresponding columns of $\boldsymbol{H}$ to make the power of all users virtually equal for the evaluation purpose. Additionally, we assume $\boldsymbol{v}_n$ has zero odd moments and is independent of inputs. Following similar procedures as used in a correlation matching context [28], it is shown in APPENDIX B that the channel MSE has the following closed-form expression

$$E\{||\delta \boldsymbol{g}_1||^2\} \approx \frac{\sigma_v^2}{N} \sum_{m_1,j_1,m_2,j_2} \gamma_{m_1,j_1,m_2,j_2} \, tr\{\boldsymbol{X}^\dagger \boldsymbol{A}_{1,m_1,j_1}^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{A}_{1,m_2,j_2} \boldsymbol{X}^\dagger\}, \tag{24}$$

where the lower and upper limits in the summation are the same as those for m and j in (20) respectively, "tr" denotes the trace of a matrix, scalar γ_{m_1,j_1,m_2,j_2} is given by

$$\gamma_{m_1,j_1,m_2,j_2} = \boldsymbol{g}_1^H \boldsymbol{A}_{1,m_2,j_2}^H \boldsymbol{Z}^\dagger \boldsymbol{R} \boldsymbol{Z}^\dagger \boldsymbol{A}_{1,m_1,j_1} \boldsymbol{g}_1. \tag{25}$$

It can be observed that the MSE is proportional to $\frac{\sigma_v^2}{N}$ which is similar to the result obtained in [25]. Meanwhile, it is determined by projections of code matrices $\boldsymbol{A}_{1,m,j}$ of the desired user onto the noise subspace according to the quantity inside the trace operator. If we examine the scalar γ_{m_1,j_1,m_2,j_2}, we can conclude that the MSE is approximately inversely proportional to the power of the transmitted signal because of the term $\boldsymbol{Z}^\dagger\boldsymbol{R}\boldsymbol{Z}^\dagger$. Based on the subspace decomposition of $\boldsymbol{Z}$ in (17), γ_{m_1,j_1,m_2,j_2} also depends on projections of signature vectors of various bits from the desired user onto the signal subspace.

V. CONNECTIONS WITH EXISTING SYSTEM MODELING AND CHANNEL ESTIMATION

We have presented a general precoding framework suitable for multirate transmission and proposed a corresponding subspace-based channel estimation method. The precoder can be chosen to satisfy requirements from a particular application, leading to much flexibility in system design. It can be used to describe a variety of multirate communication systems [7], [29]. Multicarrier systems such as orthogonal frequency-division multiplexing (OFDM) and multicarrier CDMA fit very well the proposed transmission setup. It is also clear that such a framework works for a multirate DS/CDMA system by imposing certain appropriate constaints on the structure of the precoder. In such a case, multirate communication requirement can be satisfied by either a muitlcode (MC) multirate access method or a multiple processing gain (MPG) multirate access method [30], [31]. Recalling the second sub-equation in (1), if we treat $\boldsymbol{s}_{k,m}$ (of length P) as the spreading sequence for the

m-th bit $b_{k,m}(n)$ in the n-th block of user k, then it represents a well-known CDMA spreading model [19]. The first part of (1) then suggests M_k code sequences for M_k successive bits of user k, giving rise to a MC-CDMA model. On the other hand, if each code vector $\boldsymbol{s}_{k,m}$ only has finite support of a portion of length P and all of them do not overlap each other, then a MPG-CDMA system follows. Correspondingly, the channel estimation method is well applicable to these scenarios.

The proposed setup can also be easily adapted to a system dealing with single-rate sources by setting the downsampling factor M_k to be 1. If we employ only one antenna ($L = 1$), then it degrades to an extensively studied single-rate CDMA framework. It is demonstrated that our channel estimation method is based on the subspace technique. Such a technique has been widely applied to estimate channel parameters for a single-rate CDMA system when ISI exists in the received data [17], [19]. Due to the linearity of the system and similar coding/spreading scheme, there is no doubt that our method is consistent with the existing channel estimation approaches when it is applied to a single-rate system. We will reveal this equivalence next. For simplicity and clarity of presentation, we still assume user 1 is the desired user and the receiver is synchronized to this user.

Let us take the typical subspace-based channel estimation method [19] as an example and compare its optimization criterion with the current one when $L = 1$, $M_1 = 1$. We focus on two fundamental equations (10) and (11) upon which our method is based. Both the noise subspace and the code filtering matrix $\boldsymbol{A}_{1,1,j}$ are involved in those equations. According to (5), $\boldsymbol{A}_{1,1,j}$ is simplified to

$$\boldsymbol{A}_{1,1,j} = \boldsymbol{J}^{Pj}\boldsymbol{S}_{1,1}, \tag{26}$$

where j can take any integer from $-(Q_1 + 1)$ to $Q - 1$.Then (10) becomes

$$\boldsymbol{U}_n^H \boldsymbol{J}^{Pj}\boldsymbol{S}_{1,1}\boldsymbol{g}_1 = \boldsymbol{0}. \tag{27}$$

In (27), operation $\boldsymbol{J}^{Pj}$ can be understood as being performed on either $\boldsymbol{S}_{1,1}$ (row-shifting) or $\boldsymbol{U}_n^H$ (column-shifting). The former is what has been adopted in this paper. As for the latter, post-multiplying $\boldsymbol{U}_n^H$ by $\boldsymbol{J}^{Pj}$ will shift columns in $\boldsymbol{U}_n^H$ to the left ($j > 0$) or the right ($j < 0$) by multiples of P positions. Since $\boldsymbol{U}_n$ has $\nu = QP$ rows, there are totally QP columns in $\boldsymbol{U}_n^H$. To clearly demonstrate such a shifting operation, define $\boldsymbol{G} = \boldsymbol{U}_n^H$ and partition it into Q sub-blocks with each sub-block P columns: $\boldsymbol{G} = [\underline{G}_1, \cdots, \underline{G}_Q]$, as (29) and (25) in [19] respectively. After considering all possible j in a descending order (starting from its largest value), (26) leads to a set of equations which can be stacked row by row in a matrix form

$$\begin{bmatrix} \underline{G}_Q & & & \\ \vdots & \ddots & & \\ \vdots & \underline{G}_Q & & \\ \vdots & \vdots & \ddots & \\ \underline{G}_1 & \vdots & & \underline{G}_Q \\ & \ddots & \vdots & \vdots \\ & & \underline{G}_1 & \cdots & \underline{G}_{Q-Q_1-1} \end{bmatrix} \boldsymbol{S}_{1,1}\boldsymbol{g}_1 = \boldsymbol{0}.$$

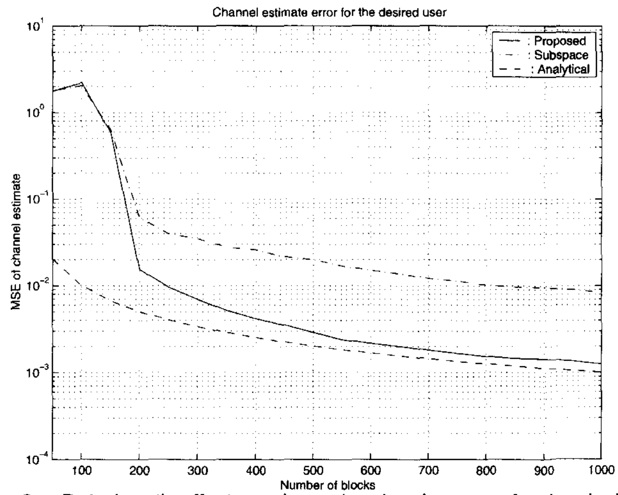

Fig. 3. Data length effect on channel estimation error for the desired user.

By appending some zero rows beneath $\boldsymbol{S}_{1,1}$ to form a matrix $\mathcal{C}_1$, we obtain

$$\mathcal{G}\mathcal{C}_1\boldsymbol{g}_1 = \boldsymbol{0}, \tag{28}$$

where $\mathcal{G}$ is a block Toeplitz matrix constructed from $\underline{G}_1, \cdots, \underline{G}_Q$ as in [19]. It is clear that (28) shows a similar form as (33) in [19]. As a consequence, our optimization criterion (11) is equivalent to (34) in [19] when it deals with a single-rate CDMA system.

Similarities between the proposed method and the existing methods stem from the application of a common subspace technique. However, our precoding framework is not restricted to a single-rate single-antenna system. It is also applicable to multirate multiuser communications.

VI. SIMULATIONS

We test the proposed method by simulating a variable data rate system with 5 equally-powered users. User 1 is assumed to be the desired user. Each user has i.i.d. binary symbols to transmit. Hadamard codes of length $P = 32$ are chosen for each of the five precoders. Different information rates are obtained by choosing downsampling factors (M_k) for different filterbanks as $[6, 3, 9, 3, 3]$ respectively. All sub-channels ($g_{k,l}(n)$) are randomly generated with unit power for different users and have orders ($q_{k,l}$) $[5, 11, 19, 19, 18]$. We employ 3 antennas ($L = 3$) at the receiver. Delays are randomly selected. The SNR is set to be 20dB. We adopt the MSE as a performance measure. It will be compared with our analytical result and the conventional subspace method (here defined as the method which uses only the signature waveform of the current symbol). The blind MMSE detector based on the proposed channel estimates is constructed to detect the desired symbols. Its performance is measured by the output signal to interference plus noise ratio (SINR).

First, we test the effect of the input data length (in blocks) on the MSE of channel estimation. Our method is compared with the conventional subspace method. It is also compared with the analytical result in (24). The average channel estimation MSEs from 50 independent realizations versus the number of data blocks (N) are presented in Fig. 3 by the solid

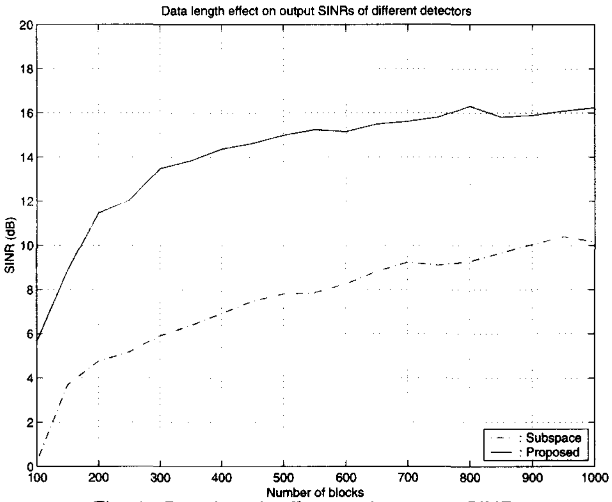

Fig. 4. Data length effect on the output SINR.

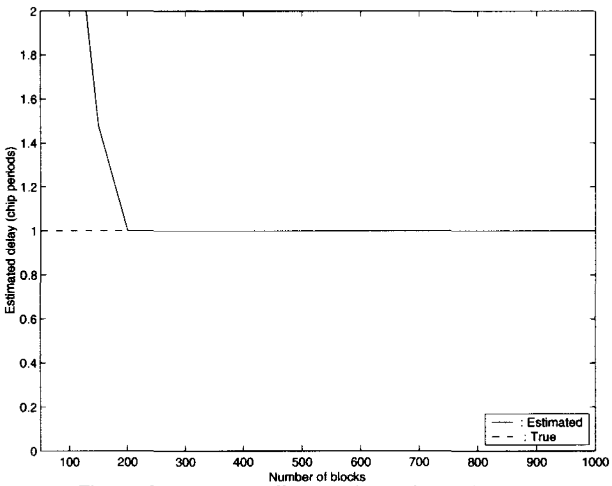

Fig. 5. Data length effect on the estimated delay.

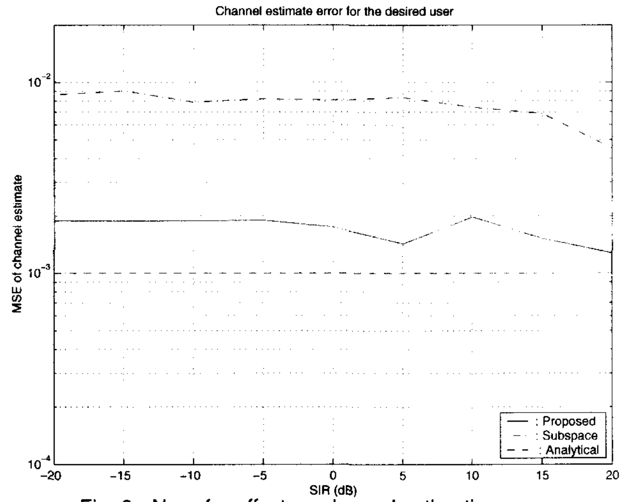

Fig. 6. Near-far effect on channel estimation error.

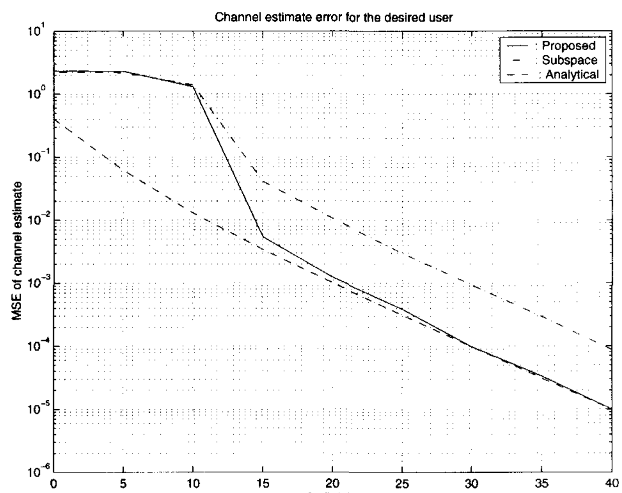

Fig. 7. Noise effect on channel estimation error.

line, dashed-dotted line and dashed line respectively. As can be seen, the MSE based on the proposed method is much smaller than that obtained from the conventional subspace method after 200 blocks. Our method also approaches the asymptotic performance for large N which validates our analysis. The output SINRs of the corresponding MMSE detectors are shown in Fig. 4. More than 7dB gain is achieved after 1000 blocks of data are received and processed. Together with channel estimates, delay is also estimated by the proposed method for the desired user in Fig. 5. It is observed that estimate is reliable after 200 blocks which coincides with Fig. 3.

Next, we study the near-far effect on the channel estimation error. The result obtained after 1000 blocks is presented in Fig. 6. We vary the power for user 1 while maintain equal constant power for all other 4 users. The SNR is 20dB. Our method is tested for different signal to interference ratios (SIR) from -20dB to 20dB. Similarly for comparison, results for the conventional subspace method and from our analysis are also provided. It can be seen that the proposed channel estimation method is near-far resistant.

We also investigate the noise effect and present the results in Fig. 7 for a large range of SNR levels from 0dB to 40dB. It is observed that both proposed and conventional subspace methods perform well when SNR is high. The experimental result for our method is also highly consistent with our analysis. However, the proposed method requires higher SNR for explicit separation of the signal subspace from noise subspace, which is true for most subspace based methods. Therefore, small noise is desired in order to achieve better estimation performance.

The proposed multirate precoding framework is also applicable to a multirate CDMA system. As an example, we consider a dual-rate synchronous CDMA system with a MPG multirate access method [31]. One group has two low-rate users while the other group has two 4-fold data rate users. The spreading sequences of length 32 are obtained by adding one more random code to the Gold sequences of length 31. Then each sequence is split into 4 subsequences to be assigned to each high-rate user [30]. All users transmit BPSK data symbols at the same power per bit. The multipath channel coefficients for each user are randomly generated. All channels are assumed to have order 20. Each time $\nu = 96$ chip data samples are collected in a data vector for processing. The bit SNR is set to be 15dB. Again, the experimental channel estimation MSE versus the number of data blocks N is compared with our analytical result and that based on the conventional subspace method. The results for one

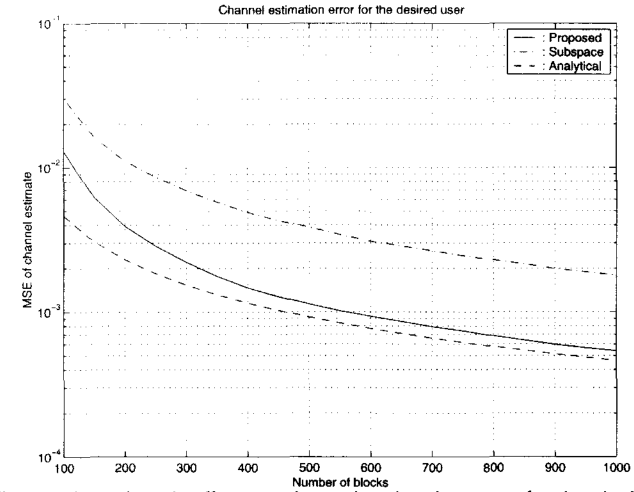

Fig. 8. Data length effect on channel estimation error for the desired user in a MPG multirate CDMA system.

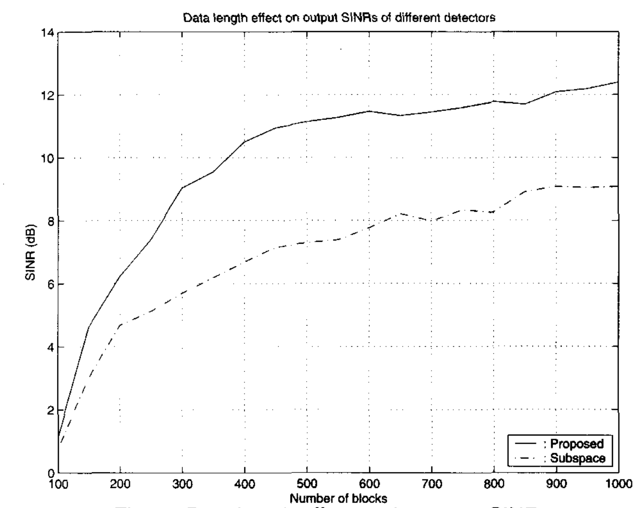

Fig. 9. Data length effect on the output SINR.

user (e.g., a low-rate user) are plotted in Fig. 8 by the solid line, dashed line and dashed-dotted line respectively. The proposed method performs better than the conventional subspace method. It is not surprising that the experimental result is highly consistent with the analytical result when SNR is high or N is large on which our perturbation analysis is based. The superiority of the proposed method can also be observed from Fig. 9, where the output SINRs of the MMSE detectors constructed from corresponding estimated channel parameters are compared. Obviously the proposed detector gives higher SINRs.

VII. CONCLUSIONS

In order to gain more design flexibility for a multiuser communication system, multirate precoding scheme can be employed before transmission. When the communication channel has a long impulse response, the received data is significantly corrupted by inter-block interference besides other interferences. In this paper, we explore the structured signature waveforms and apply the subspace technique to estimate the multipath channel parameters. The identifiability conditions are discussed. The asymptotic performance of the channel estimator is analyzed. The channel estimation mean-square-error is derived in a closed-form based on perturbation theory. Simulation results further validate our analysis.

APPENDIX A: PROOF OF LEMMA 2

The subspace decomposition of $\boldsymbol{R}$ is given by (9) where $\boldsymbol{U}_s$ spans the signal subspace and $\boldsymbol{U}_n$ spans the noise subspace. Assume the first-order perturbation of the noise subspace is $\delta\boldsymbol{U}_n$. Then the perturbed noise subspace can be expressed as [22], [32]

$$\widehat{\boldsymbol{U}}_n = \boldsymbol{U}_n + \delta\boldsymbol{U}_n = \boldsymbol{U}_n + \boldsymbol{U}_s\boldsymbol{Q},$$

where $\boldsymbol{Q}$ is a matrix whose norm is of the order of $||\delta\boldsymbol{R}||$. The matrix norm can be any submultiplicative norm such as the Euclidean 2-norm or the Frobenius norm.

Since the perturbation scenario is different from that discussed in [22], the expression for $\boldsymbol{Q}$ needs to be investigated. After perturbation, the subspace decomposition of $\widehat{\boldsymbol{R}}$ becomes

$$\widehat{\boldsymbol{R}} = \widehat{\boldsymbol{U}}_s\widehat{\boldsymbol{\Lambda}}_s\widehat{\boldsymbol{U}}_s^H + \widehat{\boldsymbol{U}}_n\widehat{\boldsymbol{\Lambda}}_n\widehat{\boldsymbol{U}}_n^H.$$

Correspondingly, $\widehat{\boldsymbol{\Lambda}}_s$ and $\widehat{\boldsymbol{\Lambda}}_n$ are perturbed versions of $\boldsymbol{\Lambda}_s$ and $\boldsymbol{\Lambda}_n$. From the orthogonality, we have

$$(\boldsymbol{U}_n+\boldsymbol{U}_s\boldsymbol{Q})^H(\boldsymbol{R}+\delta\boldsymbol{R}) = (\boldsymbol{\Lambda}_n+\delta\boldsymbol{\Lambda}_n)(\boldsymbol{U}_n+\boldsymbol{U}_s\boldsymbol{Q})^H. \quad (29)$$

By noticing $\boldsymbol{U}_n^H\boldsymbol{R} = \boldsymbol{\Lambda}_n\boldsymbol{U}_n^H$, $\boldsymbol{U}_s^H\boldsymbol{R} = \boldsymbol{\Lambda}_s\boldsymbol{U}_s^H$, and taking Hermitian on both sides of (29), we obtain

$$\delta\boldsymbol{R}\boldsymbol{U}_n+\boldsymbol{U}_s\boldsymbol{\Lambda}_s\boldsymbol{Q}+\delta\boldsymbol{R}\boldsymbol{U}_s\boldsymbol{Q} = \boldsymbol{U}_s\boldsymbol{Q}\boldsymbol{\Lambda}_n+\boldsymbol{U}_n\delta\boldsymbol{\Lambda}_n+\boldsymbol{U}_s\boldsymbol{Q}\delta\boldsymbol{\Lambda}_n, \quad (30)$$

where the Hermitian property of $\delta\boldsymbol{R}$ and $\delta\boldsymbol{\Lambda}_n$ has been used. Pre-multiplying both sides of (30) by $\boldsymbol{U}_s^H$ and ignoring second order terms, we have

$$\boldsymbol{U}_s^H\delta\boldsymbol{R}\boldsymbol{U}_n + \boldsymbol{\Lambda}_s\boldsymbol{Q} \approx \boldsymbol{Q}\boldsymbol{\Lambda}_n. \quad (31)$$

Notice that $\boldsymbol{\Lambda}_n = \sigma_v^2\boldsymbol{I}$ from (9) and $\boldsymbol{\Omega} = \boldsymbol{\Lambda}_s - \sigma_v^2\boldsymbol{I}$. Then

$$\boldsymbol{Q} \approx -\boldsymbol{\Omega}^{-1}\boldsymbol{U}_s^H\delta\boldsymbol{R}\boldsymbol{U}_n.$$

Observing $\delta\boldsymbol{U}_n = \boldsymbol{U}_s\boldsymbol{Q}$ and (17), we obtain (19). □

APPENDIX B: EVALUATION OF CHANNEL MSE

Let us begin with (20). The covariance matrix is given by

$$E\{\delta\boldsymbol{g}_1\delta\boldsymbol{g}_1^H\} \approx \sum_{m_1,j_1,,m_2j_2} \boldsymbol{X}^\dagger\boldsymbol{A}_{1,m_1,j_1}^H\boldsymbol{U}_n \boldsymbol{B}_{m_1,j_1,m_2,j_2}\boldsymbol{U}_n^H\boldsymbol{A}_{1,m_2,j_2}\boldsymbol{X}^\dagger, \quad (32)$$

where deterministic quantities $\boldsymbol{B}_{m_1,j_1,m_2,j_2}$ are defined as

$$\boldsymbol{B}_{m_1,j_1,m_2,j_2} \triangleq \boldsymbol{U}_n^H E\{\delta\boldsymbol{R}\boldsymbol{Z}^\dagger\boldsymbol{A}_{1,m_1,j_1}\boldsymbol{g}_1\boldsymbol{g}_1^H\boldsymbol{A}_{1,m_2,j_2}^H\boldsymbol{Z}^\dagger\delta\boldsymbol{R}\}\boldsymbol{U}_n$$

For notational convenience, it suffices to consider matrix $\boldsymbol{B} = \boldsymbol{U}_n^H E\{\delta\boldsymbol{R}\boldsymbol{D}\delta\boldsymbol{R}\}\boldsymbol{U}_n$ first with an arbitrary constant weighting matrix $\boldsymbol{D}$. Then replace $\boldsymbol{D}$ by $\boldsymbol{Z}^\dagger\boldsymbol{A}_{1,m_1,j_1}\boldsymbol{g}_1\boldsymbol{g}_1^H\boldsymbol{A}_{1,m_2,j_2}^H\boldsymbol{Z}^\dagger$ to achieve our goal. Substituting $\delta\boldsymbol{R}$ by $\widehat{\boldsymbol{R}} - \boldsymbol{R}$, we obtain

$$\boldsymbol{B} = \boldsymbol{U}_n^H E\{\widehat{\boldsymbol{R}}\boldsymbol{D}\widehat{\boldsymbol{R}}\}\boldsymbol{U}_n - \boldsymbol{U}_n^H\boldsymbol{R}\boldsymbol{D}\boldsymbol{R}\boldsymbol{U}_n.$$

Assume N data vectors used in obtaining $\widehat{\boldsymbol{R}}$ are mutually independent. Then

$$E\{\widehat{\boldsymbol{R}}\boldsymbol{D}\widehat{\boldsymbol{R}}\} = \frac{1}{N}E\{\boldsymbol{y}_n\boldsymbol{y}_n^H\boldsymbol{D}\boldsymbol{y}_n\boldsymbol{y}_n^H\} + (1-\frac{1}{N})\boldsymbol{R}\boldsymbol{D}\boldsymbol{R}.$$

Therefore

$$\boldsymbol{B} = \frac{1}{N}\boldsymbol{U}_n^H E\{\boldsymbol{y}_n\boldsymbol{y}_n^H\boldsymbol{D}\boldsymbol{y}_n\boldsymbol{y}_n^H\}\boldsymbol{U}_n - \frac{1}{N}\boldsymbol{U}_n^H\boldsymbol{R}\boldsymbol{D}\boldsymbol{R}\boldsymbol{U}_n. \quad (33)$$

Since $\boldsymbol{U}_n$ is orthogonal to $\boldsymbol{H}$, substitution of $\boldsymbol{y}_n$ by (23) and $\boldsymbol{R}$ by (8) yields

$$\begin{aligned}\boldsymbol{B} &= \frac{\sigma_b^2\sigma_v^2}{N}tr(\boldsymbol{H}^H\boldsymbol{D}\boldsymbol{H})\boldsymbol{I} + \frac{1}{N}\boldsymbol{U}_n^H E\{\boldsymbol{v}_n\boldsymbol{v}_n^H\boldsymbol{D}\boldsymbol{v}_n\boldsymbol{v}_n^H\}\boldsymbol{U}_n \\ &- \frac{\sigma_v^4}{N}\boldsymbol{U}_n^H\boldsymbol{D}\boldsymbol{U}_n. \quad (34)\end{aligned}$$

To evaluate $E\{\boldsymbol{v}_n\boldsymbol{v}_n^H\boldsymbol{D}\boldsymbol{v}_n\boldsymbol{v}_n^H\}$, we perform "$vec$" and then the reverse "$unvec$" operations

$$E\{\boldsymbol{v}_n\boldsymbol{v}_n^H\boldsymbol{D}\boldsymbol{v}_n\boldsymbol{v}_n^H\} = unvec\left[E\{(\boldsymbol{v}_n^*\otimes\boldsymbol{v}_n)(\boldsymbol{v}_n^T\otimes\boldsymbol{v}_n^H)\}vec(\boldsymbol{D})\right], \quad (35)$$

where superscripts "$*$" and "T" denote complex conjugate and transpose respectively, properties of "vec" and the Kronecker product "$\otimes$" have been applied [33]. $E\{(\boldsymbol{v}_n^*\otimes\boldsymbol{v}_n)(\boldsymbol{v}_n^T\otimes\boldsymbol{v}_n^H)\}$ depends on whether the $\nu\times 1$ noise vector $\boldsymbol{v}_n$ is a real or complex vector.

Case 1: $\boldsymbol{v}_n$ is real

After some straightforward algebra, it can be verified that [28]

$$\begin{aligned}E\{(\boldsymbol{v}_n\otimes\boldsymbol{v}_n)(\boldsymbol{v}_n^T\otimes\boldsymbol{v}_n^T)\} &= (m_{4v}-3\sigma_v^4)\boldsymbol{X}_1 + \sigma_v^4\boldsymbol{X}_2 \\ +\sigma_v^4 vec(\boldsymbol{I})vec^T(\boldsymbol{I}) &+ \sigma_v^4\boldsymbol{I}, \quad (36)\end{aligned}$$

where $\boldsymbol{X}_1$ is a block diagonal matrix

$$\boldsymbol{X}_1 = diag\{\boldsymbol{a}_1\boldsymbol{a}_1^T,\cdots,\boldsymbol{a}_\nu\boldsymbol{a}_\nu^T\},$$

$$\boldsymbol{a}_l^T = [0,\cdots,0,\underbrace{1}_{l\text{-th}},0,\cdots,0]_{1\times\nu},\ \ \boldsymbol{X}_2 = [\boldsymbol{a}_j\boldsymbol{a}_i^T]_{\nu\times\nu},$$

and $\boldsymbol{X}_2$ has been partitioned into $\nu\times\nu$ sub-blocks with the (i,j)-th sub-block $\boldsymbol{a}_j\boldsymbol{a}_i^T$. Since the fourth-order absolute moment m_{4v} of the AWGN is $3\sigma_v^4$, after substituting (36) in (35), we obtain

$$E\{\boldsymbol{v}_n\boldsymbol{v}_n^T\boldsymbol{D}\boldsymbol{v}_n\boldsymbol{v}_n^T\} = \sigma_v^4 unvec\left[\boldsymbol{X}_2 vec(\boldsymbol{D})\right] + \sigma_v^4 tr(\boldsymbol{D})\boldsymbol{I} + \sigma_v^4\boldsymbol{D}. \quad (37)$$

Express matrix $\boldsymbol{D}$ explicitly by columns as $[\boldsymbol{d}_1,\cdots,\boldsymbol{d}_\nu]$. The i-th block row of vector $\boldsymbol{X}_2 vec(\boldsymbol{D})$ is $\boldsymbol{a}_1\boldsymbol{a}_i^T\boldsymbol{d}_1+\cdots+\boldsymbol{a}_\nu\boldsymbol{a}_i^T\boldsymbol{d}_\nu$ which can be written as $\boldsymbol{a}_1\boldsymbol{d}_1^T\boldsymbol{a}_i+\cdots+\boldsymbol{a}_\nu\boldsymbol{d}_\nu^T\boldsymbol{a}_i$. Since $[\boldsymbol{a}_1,\cdots,\boldsymbol{a}_\nu]=\boldsymbol{I}$, it becomes $\boldsymbol{D}^T\boldsymbol{a}_i$. Therefore

$$unvec\Big[\boldsymbol{X}_2\ vec(\boldsymbol{D})\Big] = [\boldsymbol{D}^T\boldsymbol{a}_1,\cdots,\boldsymbol{D}^T\boldsymbol{a}_\nu] = \boldsymbol{D}^T. \quad (38)$$

With (38), (37) becomes

$$E\{\boldsymbol{v}_n\boldsymbol{v}_n^T\boldsymbol{D}\boldsymbol{v}_n\boldsymbol{v}_n^T\} = \sigma_v^4 tr(\boldsymbol{D})\boldsymbol{I} + \sigma_v^4(\boldsymbol{D}+\boldsymbol{D}^T). \quad (39)$$

Substituting (39) in (34), we have

$$\boldsymbol{B} = \frac{\sigma_v^2}{N}tr(\boldsymbol{R}\boldsymbol{D})\boldsymbol{I} + \frac{\sigma_v^4}{N}\boldsymbol{U}_n^H\boldsymbol{D}^T\boldsymbol{U}_n, \quad (40)$$

where $\sigma_b^2\boldsymbol{H}\boldsymbol{H}^H + \sigma_v^2\boldsymbol{I}$ has been replaced by $\boldsymbol{R}$. Setting $\boldsymbol{D}$ equal $\boldsymbol{Z}^\dagger\boldsymbol{A}_{1,m_1,j_1}\boldsymbol{g}_1\boldsymbol{g}_1^H\boldsymbol{A}_{1,m_2,j_2}^H\boldsymbol{Z}^\dagger$ and observing that $\boldsymbol{Z}^\dagger$ is orthogonal to $\boldsymbol{U}_n$, we obtain

$$\boldsymbol{B}_{m_1,j_1,m_2,j_2} = \frac{\sigma_v^2}{N}\gamma_{m_1,j_1,m_2,j_2}\boldsymbol{I}, \quad (41)$$

where γ_{m_1,j_1,m_2,j_2} is given by (25). Substituting (41) in (32) and performing the trace operation, (24) follows.

Case 2: $\boldsymbol{v}_n$ is complex

Similar to (36), we can easily find

$$\begin{aligned}E\{(\boldsymbol{v}_n^*\otimes\boldsymbol{v}_n)(\boldsymbol{v}_n^T\otimes\boldsymbol{v}_n^H)\} &= (m_{4v}-2\sigma_v^4)\boldsymbol{X}_1 \\ +\sigma_v^4 vec(\boldsymbol{I})vec^T(\boldsymbol{I}) &+ \sigma_v^4\boldsymbol{I}, \quad (42)\end{aligned}$$

when $\boldsymbol{v}_n$ is complex. Now m_{4v} equals $2\sigma_v^4$ instead of $3\sigma_v^4$ in the real case (for example, see explanations before (105) in [28]). Then applying (42), (35) becomes

$$E\{\boldsymbol{v}_n\boldsymbol{v}_n^H\boldsymbol{D}\boldsymbol{v}_n\boldsymbol{v}_n^H\} = \sigma_v^4 tr(\boldsymbol{D})\boldsymbol{I} + \sigma_v^4\boldsymbol{D}. \quad (43)$$

Substituting (43) in (34), we have

$$\boldsymbol{B} = \frac{\sigma_v^2}{N}tr(\boldsymbol{D}\boldsymbol{R})\boldsymbol{I}, \quad (44)$$

where $\sigma_b^2\boldsymbol{H}\boldsymbol{H}^H+\sigma_v^2\boldsymbol{I}$ has been replaced by $\boldsymbol{R}$. Setting $\boldsymbol{D}$ equal $\boldsymbol{Z}^\dagger\boldsymbol{A}_{1,m_1,j_1}\boldsymbol{g}_1\boldsymbol{g}_1^H\boldsymbol{A}_{1,m_2,j_2}^H\boldsymbol{Z}^\dagger$, we obtain

$$\boldsymbol{B}_{m_1,j_1,m_2,j_2} = \frac{\sigma_v^2}{N}\gamma_{m_1,j_1,m_2,j_2}\boldsymbol{I}, \quad (45)$$

where γ_{m_1,j_1,m_2,j_2} is given by (25). Substituting (45) in (32) and performing the trace operation, (24) follows.

It is worth to mention that although we have differentiated two different cases, the final expression of the MSE shows one common form. □

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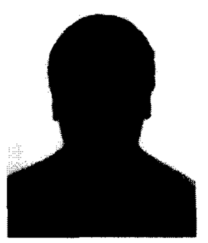

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