Architectures and Connection Probabilities for Wireless Ad Hoc and Hybrid Communication Networks

Jeng-Hong Chen and William C. Lindsey

Abstract: **Ad hoc wireless networks involving large populations of scattered communication nodes will play a key role in the development of low power, high capacity, interactive, multimedia communication networks. Such networks must support arbitrary network connections and provide coverage anywhere and anytime. This paper partitions such arbitrarily connected network architectures into three distinct groups, identifies the associated dual network architectures and counts the number of network architectures assuming there exist N network nodes. Connectivity between network nodes is characterized as a random event. Defining the *link availability* p as the probability that two arbitrary network nodes in an ad hoc network are directly connected, the *network connection probability* $f_N(p)$ that any two network nodes will be directly or indirectly connected is derived. The network connection probability $f_N(p)$ is evaluated and graphically demonstrated as a function of p and N. It is shown that ad hoc wireless networks containing a large number of network nodes possesses the same network connectivity performance as does a fixed network, i.e., for $p > 0$, $\lim_{N\to\infty} f_N(p) = 1$. Furthermore, by cooperating with fixed networks, the ad hoc *network connection* probability is used to derive the *global network connection probability* for hybrid networks. These probabilities serve to characterize network connectivity performance for users of wireless ad hoc and hybrid networks, e.g., IEEE 802.11, IEEE 802.15, IEEE 1394-95, ETSI BRAN HIPERLAN, Bluetooth, wireless ATM and the world wide web (WWW).**

Index Terms: **Network architecture, wireless ad hoc and hybrid networks, network connection probability, link availability, network connectivity, master node, slave node**

I. INTRODUCTION

Ad hoc wireless communication networks are becoming popularized by numerous applications and by the pervasive desire for wireless connectivity anytime and anywhere [1]–[5]. This is further driven by the fact that a plethora of wireless communications devices (palm pilots, mobile phones, e-tag, active badges, and laptop PCs, etc.) and services will soon characterize the future wireless information society. In this context, wireless home area networking (WHAN) and wireless small office-home office (SOHO) networking are creating new applications for global wireless communication industries. Standards such as the IEEE 802.11, ETSI/BRAN-HIPERLAN II, IEEE 1394-95 and Wireless ATM are essentially mature [6]–[12]. In this regard, the industry is working hard towards their implementation such that all users can benefit from pervasive wireless connectivity.

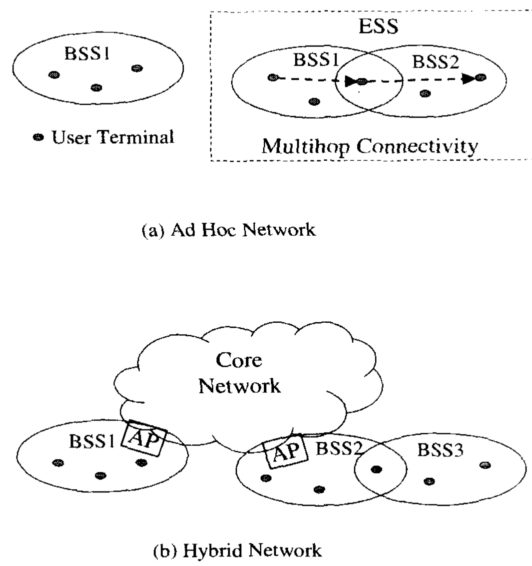


Fig. 1. Ad hoc and hybrid networks.

Ad hoc mobile networks can be made to work with fixed wired and/or wireless networks creating what is called hybrid networks [6], [13]. The major motivation for hybrid networks arises from the fact that no one technology or service can provide ubiquitous coverage or pervasive wireless connectivity anytime and anywhere. Hybrid networks involving wireless local area networks (WLANs) are also of great interest where infrastructure costs and world-wide connectivity are issues [10]–[13]. Such applications include metropolitan areas where dense populations are involved, e.g., in wireless ATM (WATM) networks, in PCS cellular networks and in SOHO/WHAN enterprise applications. Hybrid networks are also useful in extending the reach of the Internet. Additionally, such networks can be deployed instantaneously; as such they have applications in the digital battlefield, in emergency rescue operations, on university campuses and in large-scale conferencing situations where a large portion of the network nodes are mobile.

Fig. 1 conceptually illustrates the architecture of both ad hoc and hybrid networks. By introducing multi-hop communication protocols, coverage, and connectivity beyond that offered by the basic service set (BSS) can be achieved among various wireless product clusters [14]–[16]. By introducing access points (APs) into the network architecture, ad hoc networks can be connected to core networks. Use of such architectures further

Manuscript received September 27, 2000; approved for publication by H. Vincent Poor, Division I Editor, May 21, 2001.

J.-H. Chen was with LinCom Wireless, Inc., 5120 W. Goldleaf Circle, Suite 400, Los Angeles, CA 90056, e-mail: jenghong2001@yahoo.com.

W. C. Lindsey is Professor of Electrical Engineering, Communications Sciences Institute, University of Southern California and Chairman of the Board of LinCom Wireless, Inc., Los Angeles, CA, e-mail: wcl@usc.edu.

1229-2370/02/$10.00 © 2002 KICS

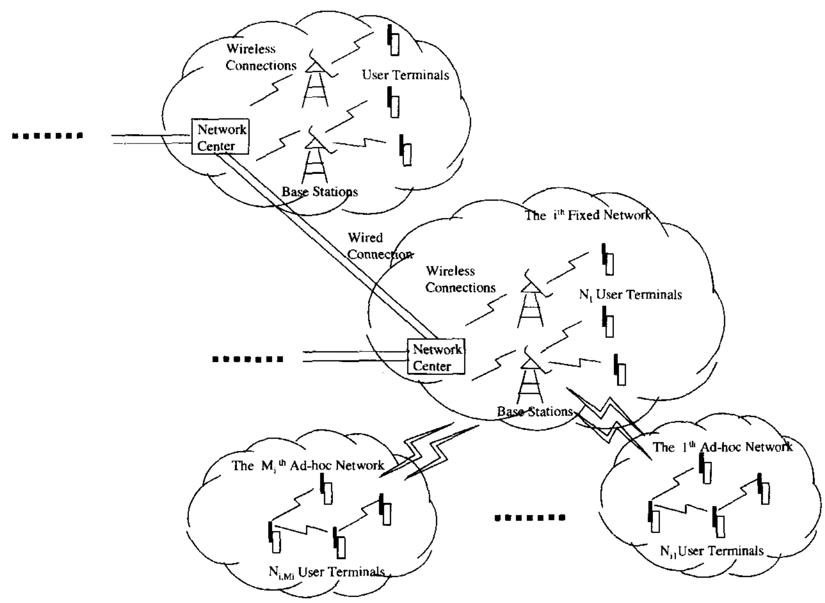


Fig. 2. Global connectivity of ad hoc networks via fixed network infrastructures.

extends the coverage region. Hence, APs serve as connecting bridges to other BSSs and to core networks, e.g., IP, ATM, UMTS, and IEEE 1394-95. In this way, BSS connectivity can be extended to provide coverage to extended service sets (ESSs) [7]–[9]. Fig. 1(b) illustrates the concept of an ad hoc network where multiple communication hops are required in order to insure connectivity between two ad hoc wireless networks.

In hybrid networks, the transmission range between BSs and MSs are both reduced by 1/n of the range accommodated in contemporary single-hop cellular networks; here, n is defined as the transmission range reduction factor [14], [15]. The analysis results in [14], [15] demonstrated that the throughput of multi-hop ad hoc networks is superior to that of single-hop networks and increases as the transmission power decreases. This is because the average hop count increases on the order of n while the average number of simultaneous transmission connections increases on the order of n squared. In addition to increased throughput, there are other advantages of multi-hop ad hoc networks; viz., (1) the number of APs (or BSs) or the transmission range can be reduced, (2) connections are still allowed without APs (or BSs), and (3) paths are less vulnerable in ad hoc networks because the APs can help reduce the hop count [14], [15]. In this paper, we provide a precise mathematical definition for hybrid and ad hoc network architectures and characterize and graphically study network connection probabilities for both architectures.

II. NETWORK CONNECTION PROBABILITY

An *arbitrarily connected network* is defined in [16] and three levels of network partitions for *ad hoc and hybrid networks* are demonstrated. This includes: (a) *connecting networks (fixed networks)* (b) the *master, network infrastructure (NI),* and *sink groups* of nodes, and (c) *irreducible subnetworks*. In order to investigate network connectivity, the *link availability* p is defined as the probability that two arbitrary network nodes are connected. For a network containing N network nodes, an *event* E_N of great interest is defined as a set of all network connectivities (*outcomes*) such that all network nodes are directly or indirectly connected. The probability *space* for this probability experiment is therefore defined as $S = E_N \cup \overline{E_N}$ where $\overline{E_N}$ is the *complement set* of E_N.

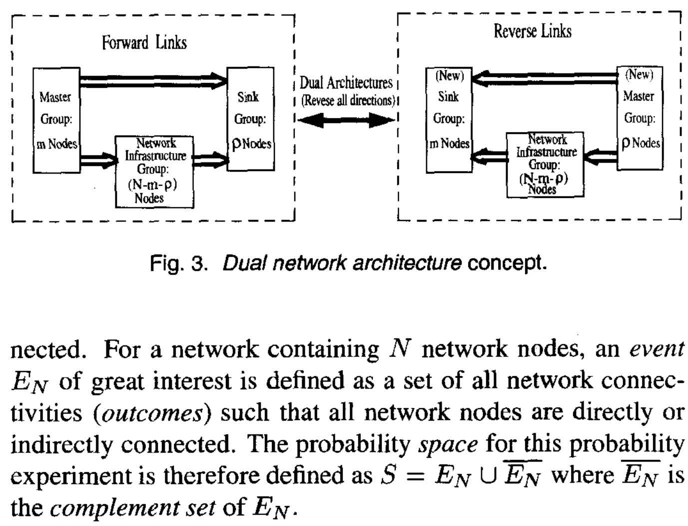
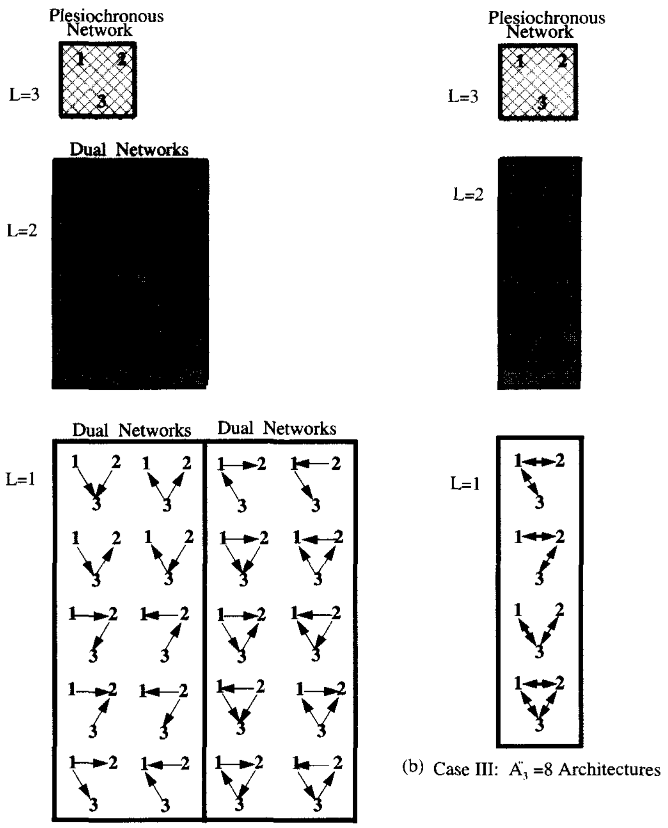


Fig. 3. *Dual network architecture* concept.

For an N-nodal network, the *network connection probability* is defined as the probability that all network nodes are directly or indirectly connected, i.e., $Pr(E_N) \equiv f_N(p)$ for a given *link availability p*. Furthermore, the concept of global connectivity via fixed network infrastructures and ad hoc networks is illustrated in Fig. 2. Herein two arbitrary users in different ad hoc networks also can be connected via the fixed network infrastructure. Such a network is called a hybrid network. Aside from the *network connection probability* $f_N(p)$, the **global** *network connection probability* for hybrid network composed of fixed and ad hoc networks is also of interest. One purpose of this paper is to characterize these probabilities as a function of the link availability p and the number of network nodes N.

III. DUAL NETWORK ARCHITECTURES

A ***dual network architecture*** is obtained by reversing the directions of communication on the transport links in the original network [16]. In practice, communication systems employing



(a) Case II: $A_3' = 27$ Architectures.

(b) Case III: $A_3'' = 8$ Architectures

Fig. 4. Possible network architectures for a three-nodal ad hocly connected networks.

Table 1. Total number of possible architectures for A_N, A_N', and A_N''.

Total Number of Architectures	Connection Relations Between Arbitrary Network Nodes i and j
Case I: $A_N = 4^{N(N-1)/2}$	1) $i \not\to j$ and $j \not\to i$, 2) $i \not\to j$ and $j \to i$, 3) $i \to j$ and $j \not\to i$, 4) $i \to j$ and $j \to i$.
Case II: $A_N' = 3^{N(N-1)/2}$	1) $i \not\to j$ and $j \not\to i$, 2) $i \not\to j$ and $j \to i$, 3) $i \to j$ and $j \not\to i$.
Case III: $A_N'' = 2^{N(N-1)/2}$	1) $i \not\to j$ and $j \not\to i$, 2) $i \to j$ and $j \to i$.

either *Frequency-Division Duplexing (FDD)* or *Time-Division Duplexing (TDD)* can be considered as examples of *dual network architectures*. As shown in Fig. 3, the forward links and reverse links form a pair of dual network architectures. This pair of dual network architectures can be considered as a duplex FDD or TDD system with two simplex architectures; the *master group* of nodes and the *sink group* of nodes are two sets of communication terminals.

The connecting relations between two nodes were defined and summarized in [16]. Table 1 summarizes the total number of achievable architectures for Cases I. II. and III.

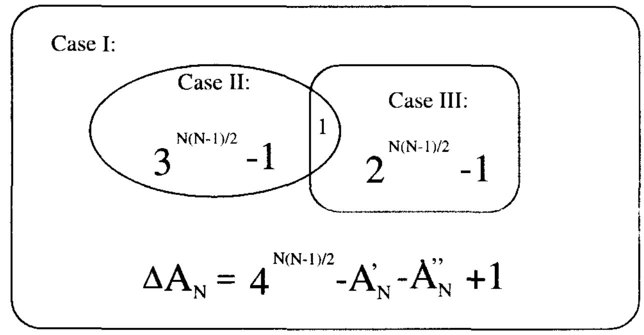


Fig. 5. Venn diagram illustrating architectures for Cases I, II, and III (A_N, A_N', and A_N'').

A dual network architecture exists for any *arbitrarily connected network*. If the original network architecture is a *connecting network*, its dual network architecture is also a *connecting network*. Since the network nodes are distinct, the dual network architecture is another distinguishable architecture if there exist arbitrary nodes i and j which are *one-way connected* [16]. Because of the symmetry of the connections in Case III, the dual network architecture has the same architecture as the original network architecture. Therefore, there are $(A_N - A_N'')$ architectures whose dual network architectures are distinguishably related to their original forms.

A special property is found for Case II. Suppose that two arbitrary nodes are either *one-way connected* or not connected in any simplex system. One observes that there are $\frac{1}{2}(A_N' - 1)$ pairs[1] in Case II. The architecture which contains no connections (plesiochronous network) is the only architecture which has no distinguishable dual network architecture as shown in Fig. 4. In addition, the plesiochronous network is the intersection between Cases II and III. A Venn diagram for Cases I, II, and III is illustrated in Fig. 5.

IV. COUNTING THE NUMBER OF NETWORK ARCHITECTURES

It is shown in [16] that an arbitrarily connected network $\mathfrak{B}$ is the union of L isolated connecting networks ($\mathfrak{B} = \cup_{i=1}^{L} \mathfrak{B}_i$). The number of possible network architectures in which all network nodes are directly or indirectly connected is defined as C_N. The number of possible network architectures for $\mathfrak{B}$ in Cases I, II, and III is given by

$$X_N = \sum_{L=1}^{N} \{ \sum_{(\sum_{i=1}^{L} N_i = N, 1 \le N_i \le N)} [\frac{N!}{N_1! N_2! \cdots N_L!}] \cdot [\frac{1}{\nu_1! \nu_2! \nu_3! \cdots \nu_N!}] [Y_{N_1} Y_{N_2} \cdots Y_{N_L}] \}, \quad (1)$$

where $(X_i, Y_i) = (A_i, C_i)$ or (A_i', C_i') or (A_i'', C_i'') which is defined in (1) and (2) from [16] for Cases I, II, and III respectively, N_i is the total number of nodes in $\mathfrak{B}_i$, Y_k is the total number of possible network architectures for a connecting network having

[1] In Case II, the number of possible network architectures A_N' is an odd number.

Table 2. Distribution of possible network architectures for an arbitrarily connected network $\mathfrak{B}$ containing $N = 6$ nodes.

L	$N_1, N_2, \cdots, N_L$	Number of Possible Network Architectures for $\mathfrak{B}$ as a Function of L. $A_6' = 14348907$	$A_6'' = 32768$
6	1,1,1,1,1,1	1	1
5	1,1,1,1,2	30	15
4	1,1,1,3	400	80
	1,1,2,2	180	45
3	1,1,4	9360	570
	1,2,3	2400	240
	2,2,2	120	15
2	1,5	331488	4368
	2,4	18720	570
	3,3	4000	160
1	6	13982208	26704

Table 3. Some specific values for A_N', C_N', A_N'', C_N'' and their ratios in Cases II and III.

N	Case II A_N'	C_N'	C_N'/A_N'	A_{N-1}'/A_N'	C_{N-1}'/C_N'
1	1	1	100%	—	—
2	3	2	66.67%	33.33%	50.00%
3	27	20	74.07%	11.11%	10.00%
4	729	624	85.60%	3.70%	3.21%
5	59049	55248	93.56%	1.23%	1.13%
6	14348907	13982208	97.44%	0.41%	0.40%
N	Case III A_N''	C_N''	C_N''/A_N''	A_{N-1}''/A_N''	C_{N-1}''/C_N''
1	1	1	100.00%	—	—
2	2	1	50.00%	50.00%	100.00%
3	8	4	50.00%	25.00%	25.00%
4	64	38	59.38%	12.50%	10.53%
5	1024	728	71.09%	6.25%	5.22%
6	32768	26704	81.49%	3.13%	2.73%

k nodes, and ν_j is the number of connecting networks $\mathfrak{B}_i$ in $\mathfrak{B}$ which have node number j, $1 \leq j \leq N$. Finding the number Y_N directly is a difficult problem and there appears to be no closed form for Y_N. An easier way to specify Y_N is to use a recursion method starting from $Y_1 = X_1 = 1$. Since the number Y_N is a special case of X_N when all N nodes are directly or indirectly connected (the case $L = 1$), one can readily show from (1) that if $N \geq 2$, then

$$Y_N = X_N - \sum_{L=2}^{N}\{ \sum_{(\sum_{i=1}^{L} N_i = N, 1 \leq N_i \leq N)} [\frac{N!}{N_1!N_2!\cdots N_L!}] \cdot [\frac{1}{\nu_1!\nu_2!\nu_3!\cdots \nu_N!}][Y_{N_1}Y_{N_2}\cdots Y_{N_L}]\}. \quad (2)$$

By using (2) recursively, Y_N is characterized in terms of X_N and Y_i, $i = 1, 2, \cdots, N-1$. Several examples of the distribution of possible network architectures in an arbitrarily connected network containing $N = 2, 3, \cdots, 6$ nodes as a function of $L = 1, 2, \cdots, N$, are provided in Tables 2 and 3. Examples for Case I are provided in [16].

All possible architectures in an arbitrarily connected network containing $N = 3$ nodes are illustrated in Fig. 4. The architecture in the top-left-hand corner for Case II or Case III is the only architecture valid for $L = 3$. Architectures for $L = 2$ are indicated by the dark blocks. The remaining architectures are for $L = 1$.

Tables 2 and 3 demonstrate that the case $L = 1$ contains most of the total number X_N of possible network architectures. When the total number of nodes N approaches infinity, one readily notes from (2) the interesting limiting ratios:

$$\lim_{N\to\infty} \frac{C_N}{A_N} = \lim_{N\to\infty} \frac{C_N'}{A_N'} = \lim_{N\to\infty} \frac{C_N''}{A_N''} = 1, \quad (3)$$

$$\lim_{N\to\infty} \frac{A_K}{A_N} = \lim_{N\to\infty} \frac{A_K'}{A_N'} = \lim_{N\to\infty} \frac{A_K''}{A_N''} = 0, \quad (4)$$
$$\forall\, 1 \leq K \leq N-1,$$

$$\lim_{N\to\infty} \frac{C_K}{C_N} = \lim_{N\to\infty} \frac{C_K'}{C_N'} = \lim_{N\to\infty} \frac{C_K''}{C_N''} = 0, \quad (5)$$
$$\forall\, 1 \leq K \leq N-1.$$

A conceptual diagram used to demonstrate (3)–(5) is shown in Fig. 6. Suppose that for all $K, 1 \leq K \leq N$, the case $L = 1$ contains most of the $X_N = a^{N(N-1)/2}$ possible architectures ($Y_N/X_N \approx 1$); where a =4, 3, and 2 for Cases I, II, and III, respectively. Suppose that the $(N+1)^{th}$ node is added to this N-nodal network, there are $Y_N \times a^N$ and $(X_N - Y_N) \times a^N$ architectures produced from Group 1 and Group 2, respectively. Therefore, the possible number of architectures containing $(N+1)$ nodes is given by

$$X_{N+1} = Y_N a^N + (X_N - Y_N)\, a^N = X_N\, a^N = a^{(N+1)N/2}. \quad (6)$$

Furthermore, an $(N+1)$-nodal connecting network is produced from Group 1 unless the $(N+1)^{th}$ node is not connected to all N nodes. Therefore, produced from Group 1, there are at least $Y_N(a^N - 1)$ connecting networks and Y_N non-connecting networks. Similarly, there are $(X_N - Y_N)a^N$ architectures produced from Group 2. Therefore, the ratio of the number of connecting networks to non-connecting networks in an $(N+1)$-nodal network is

$$\frac{\text{Number of connecting networks}}{\text{Number of non-connecting networks}} \geq \frac{Y_N(a^N-1)}{Y_N + (X_N - Y_N)a^N} \xrightarrow{N\to\infty} \infty. \quad (7)$$

Therefore, the probability that one obtains a new connecting network containing $(N+1)$ network nodes approaches one as N approaches infinity.

The implications of (3)–(7) on network architecture is discussed as follows: From (5), one observes that by adding one network node to the original network architecture, one has many more options to choose from the new network configuration and hence network performance can be optimized. From (7), once a network node fails (disconnected from all other nodes), it is highly probable that the remaining network architecture corresponds to a new connecting network.

V. NETWORK CONNECTION PROBABILITIES FOR AD HOC WIRELESS NETWORKS

An interesting question involves determining the *network connection probability* $f_N(p)$ in an N-nodal ad hoc wireless net-

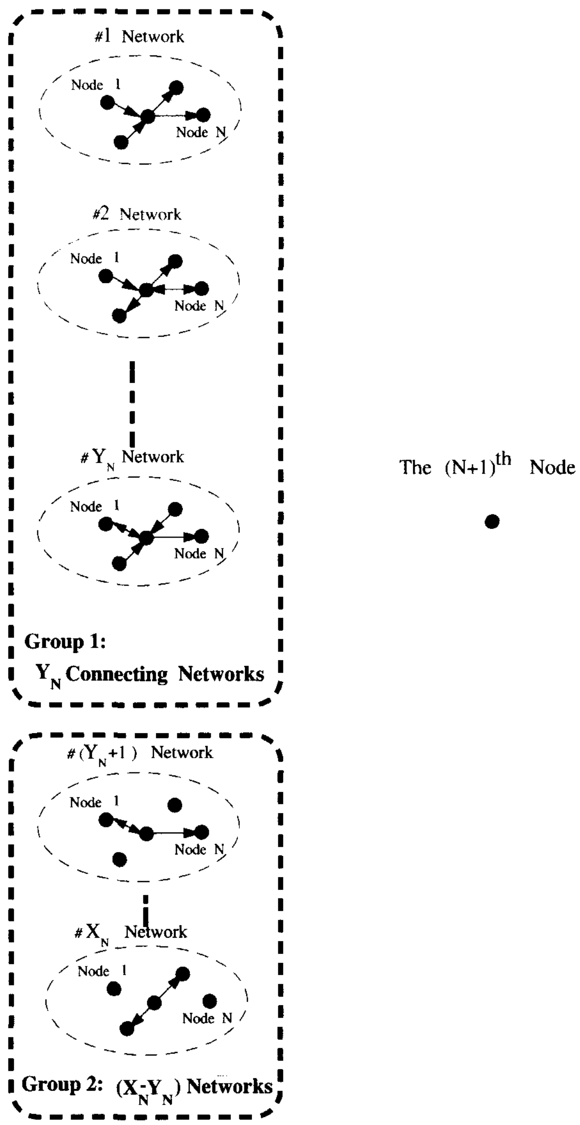


Fig. 6. Conceptual digram demonstrating (3)–(5).

work. For simplicity, the *link availability* p is assumed to be a fixed constant in the interval [0,1]. For a network with a large number of nodes N, this assumption is questionable since the network nodes which are close to each other may have larger link availability. However, we shall later see that the value of p is not a critical factor as long as it is non-zero when N is very large. If the probability p is equal to or greater than 0.5, then from (3), it is highly probable that all nodes are connected directly or indirectly; especially, for a large N. In probability theory, if one randomly connects each of the $N(N-1)/2$ pairs of network nodes with probability p, there are $2^{N(N-1)/2}$ outcomes (network connectivities) as discussed earlier. A special event E_N is defined as that event where all N network nodes are directly or indirectly connected and the *ad hoc network connection probability* is defined as $Pr(E_N) = f_N(p)$. Using (2), this probability can be derived recursively and shown to be given by (8) as shown at the bottom of this page. The first term includes all $2^{N(N-1)/2}$ possible connectivities of an N-nodal network and the last term $(1-p)^{N(N-1)/2-\sum_{j=1}^{L} N_j(N_j-1)/2}$ indicates that there is no connection between *isolated* subnetworks [16]. Examples are found in Table 4. From Table 4, if the total number of connections in an N-nodal network is less than N, it is not possible to form an N-nodal connecting network. On the other hand, if the total number of connections is greater than $\binom{N-1}{2}$, then an N-nodal connecting network is a certain result with probability one. The coefficients of $f_N(p)$ grows exponentially and it is not possible to find the coefficients from searching all exhaustive $2^{N(N-1)}$ possible connectivities. However, the probability that all network nodes are connected for small p and large N is of primary interest and the results obtained from computer simulation is provided in the next section.

Table 4. $f_N(p) \equiv a_0 p^0 (1-p)^{N(N-1)/2} + a_1 p^1 (1-p)^{N(N-1)/2-1} + \cdots + p^{N(N-1)/2}$

N	The coefficients $a_0, a_1, a_2, \cdots, a_{N(N-1)/2}$
1	1
2	0,1
3	0,0,3,1
4	0,0,0,16,15,6,1
5	0,0,0,0,125,222,205,120,45,10,1
6	0,0,0,0,0,1296,3660,5700,6165,4945,2997,1365,455,105,15,1
7	0,0,0,0,0,0, 16807,68295,156555,258125,331506,343140, 290745,202755,116175,54257,20349,5985,1330,210,21,1
⋮	⋮
18	$\cdots$,714087120763134175370088434951088453946703872, 732890586455815716102878271253691423803834368, 732899854765267971940293413975585746340282368, 714114680257758622535887348643709595473149952, 677962299457289879507791552238707829072986112, $\cdots$, 654045930, 21947850, 585276, 11628, 153,1

VI. DISCUSSION AND SIMULATION RESULTS

Several computer programs have been developed to count the coefficients which characterize the network connection probability $f_N(p)$. The exhaustive search for all $2^{N(N-1)/2}$ connectivities is limited to $N=6$ by using *Matlab* software and $N=9$ by using the C program on a Pentium 300 MHz personal computer. By applying (8), one is able to find these coefficients up to $N=18$ from the recursive method using the C program. Unfortunately, for $N=19$, all coefficients are found to be infinite

$$f_N(p) = \sum_{k=0}^{N(N-1)/2} \binom{N(N-1)/2}{k} p^k (1-p)^{N(N-1)/2-k} - \sum_{L=2}^{N} \sum_{(\sum_{i=1}^{L} N_i = N, 1 \le N_i \le N)} [\frac{N!}{N_1! N_2! \cdots N_L!}]$$
$$\times [\frac{1}{\nu_1! \nu_2! \nu_3! \cdots \nu_N!}][f_{N_1}(p) f_{N_2}(p) \cdots f_{N_L}(p)] \times (1-p)^{N(N-1)/2-\sum_{j=1}^{L} N_j(N_j-1)/2} \quad (8)$$

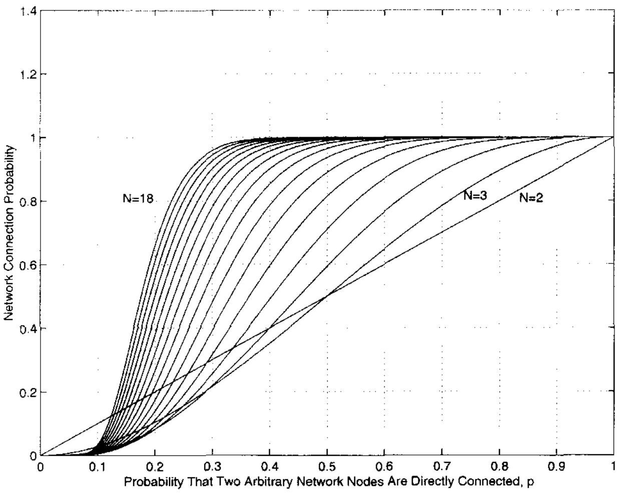


Fig. 7. Network connection probability $f_N(p)$ for various N.

Table 5. The unique real root of $\frac{d^2}{dp^2}[f_N(p)]$ in [0,1] for $N = 3$ to $N = 18$.

N	p^*	N	p^*	N	p^*	N	p^*
3	0.50000	7	0.30761	11	0.22598	15	0.18195
4	0.43195	8	0.28140	12	0.21265	16	0.17386
5	0.38019	9	0.25970	13	0.20106	17	0.16690
6	0.33982	10	0.24148	14	0.19090	18	0.16019

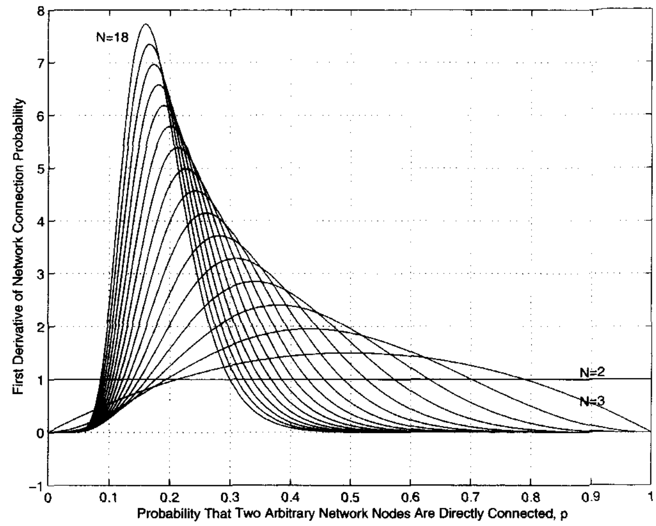


Fig. 8. First derivative of network connection probability $f_N(p)$ for various N.

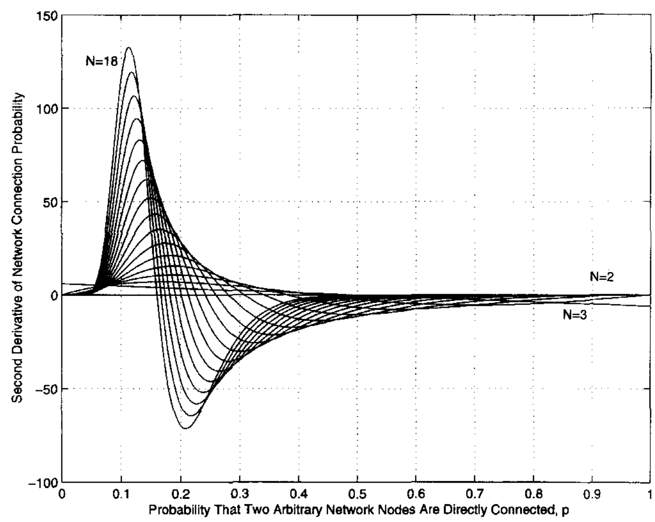


Fig. 9. Second derivative of network connection probability $f_N(p)$ for various N.

when using the computer program since they exceed the maximum number a variable can store[2]. The network connection probability $f_N(p)$ is illustrated in Fig. 7 for various N from two to 18. By increasing the number of network nodes N, the network connection probability $f_N(p)$ approaches one with smaller p. This indicates that, although the connecting probability between two arbitrary nodes (p_{ij}) is small, the probability that all network nodes are directly or indirectly connected is highly probable if N is a large number.

The limiting case (when N approaches infinity) is of great interest but it is very difficult to find the relationship between N and p using the computer for $N > 50$ ($2^{25\times 49}$ connectivities). Two approaches are suggested. First, one can use the C program trying to store a real number of more than 8 bytes. For example, by defining a data type which has 10000 bytes, one can store a real number of up to 2^{80000}. This approach may be able to be used calculate the coefficients for $N > 100$. However, it becomes extremely time-consuming when N becomes large. In addition, when plotting the function $f_N(p)$ for small values of p, round-off error is unavoidable. For example, suppose that N is 20 and p is 10^{-2}, the first term of $f_N(p)$ is $(1 - 10^{-2})^{190}$. This increases the difficulty in plotting $f_N(p)$ with accuracy and finding the relationship between $f_N(p)$ and p for large values of N. An alternative suggestion is to find a known function $h_N(p)$ which is a close approximation to the function $f_N(p)$ and take the limit as N approaches infinity. It is the authors' *conjecture* that as N approaches infinity, the function $f_N(p)$ approaches a ***step function*** with the transition point at $p = 0^+$. If this conjecture is valid, this may explain a rule of nature. In nature, cells, particles, and even atoms are connected in a large numbers on the order of 10^{23}. Although the link availability (strength of connection) between two arbitrary nodes is extremely small ($10^{-k}, k >> 1$), the "nodes" have a tendency to be connected when N is extremely large.

From Fig. 7, one observes that the network connection probability $f_N(p)$ increases slowly when p is small and increases rapidly after p is larger than a certain *inflection* point $p*$. In the limiting case, it is the authors' conjecture that this certain point $p*$ is 0^+ when N approaches infinity. The curves for the first and second derivative of the network connection probability $f_N(p)$ are illustrated in Fig. 8 and Fig. 9. It is interesting to find that, although $f_N(p)$ is a polynomial with degree $N(N-1)/2$ and with coefficients as large as $\binom{N(N-1)/2}{k}$, the second derivative of $f_N(p)$ has a unique real root $p*$ between 0 and 1 for $3 \le N \le 18$. Table 5 shows the values of p^* for $3 \le N \le 18$ which are solved by software *Macsyma* with 10^{-6} precision.

[2] In the C program, *double precision* which is using 4 or 8 bytes to store a real number is the variable type containing the largest memory.

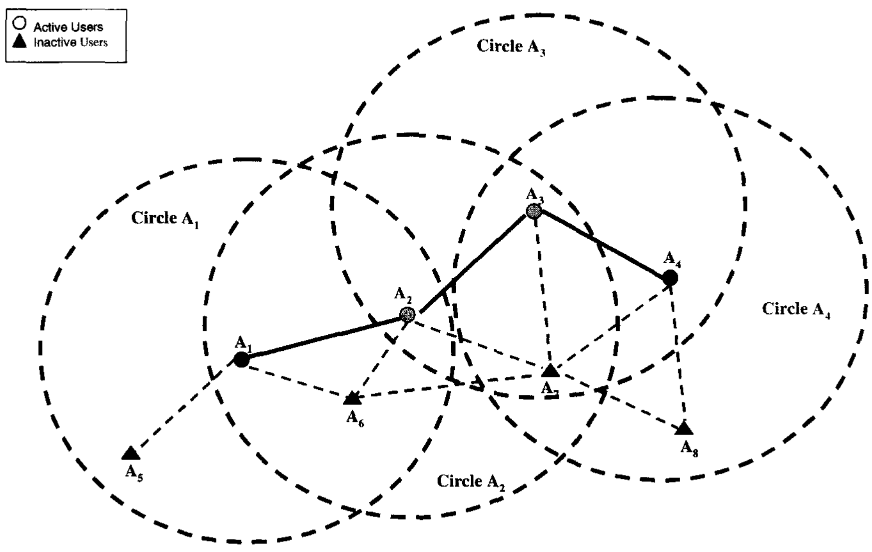


Fig. 10. Example of an ad hoc network.

One observes that the *inflection point* p^* of $f_N(p)$ decreases with values of increasing N. It is authors' conjecture that the *inflection point* p^* of $f_N(p)$ is 0^+ as N approaches infinity. In other words, if the link availability p is larger than p^*, nodes are more probable to be connected with a large numbers of nodes; if the link availability p is smaller than p^*, all nodes are more probable to be connected when N is small.

A. User Connection Probability for Ad Hoc Wireless Networks

It is envisioned that ad hoc networks will be required to support network dynamics with random connections (in time and space) for thousands of scattered mobile terminals with and without fixed (wired or wireless) network infrastructures. Therefore, the network connection probability $f_N(p)$ can be used as the metric to indicate "user connection probability" for *terminals* [6] in ad hoc networks. For example, suppose that all students have mobile terminals on future campuses. Furthermore, we assume that each mobile user (on the average) randomly calls 10 out of 1000 mobile users in a local area. Then it is reasonable to assume that the *link availability* p is 0.01 for all mobile users in this ad hoc network. According to the metric, the *network connection probability* approaches 100% in this ad hoc network as N becomes large. Therefore, at least one direct or indirect connection from one mobile user to another is almost guaranteed when no fixed network infrastructure (APs or base stations) is available. If small power (short range) wireless communication terminals are available for a small local area with a large number of mobile users, then with high probability all user communications can be forwarded to their destinations without the use of a fixed network infrastructure.

Example 1: Fig. 10 serves to illustrate an example of an ad hoc network suggested in IEEE 802.11 standard [6]. A solid line indicates the establishment of a direct connection between two network nodes. For a particular circle, it is assumed that a direct connection can be established between the center network node and an arbitrary network node within its circle. An **active** node is defined as a network node which is currently connected to any other network node; otherwise, a network node is called *inactive*. A dotted line indicates a connection is available (e.g., one network node is within the circle which is centered at another network node) between two network nodes but currently is not connected. If one applies this model to a mobile network, a particular circle is the **reachable region** of the center mobile station which can serve as a *master node* for all *inactive* nodes within this circle. In other words, the connecting probability between an active node and any nodes inside its *reachable region* is assumed to be 100%. This region may not be a circle and usually it is determined by the surrounding environment, mobile power and interference. A larger *reachable region* may cover more mobile stations inside and more connections may become available. The union of all *reachable regions* from *active* mobile stations provides the **coverage** of the mobile network. *Network coverage* changes with time according to the movements of mobile stations. In order to obtain maximum *network coverage*, each *active* network node should establish all available connections with other *active* nodes. Furthermore, *inactive* mobile stations may be connected to other mobile stations from the dotted lines and *active* mobile stations may be disconnected because the mobile stations have moved outside of the *network coverage area* or the connections have been terminated by mobile stations. From discussions in previous sections, if network nodes are geographically distributed in a small region and the *reachable region* of arbitrary node is large enough to cover a considerable number of network nodes, then it is very probable that most of the network nodes can be directly or indirectly connected.

Specifically, the *active* nodes A_1 to A_4 in Fig. 10 provide three active connections plus nine available (inactive) connections. The *network connection probability* $f_8(12/28) = 1$; i.e., a direct or indirect connection can be established between two arbitrary nodes from 12 available (solid or dotted) connections. A direct or indirect connection can be established between

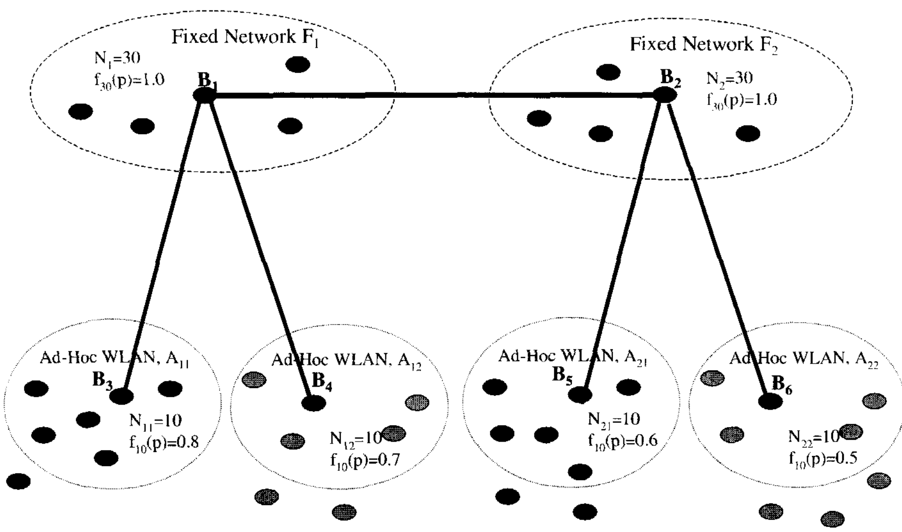


Fig. 11. Example of a hybrid network with fixed and ad hoc networks.

Table 6. The relationship between number of nodes and network connection probability.

$N = 8, f_8(p) \geq 0.9 \Rightarrow p \geq 0.45$ $\Rightarrow$ number of connections $\geq 0.45 \times C_2^8 = 12.6$
$N = 10, f_{10}(p) \geq 0.9$ $\Rightarrow p \geq 0.4 \Rightarrow$ number of connections $\geq 0.4 \times C_2^{10} = 18$
$N = 15, f_{15}(p) \geq 0.9$ $\Rightarrow p \geq 0.3 \Rightarrow$ number of connections $\geq 0.3 \times C_2^{15} = 31.5$

two arbitrary nodes using the following procedure: suppose that node A_5 is requesting to make a connection with node A_8, then a connection between A_1 and A_5 can be established first. Secondly, the message that node A_8 is requested for use can be broadcasted among *active* nodes (A_1 to A_5). Finally, a connection between nodes A_4 and A_8 can be established since node A_8 is inside Circle A_4. Therefore, a indirect connection is established between nodes A_5 and A_8. In this example, eight network nodes can be connected using 3 connections.

As shown in this example, the *reachable region* of a particular user terminal may cover lots of other user terminals in a small area with a large number of user terminals (e.g., a university campus) and the *coverage* of network is the union of *reachable regions* of the active nodes. Therefore, only the available connections in the neighborhood, but not the total available connections (which defines p), is of interest for any user terminal.

Example 2: After understanding Example 1, the values of link availability p and *network connection probability* $f_N(p)$ should be further discussed. From Fig. 7, the relationship between the number of nodes and connections for a given network connection probability $f_N(p) = 0.9$ is listed in Table 6.

The *link availability* defined in Section II includes all *active* and *inactive* links. However, one observes from Example 1 that only three active connections provide $p = 12/C_2^8 = 12/28$ and the network connection probability is 100%, $f_8(p) = 1$. Since each active node can serve as a master node to connect other *inactive* nodes, the number of (**active**) connections for a required network connection probability $f_N(p)$ will be much less than the numbers given in Table 6. In other words, the number of active connections can be a small portion of the total number of available connections (active plus inactive).

B. Global User Connection Probability for the Union of Fixed and Ad Hoc Networks

As illustrated in Fig. 2, there are M fixed (wired or wireless), structured communication network clusters denoted as F_i which are directly or indirectly connected to each other. Furthermore, we assume that the i^{th} fixed network is connected with M_i ad hoc networks denoted as A_{ij}. Then the set including all user terminals which are directly or indirectly connected by cooperating with fixed and ad hoc networks is given by

$$\bigcup_{i=1}^{M}\{\text{terminals in } F_i\} \bigcup_{j=1}^{M_i}\{\text{terminals in ad hoc network, } A_{ij}\}. \tag{9}$$

Furthermore, we assumed that N_i is the number of user terminals supported only by the i^{th} fixed network[3], N_{ij} and p_{ij} are the number of user terminals and the *link availability* in the j^{th} ad hoc network which is also connected to the i^{th} fixed network. Then the ratio of the average number of user terminals directly or indirectly connected to the total number of user terminals is defined to be the **global** *network connection probability*

$$g_N(\underline{P}) = \frac{1}{N}\{\sum_{i=1}^{M}[N_i + \sum_{j=1}^{M_i} N_{ij} f_{N_{ij}}(p_{ij})]\}, \tag{10}$$

[3]Network connection probability is assumed to be one for a fixed network.

where $\underline{P} \equiv [p_{ij};\ i = 1, \cdots, M, j = 1, \cdots, M_i]$ is the probability vector of *link availabilities* for ad hoc networks A_{ij}, $N = \sum_{i=1}^{M}(N_i + \sum_{j=1}^{M_i} N_{ij})$ is the total number of user terminals and $f_{N_{ij}}(p_{ij})$ is defined in (8) for the ad hoc network A_{ij}. Therefore, by cooperating with fixed networks, ad hoc networks with random connections can be connected globally with high probability. This provides the characterization for the *network connection probability* in today's world wide website (WWW) where millions of scattered user terminals are connected through fixed networks and ad hoc networks.

It is the authors' vision that future wireless mobile networks will evolve to "random personal networks" available to everyone in the local area with random connections among mobile users with few controls from fixed network infrastructures. State-of-the-art computer technology will allow private users to establish "private basestations" using personal computers (PCs). Communications among thousands of mobile or fixed terminal users with random connections in a local area is supported both by fixed networks or ad hoc networks. These "private basestations" are similar to today's world wide websites, which are available for much personalization by individuals yet are not subject to much governance by any particular organizations. According to this vision, it is obvious that the *network connection probability* for the Internet can approach 100% of populations geographically separated by individuals using arbitrary random connections.

Example 3: Fig. 11 serves to illustrate an example of a hybrid network with two fixed networks and four ad hoc networks. It is assumed that five reliable connections amongst always active nodes B_1 to B_6 are established 100% of the time and hence provide the connections for users to communicate beyond their local area networks. This network architecture is similar to an *Extended Service Set (ESS)* defined in IEEE 802.11 Standard [6] and nodes B_1 to B_6 can be viewed as *access points (APs)*. Suppose that 80% of networks nodes in ad hoc network A_{11} is inside the *coverage* of node B_3, there are $10 \times 0.8 = 8$ nodes connected with node B_3. In addition, each of the 8 nodes in ad hoc network A_{11} can establish a connection with any node in the fixed network F_1 from node B_1. Therefore, the *global network connection probability* is given by $g_{100}(\underline{P}) = [30+30+10\times(0.8+0.7+0.6+0.5)]/100 = 0.86$. In conclusion, by using hybrid networks, the network nodes can establish connections beyond their own ad hoc network capability and network connectivity evolves globally.

VII. CONCLUSIONS

This paper has derived network connection probabilities for users of wireless ad hoc and hybrid networks, e.g., IEEE 802.15, IEEE 802.11 ETSI BRAN HIPERLAN, Bluetooth, Wireless ATM, and the world wide web (WWW). These probabilities are graphically demonstrated and indicate that global connectivity can be achieved with high probability. In other words, "wired equivalent network connectivity" can be achieved by deploying wireless ad hoc and hybrid networks. It would be of interest to find the asymptotic relationship between $f_N(p)$ and p when N approaches infinity. This limiting case remains an open issue for further study.

REFERENCES

[1] C.-K. Toh, *Ad Hoc Mobile Wireless Networks*, Prentice-Hall, Englewood Cliffs, New Jersey, 2001.

[2] C.-K. Toh, *Wireless ATM and Ad Hoc Networks*, Kluwer Academic Press, New York, 1996.

[3] E. Royer and C.-K. Toh, "A review of current routing protocols for ad hoc mobile wireless networks," *IEEE Personal Commun.*, pp. 46–55, Apr. 1999.

[4] D. Kim, S. Ha, and Y. Choi, "K-hop cluster-based dynamic source routing in wireless ad-hoc packet radio networks," in *Proc. IEEE VTC '98*, 1998.

[5] C.-K. Toh, "Associativity based routing for ad hoc mobile networks," *Wireless Personal Commun. J.*, Mar. 1997.

[6] B. Ohara and A. Petrick, *IEEE 802.11 Handbook*, Standards Information Network, IEEE Press, 1999.

[7] ETSI TC-RES, "Radio equipment and systems: High performance radio local area network (HIPERLAN); type I; functional specification," *Technical Correction*, France, Dec. 1996.

[8] ETSI TC-RES, "Radio equipment and systems: HIPERLAN; system definition," *Technical Correction*, France, July 1994.

[9] T. Wilkinson, "HIPERLAN," *2nd IEEE Workshop*, Wireless LANs, Worcester Polytech Inst., Oct. 1996.

[10] E. Ayanagla *et al.*, "Mobile information infrastructure," *Bell Labs Tech. J.*, pp. 143–163, 1996.

[11] P. Agrawal *et al.*, "SWAN: A mobile multimedia wireless network," *IEEE Personal Commun.*, pp. 18–33, Apr. 1996.

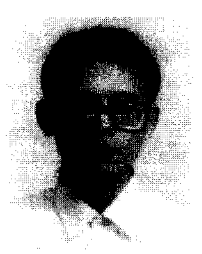
[12] K. Y. Eng *et al.*, "BAHAMA: A broadband ad hoc wireless ATM local area networks," in *Proc. IEEE ICC'95*, 1995, pp. 1216–1223.

[13] K. Pohlovan, "Handoff in hybrid mobile data networks," *IEEE Personal Commun.*, pp. 34–47, Apr. 2000.

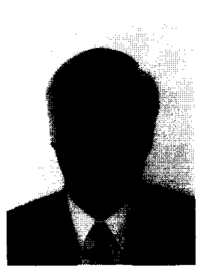
[14] Y.-D. Lin *et al.*, "Multihop wireless IEEE 802.11 LANs: A prototype implementation," *J. Commun. Networks*, Dec. 2000.

[15] D. Kim, C.-K. Toh, and Y. Choi, "TCP-Bus:Improving TCP performance in wireless ad hoc networks," *J. Commun. Networks*, pp. 175–185, June 2001.

[16] W. C. Lindsey and J.-H. Chen, "Architectures for arbitrarily connected synchronization networks," *J. Commun. Networks*, vol. 1, no. 2, pp. 89–98, June 1999.



Jeng-Hong Chen was born in Kaohsiung, Taiwan, R.O.C. in 1966. He received the B.S. degree in communication engineering from National Chiao Tung University in 1988, the M.S. and Ph.D. degrees in electrical engineering from National Taiwan University, Taipei, in 1990, and University of Southern California, Los Angeles, in 1999, respectively. His research interests are network synchronization, analog/digital phase/frequency locked loops, DS/FH CDMA code acquisition/tracking, mobile satellite communications, and cellular communication networks.



William C. Lindsey is Professor of Electrical Engineering at the University of Southern California and served as Chairman of the Board and CEO of LinCom Corporation, which he founded in 1974. He has been a frequent consultant to the US government and various industrial organizations. He has published over 150 papers on diverse topics in communication theory and holds numerous patents. He has written three books: Synchronization Systems in Communication and Control (Englewood Cliffs, NJ: Prentice-Hall, 1972); Telecommunication Systems Engineering, coauthored with M.K. Simon (Englewood Cliffs, NJ: Prentice Hall, 1973; revised edition, Dover Publications, 1991); Digital Communication Techniques: Signal Design and Detection, co-authored with Marvin K. Simon and Sami M. Hinedi (Englewood Cliffs, NJ: Prentice Hall, 1995). He is also coauthor of the IEEE Press Book Phase-Locked Loops and Their Applications. Currently, he serves as Chairman of the Board and CTO of LinCom Wireless Corporation. He serves on Commission C, Signals and Systems of the International Scientific Radio Union (URSI) and was Vice President for Technical Affairs of the IEEE Communications Society. He also served as the second Vice Chairman of the Communications Society (circa 1963) and is a former Editor of the Communications Society journal. Currently he serves as an editor for the Journal on Communications and Networks. He is a Life Fellow of the IEEE and Fellow of the International Engineering Consortium. He is also a member of the National Academy of Engineering (NAE).