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시변 시간지연을 가지는 이산 선형 불확실성 시스템에 대한 보장 비용 제어

(Guaranteed Cost Control for Discrete-time Linear Uncertain Systems with Time-varying Delay)

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요 약

본 논문에서는 시변 시가지연을 가지는 이산 선형 불확실성 시스템에 대한 보장 비용 제어문제를 다룬다. 본 논문에서 다루는 불확실성 시스템은 시변 노음 한정 파라미터 불확실성을 가진다. 모든 허용 가능한 불확실성에 대해 폐루프 시스템이 자승적으로 안정하고 성능을 보장하는 제어기가 존재할 충분조건과 설계방법에 대해 논의한다. 또한, 변수 치환과 Schur 여수정리 등을 이용하여 충분조건을 모든 변수에 대한 선형행렬 부등식(linear matrix inequality)으로 표현한다.

Abstract

This paper deals with the guaranteed cost control problems for a class of discrete-time linear uncertain systems with time-varying delay. The uncertain systems under consideration depend on time varying norm-bounded parameter uncertainties. We address the existence condition and the design method of the memoryless state feedback control law such that the closed loop system not only is quadratically stable but also guarantees an adequate level of performance for all admissible uncertainties. Through some changes of variables and Schur complement, It is shown that the sufficient condition can be rewritten as an LMI(linear matrix inequality) form in terms of all variables.

Keyword: guaranteed cost control, discrete-time linear uncertain system, time-varying delay, linear matrix inequality

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I. Introduction

Recently time delay is main concerns because time delays often are the causes for instability and poor performance of control systems. Since some works of controller design methods have been developed, many state feedback controller design algorithms of time delay systems were presented^[1,9]. The problems of the quadratic stabilization of linear uncertain systems with norm-bounded parameter uncertainties have

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received considerable attention in the recent vears [1:6:10].

The guaranteed cost control approach to the design of the state feedback control law for linear uncertain systems has received a considerable amount of interest. Many related works treated the guaranteed cost control for the discrete-time linear uncertain systems with norm-bounded parameter uncertainties^[29,11,12]. But most of the results are related to uncertain systems without time delay or with constant time delay. Hence, increasing attention has been paid to the guaranteed cost control for the discrete-time linear uncertain systems with time-varying delay.

Guan et al. [2] dealt with the guaranteed cost control for the discrete-time linear uncertain system with constant time delay. They showed a sufficient condition for guaranteeing not only the quadratic stability of the closed loop system but also the cost function bound constraint. However, they did not consider the discrete-time linear uncertain system with time-varying delay. Therefore, we deal with the guaranteed cost control for the discrete-time linear uncertain systems with time-varying delay.

In this paper, we propose the guaranteed cost control problem of a class of discrete-time linear uncertain systems with time-varying delay. We address the existence condition and the design method of the memoryless state feedback control law such that the closed loop system is not only quadratically stable but also guarantees an adequate level of performance for all admissible uncertainties. Through some changes of variables and Schur complement, the obtained condition can be rewritten as an LMI form in terms of all variables. Using LMI toolbox, the solutions can be easily obtained.

The notations in this paper are quite standard. \Re , \Re^p , and $\Re^{p\times q}$ denote, respectively, the set of integer numbers, the set of P-dimensional Euclidean space and the set of all $P\times q$ real matrices. The superscript "T" denotes the matrix transpose and the notation $X \geq Y$ (respectively, X > Y) where X and Y

are symmetric matrices, means that X-Y is positive semi-definite(respectively, positive definite). I is the identity matrix with compatible dimension.

II. Robust Performance Analysis

We consider the discrete-time linear uncertain system with time-varying delay described by the difference delay equation,

$$x(k+1) = \hat{A}(k)x(k) + \hat{A}_{d}(k)x(k-d(k))$$

$$= (A + \Delta A(k))x(k) + (A_{d} + \Delta A_{d}(k))x(k-d(k)),$$

$$x(k) = \phi(k), \quad -m \le k \le 0,$$
(1)

where $x(k) \in \Re^n$ is the state, the time-varying delay $d(k) \in \Re$ is the positive integer satisfying

$$0 < d(k) \le m, \quad \forall k \ge 0, \tag{2}$$

and $\phi^{(k)}$ is an initial value at k. $A \in \Re^{n \times n}$ and $A_d \in \Re^{n \times n}$ represent constant matrices with appropriate dimensions, $\Delta A(k) \in \Re^{n \times n}$ and $\Delta A_d(k) \in \Re^{n \times n}$ denote real-valued matrix functions representing parameter uncertainties. It is assumed that all states are measurable for state feedback and the uncertainties are norm-bounded of the following form:

$$\Delta A(k) = H_1 F(k) E_1, \quad \Delta A_d(k) = H_2 F(k) E_2,$$
 (3)

where $H_i \in \Re^{n \times p}$, i = 1, 2 and $E_i \in \Re^{q \times n}$, i = 1, 2 are known constant matrices with appropriate dimensions, and $F(k) \in \Re^{p \times q}$ is unknown but normbounded as

$$F^{T}(k)F(k) \le I. \tag{4}$$

Associated with the system (1) is the quadratic cost function:

$$J = \sum_{k=0}^{\infty} x^{T}(k)Qx(k), \tag{5}$$

where Q > 0 is a positive definite matrix.

We discuss Schur complement used in this paper. One of the basic ideas of LMI problem is that the nonlinear (convex) inequalities are converted to LMI form using Schur complement.

Lemma 1^[13]: For the symmetric matrix $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix}$, the following are equivalent as follows:

L < 0

$$|I_{11}| < 0, \quad L_{22} - L_{12}^T L_{11}^{-1} L_{12} < 0,$$
 (6)

iii)
$$L_{22} < 0$$
, $L_{11} - L_{12}L_{22}^{-1}L_{12}^{T} < 0$.

We introduce Lyapunov functional to give an upper bound on the quadratic cost function (5),

$$V(x(k)) = x^{T}(k)Px(k) + \sum_{i=1}^{m} \sum_{j=k-i}^{k-1} x^{T}(j)Sx(j).$$
 (7)

Definition 1: A positive definite matrix P is a quadratic cost matrix for the system (1), the quadratic cost function (5), and the given values m>0 and $0<\tau<1$ if there exists a positive definite matrix S such that the following LMI is feasible:

$$\begin{bmatrix} (A + \Delta A(k))^T P(A + \Delta A(k)) - P + Q + mS \\ (A_d + \Delta A_d(k))^T P(A + \Delta A(k)) \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{(A + \Delta A(k))^{T} P(A_{d} + \Delta A_{d}(k))}{(A_{d} + \Delta A_{d}(k))^{T} P(A_{d} + \Delta A_{d}(k)) - \tau S} \quad 0 \quad 0 \\ 0 \quad -\Phi_{m} \quad \Phi_{m} \\ 0 \quad \Phi_{m} \quad -\tau^{-1}\Phi_{m}$$
 (8)
$$+\tau [x^{T} (k - d(k)) Sx(k - d(k)) - \sum_{i=1}^{m} x^{T} (k - i) Sx(k - i)].$$

for all $F^{T}(k)F(k) \le I$. In here, Φ_m is defined as follows:

$$\Phi_m = diag\{\widehat{S, S, \cdots S, S}\},\tag{9}$$

where the dimension of Φ_m is m (the upper bound of time-varying delay) times the dimension of S.

From Definition 1, we will show that the system (1) is quadratically stable for a guaranteed level of performance.

Theorem 1: Consider the system (1) with the cost function (5) and suppose that P>0 is a quadratic cost matrix. Then the system (1) is quadratically stable and the cost function satisfies the bound

$$J \le x^{T}(0)Px(0) + \sum_{i=1}^{m} \sum_{j=-i}^{-1} x^{T}(j)Sx(j).$$
 (10)

Proof:

Suppose P>0 is a quadratic cost matrix for the system (1) and the quadratic cost function (5). Then it follows from Definition 1 that there exists a positive definite matrix S satisfying the LMI (8).

Taking the difference of the Lyapunov functional (7) yields,

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k))$$

$$= x^{T}(k)\hat{A}^{T}(k)P\hat{A}(k)x(k)$$

$$+ x^{T}(k)\hat{A}^{T}(k)P\hat{A}_{d}(k)x(k-d(k))$$

$$+ x^{T}(k-d(k))\hat{A}_{d}^{T}(k)P\hat{A}(k)x(k)$$

$$+ x^{T}(k-d(k))\hat{A}_{d}^{T}(k)P\hat{A}_{d}(k)x(k-d(k))$$

$$- x^{T}(k)Px(k) + mx^{T}(k)Sx(k)$$

$$- \sum_{i=1}^{m} x^{T}(k-i)Sx(k-i).$$
(11)

It follows from the LMI (8) that

$$\Delta V(x(k)) < -x^{T}(k)Qx(k)$$

$$+\tau[x^{T}(k-d(k))Sx(k-d(k)) - \sum_{i=1}^{m} x^{T}(k-i)Sx(k-i)]. \tag{12}$$

and the upper-bound the time-varying delay, this implies that the system (1) is quadratically stable. Furthermore, it follows from (12) that

$$x^{T}(k)Qx(k) \le -\Delta V(x(k))$$

$$+\tau[x^{T}(k-d(k))Sx(k-d(k)) - \sum_{i=1}^{m} x^{T}(k-i)Sx(k-i)]$$

$$\le -\Delta V(x(k)) = V(x(k)) - V(x(k+1)).$$
(13)

Summing from k=0 to ∞ on the two side of the

inequality (13) leads to

$$\sum_{k=0}^{\infty} x^{T}(k)Qx(k) \le V(x(0)) - V(x(\infty)). \tag{14}$$

Since the quadratic stability of the system has already been established, we conclude that $V(x(k)) \to 0$ as $k \to \infty$. Hence, inequality (10) is satisfied.

The following theorem gives a characterization of all quadratic cost matrices in terms of LMI.

Theorem 2: A positive definite matrix P is a quadratic cost matrix for the system (1), the quadratic cost function (5), and the given values, m>0 and $0<\tau<1$ if there exist a positive definite matrix S and a parameter $\varepsilon_i>0$ (i=1,2), such that the following LMI is feasible:

Proof:

Using Lemma 1, it follows from (8) that

$$\begin{bmatrix} -P + Q + mS & 0 & \hat{A}^T(k) & 0 & 0 \\ 0 & -\tau S & \hat{A}_d^T(k) & 0 & 0 \\ \hat{A}(k) & \hat{A}_d(k) & -P^{-1} & 0 & 0 \\ 0 & 0 & 0 & -\Phi_m & \Phi_m \\ 0 & 0 & 0 & \Phi_m & -\tau^{-1}\Phi_m \end{bmatrix}$$

$$\approx \begin{bmatrix} -P + Q + mS & 0 & A^T & 0 & 0 \\ 0 & -\tau S & A_d^T & 0 & 0 \\ A & A_d & -P^{-1} & 0 & 0 \\ 0 & 0 & 0 & -\Phi_m & \Phi_m \\ 0 & 0 & 0 & \Phi_m & -\tau^{-1}\Phi_m \end{bmatrix}$$

$$\approx \begin{bmatrix} -P + Q + mS & 0 & A^T & 0 & 0 \\ 0 & 0 & 0 & \Phi_m & -\tau^{-1}\Phi_m \end{bmatrix}$$

$$\approx \begin{bmatrix} -P + Q + mS & 0 & A^T & 0 & 0 \\ 0 & -\tau S & A_d^T & 0 & 0 \\ 0 & 0 & -\Phi_m & \Phi_m \\ 0 & 0 & 0 & -\Phi_m & \Phi_m \\ 0 & 0 & 0 & \Phi_m & -\tau^{-1}\Phi_m \end{bmatrix}$$

$$\begin{vmatrix}
0 & 0 & \Delta A^{T}(k) & 0 & 0 \\
0 & 0 & \Delta A^{T}_{d}(k) & 0 & 0 \\
\Delta A(k) & \Delta A_{d}(k) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{vmatrix}$$

$$+ \begin{bmatrix}
0 & 0 & E_{1}^{T}F^{T}(k)H_{1}^{T} & 0 & 0 \\
0 & 0 & E_{2}^{T}F^{T}(k)H_{2}^{T} & 0 & 0 \\
0 & 0 & E_{2}^{T}F^{T}(k)H_{2}^{T} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} < 0. \tag{16}$$

From (16), using lemma of Petersen^[10] we have inequality (17).

$$\begin{bmatrix} -P + Q + mS & 0 & A^{T} & 0 & 0 \\ 0 & -\tau S & A_{d}^{T} & 0 & 0 \\ A & A_{d} & -P^{-1} & 0 & 0 \\ 0 & 0 & 0 & -\Phi_{m} & \Phi_{m} \\ 0 & 0 & 0 & \Phi_{m} & -\tau^{-1}\Phi_{m} \end{bmatrix}$$

$$+\varepsilon_{1}\begin{bmatrix} 0 \\ 0 \\ H_{1} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & H_{1}^{T} & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_{1}}\begin{bmatrix} E_{1}^{T} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_{1} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+\varepsilon_{2}\begin{bmatrix} 0 \\ 0 \\ H_{2} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & H_{2}^{T} & 0 & 0 \end{bmatrix} + \frac{1}{\varepsilon_{2}}\begin{bmatrix} 0 \\ E_{2}^{T} \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & E_{2} & 0 & 0 & 0 \end{bmatrix} < 0. \quad (17)$$

From (17) and Lemma 1, we obtain the inequality (15).

From Theorem 1 and 2, we can solve the problem designing the memoryless state feedback control law such that the closed loop system is not only quadratically stable but also guarantees an adequate level of performance for all admissible uncertainties. Therefore, we will design the state feedback controller using some changes of variables and Schur complement.

III. Robust Guaranteed Cost Control

We consider the discrete-time uncertain system with time-varying delay described by the difference delay equation,

$$x(k+1) = \hat{A}(k)x(k) + \hat{A}_{d}(k)x(k-d(k)) + \hat{B}(k)u(k)$$

$$= (A + \Delta A(k))x(k) + (A + \Delta A(k))x(k-d(k))$$

$$+ (B + \Delta B(k))u(k),$$

$$x(k) = \phi(k), \quad -m \le k \le 0,$$
(18)

where $x(k) \in \Re^n$ is the state, $u(k) \in \Re^l$ is the control input, the time-varying delay $d(k) \in \Re$ is the positive integer satisfying (2), and $\phi(k)$ is an initial value at k. $A \in \Re^{n \times n}$, $A_d \in \Re^{n \times n}$, and $B \in \Re^{n \times d}$ represent constant matrices with appropriate dimensions, and $\Delta A(k) \in \Re^{n \times n}$, $\Delta A_d(k) \in \Re^{n \times n}$, and $\Delta B(k) \in \Re^{n \times d}$ denote real-valued matrix functions representing parameter uncertainties satisfying

$$\Delta A(k) = H_1 F(k) E_1, \quad \Delta A_d(k) \approx H_2 F(k) E_2, \quad \Delta B(k) = H_1 F(k) E_3, \eqno(19)$$

where $H_i \in \Re^{n \times p}$, i = 1, 2, $E_i \in \Re^{q \times n}$, i = 1, 2, and $E_3 \in \Re^{q \times d}$ are known constant matrices with appropriate dimensions, and $F(k) \in \Re^{p \times q}$ is unknown but norm-bounded as inequality (4). It is assumed that all states are measurable for state feedback.

Associated with the system (18) is the quadratic cost function:

$$J = \sum_{k=0}^{\infty} \left[x^{T}(k)Qx(k) + u^{T}(k)Ru(k) \right]$$
 (20)

where Q > 0 and R > 0 are positive definite matrices.

Definition 2: For the system (18) and the cost function (20), a state feedback control law u(k) = Kx(k) is said to be a quadratic guaranteed cost control with the cost matrix P > 0 if there exists a positive definite matrix S such that the following LMI is feasible:

$$\begin{bmatrix} \hat{A}_{K}^{T}(k)P\hat{A}_{K}(k) - P + Q + mS + KRK & \hat{A}_{K}^{T}(k)P\hat{A}_{d}(k) & 0 & 0 \\ \hat{A}_{d}^{T}(k)P\hat{A}_{K}(k) & \hat{A}_{d}^{T}(k)P\hat{A}_{d}^{T}(k) - \tau S & 0 & 0 \\ 0 & 0 & -\Phi_{m} & \Phi_{m} \\ 0 & 0 & \Phi_{m} & -\tau \Phi_{m} \end{bmatrix} < 0,$$
(21)

where $\hat{A}_{K}(k)$ is defined as follows:

$$\hat{A}_K(k) = \hat{A}(k) + \hat{B}(k)K. \tag{22}$$

From Definition 2, we will design the state feedback controller such that the closed loop system (18) is not only quadratically stable but also guarantees an adequate level of performance.

Theorem 3: Consider the system (18) with the cost function (20). There exists a state feedback controller u(k) = Kx(k) if there exist a positive definite matrix P, S, a matrix M and a parameter $\varepsilon_i > 0$, (i = 1, 2) such that the following LMI is feasible:

$$\begin{bmatrix}
-P^{-1} & P^{-1} & P^{-1} & M^{T} & 0 & P^{-1}A^{T} + M^{T}B^{T} \\
P^{-1} & -m^{T}S^{-1} & 0 & 0 & 0 & 0 \\
P^{-1} & 0 & -Q^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -R^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & -R^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -R^{-1} & 0
\end{bmatrix}$$

$$\begin{bmatrix}
AP^{-1} + BM & 0 & 0 & 0 & A_{ij} & -P^{-1} \\
0 & 0 & 0 & 0 & 0 & 0 & H_{ij}^{T} \\
0 & 0 & 0 & 0 & 0 & 0 & E_{2} \\
0 & 0 & 0 & 0 & 0 & 0 & E_{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & P^{-1}E_{i}^{T} + M^{T}E_{3}^{T} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & P^{-1}E_{i}^{T} + M^{T}E_{3}^{T} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & P^{-1}E_{i}^{T} + M^{T}E_{3}^{T} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & P^{-1}E_{i}^{T} + M^{T}E_{3}^{T} & 0 & 0 & 0 \\
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\end{bmatrix}$$

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\end{bmatrix}$$

$$\begin{bmatrix}
0 & 0 & P^{-1}E_{i}^{T} + M^{T}E_{3}^{T$$

where $M = KP^{-1}$. Moreover, the cost function satisfies the bound,

$$J \le x^{T}(0)Px(0) + \sum_{j=1}^{m} \sum_{j=-j}^{-1} x^{T}(j)Sx(j).$$
(24)

Proof:

Proof is omitted.

IV. Example

Consider a discrete time-varying delay system^[2],

$$x(k+1) = \left(\begin{bmatrix} 0.95 & 0.78 \\ 0.76 & 0.87 \end{bmatrix} + \begin{bmatrix} 0.1 & 0.2 \\ 0.09 & 0.1 \end{bmatrix} F(k) \begin{bmatrix} 0.2 & 0.17 \\ 0.15 & 0.12 \end{bmatrix} \right) x(k)$$

$$+ \left(\begin{bmatrix} 0.12 & 0.09 \\ 0.11 & 0.07 \end{bmatrix} + \begin{bmatrix} 0.11 & 0.19 \\ 0.1 & 0.09 \end{bmatrix} F(k) \begin{bmatrix} 0.08 & 0.05 \\ 0.6 & 0.07 \end{bmatrix} \right) x(k-d(k))$$

$$+ \left(\begin{bmatrix} 0.5 \\ 0.45 \end{bmatrix} + \begin{bmatrix} 0.1 & 0.2 \\ 0.09 & 0.1 \end{bmatrix} F(k) \begin{bmatrix} 0.1 \\ 0.09 \end{bmatrix} \right) x(k),$$

$$x(k) = \begin{bmatrix} e^{-\frac{k}{6}} \\ 0 \end{bmatrix}, \quad -m \le k \le 0,$$

$$(25)$$

where m=6. Associated with this system is the quadratic cost function (20) with

$$Q = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.09 \end{bmatrix}, \quad R = 0.05, \quad \tau = 0.99, \tag{26}$$

Applying Theorem 3, we obtain that the system is robustly stabilizable for

$$P = \begin{bmatrix} 0.4897 & 0.3459 \\ 0.3459 & 0.3678 \end{bmatrix}, S = \begin{bmatrix} 0.0319 & 0.0214 \\ 0.0214 & 0.0163 \end{bmatrix}, M = \begin{bmatrix} -1.1652 & -3.5647 \end{bmatrix}$$

$$\varepsilon_1 = 2.4596, \quad \varepsilon_2 = 2.5784,$$

(27)

and the cost function satisfies the following bound $J \le 1.4042$. Therefore, we obtain the quadratic cost control:

$$K = [-1.8036 -1.7143]$$
 (28)

V. Conclusion

In this paper, we presented the guaranteed cost control problem of a class of discrete-time linear uncertain systems with time-varying delay and proposed the existence condition and the design method of the memoryless state feedback control law such that the closed loop system is not only quadratically stable but also guarantees an adequate level of performance for all admissible uncertainties. The assumption of state availability is made, while in some problems only output information is available. The guaranteed cost output control problem of the discrete-time linear uncertain systems with time-varying delay will be addressed.

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