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Optimal Target Reliability of Bridges Based on Minimum Life-Cycle Cost Consideration

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ABSTRACT

Cost-effectiveness in design is considered for determining the target reliability of concrete bridges under seismic actions. This objective can be achieved based on the economic optimization of the expected life-cycle cost of a bridge, which includes initial cost, direct losses, and indirect losses of a bridge due to strong earthquakes over its lifetime. A *separating factor* is defined to consider the redundancy of a transportation network. The Park-Ang damage model is employed to define the damage of a bridge under seismic action, and a Monte Carlo method based on the DRAIN-2DX program is developed to assess the failure probability of a bridge. The results for an example bridge analyzed in this paper show that the optimal target failure probability depends on the traffic volume carried by the bridge and is between 1.0×10^{-3} to 3.0×10^{-3} over a life of 50 years.

Keywords: bridge, seismic target reliability, life-cycle cost

1. Introduction

Structural reliability is now widely applied for the assessment of safety of structures and the development of building code provisions. However, for the latter application, reliability is generally used simply to provide consistency among different construction materials and or construction quality. That is the target reliability underlying a design or code requirement is calibrated to remain the same as the safety level underlying traditional designs or existing codes. In other words, thus far there has not been any new or innovative approach for the determination of the target reliability for structural design.

The issue has been addressed, directly or indirectly, for earthquake resistant designs, including Whitman, *et al.* (1975), Grandori (1982), and Rosenblueth (1986). In recent years, Ang and his co-workers have contributed to the development of cost-effective aseismic design criteria for building structures based on life-cycle cost consideration (Ang and De Leon, 1996; Ang *et al.*, 1996; Kunnath *et al.*, 1990).

The life-cycle cost concept has also been investigated for bridge designs (ASCE, 1999; Frangopol *et al.*, 2001).

In this paper, a procedure for determining the target reliability of bridges under seismic action is proposed based on minimizing the expected life-cycle cost. The redundancy of a transportation network is taken into account approximately in considering single bridges within the network.

2. Methodology

The total expected life-cycle cost function can be expressed as,

$$E[C_T] = C_I + E[C_D^0] \tag{1}$$

in which C_I is the initial cost of the structure, and C_D^0 is the cumulative damage cost, in present worth, which includes the direct damage cost and indirect loss under all earthquakes that are likely to occur over the life of the structure. C_D^0 is a random variable $E[C_D]$ is its expected value.

The optimal aseismic design problem involving socioeconnomics may then be stated as follow,

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Minimise $E[C_T]$ (2)

Eq. 2 involves a trade-off between the initial cost of the bridge and the expected damage cost to determine the optimal design and a target reliability that minimizes the expected total life-cycle cost. Although the initial cost is normally expressed in present worth, damage costs are associated with the possible occurrence of the future earthquakes and therefore, the present worth of the damage cost will necessarily depend on the occurrence time of future earthquakes. Following Lee's work (Lee *et al.*, 1997), the expected present worth of the cumulative damage cost from future earthquakes over the life L is obtained as follows:

$$E[C_D^0] = \int_0^L E[C_D]\cdot\left(\frac{1}{1+q}\right)^t v dt = \lambda \cdot vL \cdot E[C_D] \quad (3)$$

where $\lambda = \{1-\exp(-\alpha L)\}/(\alpha L)$, is the discount factor, $\alpha = \ln(1+q)$, q is the annual discount rate; v is the annual mean occurrence rate of earthquakes with significant intensities, vL is the expected number of significant earthquakes during the life L; and $E[C_D]$ is the expected current damage cost due to an earthquake. The damage cost consists of two components, direct losses and indirect losses, and the method to estimate the damage cost may be different from one type of structure. The expected damage cost $E[C_D]$ is the sum of the expected costs for all damage components and is given by

$$E[C_D] = \sum_i \int_{y_{min}}^{y_{max}} \int_0^{\infty} C_{Di}(x) f_{X|Y}(x) f_Y[y] dx dy \quad (4)$$

in which X is the damage level of structure; Y is the expected maximum ground intensity conditional on the occurrence of an earthquake ; $f_{X|Y}(x)$ is the probability density function of X conditional on $Y = y$; and $f_Y(y)$ is the probability density function of Y at the site. C_{Di} =cost function for the damage component i.

2.1 Initial Cost

The initial cost function is obtained by designing bridges using a current seismic design code, e.g., the Caltrans Specification of Bridges, but with varying seismic load capacities.. For each design, the corresponding cost is estimated and includes the costs of materials, design and construction. The reliability for each design, i.e., the probability of exceeding a given damage level, is then calculated and the initial cost is expressed as a function of the damage probability, P_f, as

$$C_I = C_I(P_f) \quad (5)$$

2.2 Direct Losses

The direct losses include the cost of repair or replacement of the bridge, and the physical losses directly associated with the damage to the bridge and is given by,

$$C_{DL} = C_R + C_{fo} \quad (6)$$

where C_R is the repair or replacement cost, and C_O is other direct losses (e.g., loss due to damage to vehicles, injury of people etc). In this paper, only the repair component, C_R is considered.

The repair cost function may be developed on the basis of available repair cost data of bridges damaged under previous earthquakes. For this purpose, the median damage of a bridge may be calculated using the Park-Ang damage model (Park and Ang, 1985), and related to the corresponding repair cost, resulting in a regression relation for the normalized repair cost as a function of the median global damage index as follows (Lee and Piers and Ang, 1997):

$$\begin{cases} C_R = \alpha\cdot(d_m)^{\beta} C_I \; ; & 0 \le d_m \le d_o \\ C_R = C_I & ; \; d_m > d_o \end{cases} \quad (7)$$

where d_m is the median global damage index of the bridge and α, β, and d_0 are constants to be evaluated from the damage data. Eq. (7) means that a bridge is not repairable when $d_m > d_0$ and must be replaced.

2.3 Indirect Losses

Indirect losses caused by structural damage from a significant earthquake can be divided into the *first round* loss and the *second round* losses,

$$C_{ID} = C_{ID}^1 + C_{ID}^2 \quad (8)$$

where C_{ID}^1 and C_{ID}^2 denote the first round and second round losses, respectively.

I-O Table

In order to examine the economic impact of earthquake, it is necessary that certain kinds of economic models be developed. In this study, the indirect economic loss caused by earthquake damages will be estimated using the so-called input-output model.

The relationship between the input and the output for industrial sectors can be expressed in the following matrix form,

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{d} \quad (9)$$

where $\boldsymbol{x}$ is the output vector; $\boldsymbol{d}$ is the final demand vector;

and A is the I-O Table for a country or a region.

If the demands of the exogenous sectors were forecast to be some specific amounts next year, we can determine how much output from each of the sectors would be necessary to supply the final demands from Eq. (9). This makes clear the dependence of each of the gross outputs on the values of each of the final demands.

The First Round Loss

The first round loss comes from the reduction in output related specifically to loss of function due to the damage of the bridge. The restoration function relates structural damage to the loss of function of the bridge. The loss of function depends on the level of structural damage and the social class of the facility. For a particular social class at a particular damage state, the restoration function is presented as a time-to-restore curve $F_R(t)$. The loss of function for the damage state, t_{loss}, measured in time, may be calculated as the area above the time-to-restore curve to the 100% functionality level, i.e.,

$$t_{loss} = \int_0^{t_1}\{1 - F_R(t)\}dt \tag{10}$$

The production loss for a given sector may be estimated as the sectoral output of the commodity flow passing through the bridge multiplied by the ratio of the loss of function (measured in time) to the time interval of an I-O table, namely,

$$x_i^{loss} = r \cdot \frac{t_{loss}}{t_{IO}} v_i x_i^p \tag{11}$$

where t_{loss} = loss of function measured in time; t_{IO}=time interval of the I-O table; x_i^p = total output of sector i without any disaster (i.e,x_i^p is a component of $\boldsymbol{x}$ before the earthquake); v_i = sectoral participation factor, i.e., the ratio of the amount of commodity flow passing through the bridge in sector i to the total commodity flow in sector i in the region for trucks, and the ratio of the workers passing through the bridge in sector i to the number of total workers of the regional economy participating in sector i for cars; and r = ratio of volume of traffic delay to total volume passing through the bridge.

For a given global damage level of the bridge, the first round loss may then be obtained as,

$$C_{ID}^1 = \sum_{i=1}^{n} \varepsilon_i x_i^{loss} \tag{12}$$

where ε_i is the economic surplus per unit total output of sector i in the I-O table.

The Second Round Loss

The loss of capacity in one industry would likely reduce the productivity of other industries which obtain inputs from the first industry. This reverberation would lead to the second round loss. Using the estimated change in gross output, the new post-earthquake level of demand is obtained as,

$$\boldsymbol{d}^* = (\boldsymbol{E} - \boldsymbol{A})\boldsymbol{x}^* \tag{13}$$

with

$$\boldsymbol{x}^* = \boldsymbol{x}^p - \boldsymbol{x}^{loss} \tag{14}$$

In conventional inter-industry studies, it is assumed that the intermediate input requirements, the matrix $\boldsymbol{A}$, is invariant. This, however, cannot be assumed after an earthquake because of reduction in productivity capacity caused by the damage to structures and interruption in service. Therefore, the changing structure of the economy must be accounted for through changes in the matrix $\boldsymbol{A}$ in order to adequately estimate the indirect business losses. Matrix $\boldsymbol{A}$ in the post-disaster economy may be approximated by assuming that the direct input requirements of sector i per unit of output j are reduced in proportion to the reduction in output i. This means that local purchases of sector j from sector i to meet the reduced levels of final demand are reduced by the proportion of damage in sector i. The new level of production is thus estimated as,

$$\boldsymbol{x}^r = (\boldsymbol{E} - \boldsymbol{A}^*)\boldsymbol{d}^*$$

$$= (\boldsymbol{E} - \boldsymbol{A}^*)(\boldsymbol{E} - \boldsymbol{A})\boldsymbol{x}^* \tag{15}$$

where $\boldsymbol{A}^*$ is the input coefficient matrix for the post-earthquake economy whose elements are calculated as follows,

$$a_{ij}^* = \frac{x_i^*}{x_i^p} a_{ij} \tag{16}$$

For a given damage level of the bridge, the second round loss is thus obtained as follows,

$$C_{ID}^2 = \sum_{i=1}^{n} \varepsilon_i (x_i^* - x_i^r) \tag{17}$$

2.4 Parameter v_i and r

According to definition of v_i in Eq. (11), v_i may be calculated as follows,

$$n_i = \frac{\text{Values Distributed by Trucks Through the Model Bridge}}{\text{Values Distributed byTrucks Over the U.S.}} \tag{18}$$

It is clear that v_i is different for different traffic volume passing through the model bridge. In this paper only the economic losses due to transportation interruption of commodities is taken into account. However, it is not difficult to extend Eq. (18) to consider the economic losses caused by other types of transportation interruption.

Because of redundancy in the road network, only part of the traffic would be delayed as a result of the damage to a particular bridge. The study of traffic delays is part of transportation engineering, which is beyond the scope of this paper. Werner *et al.* (1995) presents some results of delays of travel time caused by earthquakes. The traffic delay is 19.54 percent (i.e., r = 19.54%) of the total vehicle hours without earthquake damage (for an example earthquake). However, correctly the ratio, r, should be related to the earthquake damage level of the bridge. Due to the lack of data for evaluating r, this ratio is assumed in this paper as,

$$r = \begin{cases} 0.6d_m, & d_m \leq 0.5 \\ 0.30, & d_m \leq 0.5 \end{cases} \tag{19}$$

where d_m is the median damage level of the bridge.

3. Realiability Assessment

To properly assess a component damage under random seismic loads, the bridge must be adequately modelled and analyzed in order to obtain its response under simulated or recorded earthquake ground motions. To assess the damage of a bridge, a damage model for a reinforced concrete component, column and girder proposed by Park and Ang (1985, 1987) is adopted as follows,

$$D = \frac{\delta_m}{\delta_u} + \frac{\beta_0 E}{Q_y \delta_u} \tag{20}$$

where δ_m is the maximum response displacement; δ_u is the displacement capacity; β_0 is a constant; Q_y is the yielding force; and E is the hysteretic energy dissipated. The global damage index, D_g, of the bridge may be defined as,

$$(D_g > d) = \bigcup_i (D_i > d) \tag{21}$$

where D_i is the damage index of the ith critical component of the structure. The determination of the parameters in Eq. 20 is detailed in references (Kunnath *et al.*, 1990; Park *et al.*, 1985; Park *et al.*, 1987).

Since the bridge response under a severe earthquake is non-linear and hysteretic, the computation of the response statistics under random seismic ground motions using appropriate bridge models and capacities becomes an extremely complex task. Therefore, Monte Carlo simulation is used to compute the desired response statistics. In this regard, the computer program DRAIN-2DX has been modified to perform the desired Monte Carlo simulations, with the critical bridge components modelled using the beam-column element in DRAIN-2DX with a tri-linear elasto-plastic hysteresis. Seismic ground motions used as input for the simulation can be actual earthquake records or samples of non-stationary, filtered Gaussian processes.

4. Example Bridge

4.1 Configuration of The Example Bridge

A 4-span reinforced concrete bridge, as shown in Fig. 1, is taken as an example to demonstrate the procedure suggested in this paper. The bridge has a total length of 16+20+20+16 meters with three single piers of circular sections. The depth of the girder is 110 cm, and the diameter of the column piers is 120 cm. The height of each column pier is 8.5 m. The connection joints between the girder and the column piers are assumed to be fixed. At the two ends of the girder, the girder can move in the longitudinal direction but is fixed in the vertical direction.

4.2 Target Probability

In this paper, the response of the bridge in the longitudinal direction is considered; the response in the lateral direction is neglected. A total of seven designs of the model bridge were performed following the CALTRANS Bridge Design Specifications (CALTRANS, 1990) using seven different *ductility/risk* reduction factors, i.e., $1/Z = C \cdot C_0$; where C= 0.50, 0.70, 0.85, 1.00, 1.15, 1.30, 1.50, and C_0=1/6 is the current *ductility/risk* reduction factor in the *Bridge Specifications of Caltrans*. To calculate the response statistics of the bridge, the peak ground acceleration (PGA) are graded into 10 levels, namely 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0g; ground motions are generated as nonstationary filtered Gaussian processes. For each design and each PGA level, the global median damage index, d_m, and the probability of collapse,

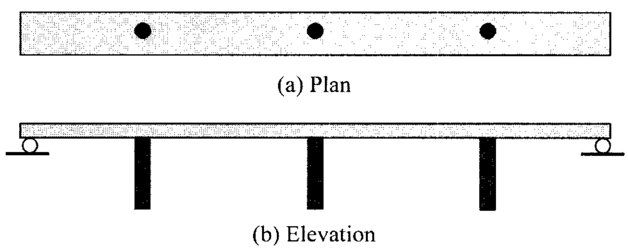

Fig. 1. Configuration of the Example Bridge

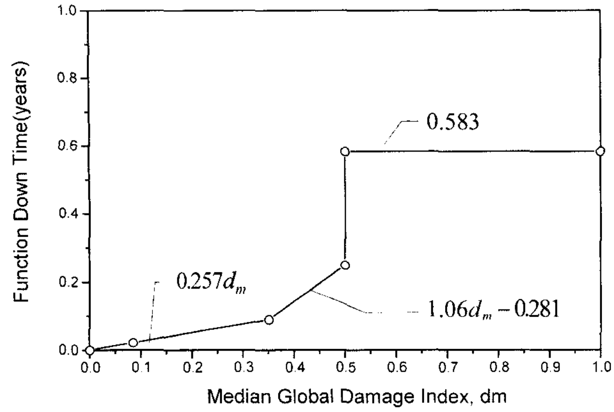

Fig. 2. Function Down Time vs d_m.

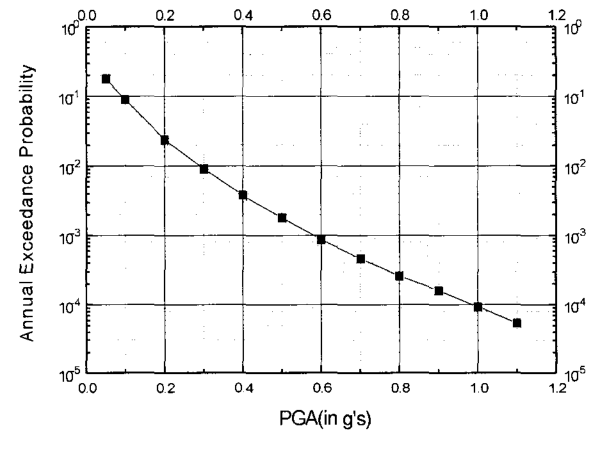

Fig. 3. The Annual Seismic Hazard Curve in Los Angeles Area.

Table 1. Breakdown of Traffic in Western U.S.A

	Passenger Cars (%)	Trucks (%)	Others (%)
21293	52.21	14.69	33.10

$P(D_g>1.0)$, are calculated using MCS method, as stated earlier.

Following Lee (1997), the relationship between the function down time and the median global damage index, d_m, used in this paper, are shown in Fig. 2. The annual seismic hazard used in this paper is shown in Fig. 3, which is for soil sites in Los Angeles. The results of d_m and $P(D_g>1.0)$ are shown in Fig. 4 as respective functions of the PGA. The median damage indices and the probability of damage are then used to calculate the expected life-cycle cost as a function of the life-cycle probability of collapse.

To estimate the parameter v_i for the economic losses, transportation statistics is needed. Table 1 is the traffic statistics in the west area of the U.S., and Table 2 lists the commodity distribution for the United States.

5. Results and Conclusions Median

5.1 Results

The results obtained for the target failure probability and life-cycle cost of the model bridge are shown in Fig. 5 for various traffic volumes (TV), where C_i is the initial cost, C_r is the repair cost and C_{id} is the indirect loss. For a given

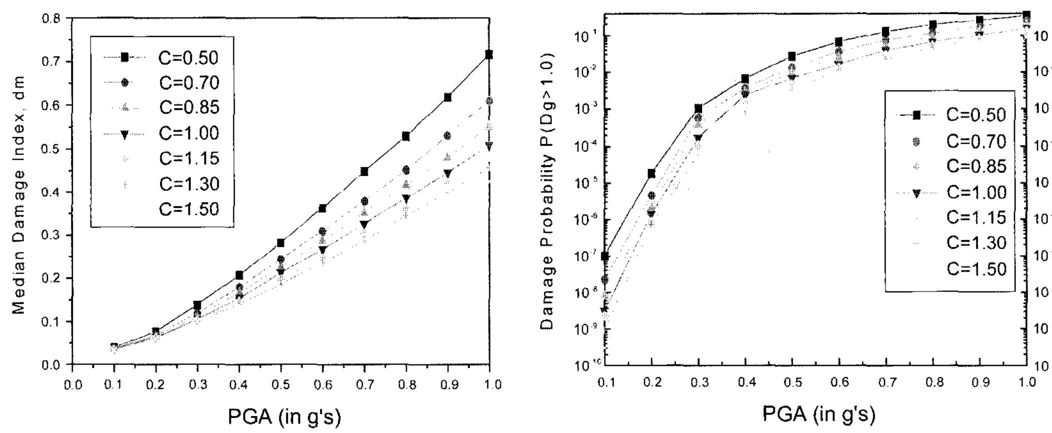

Fig. 4. Median Damage index, d_m, and Probability of Collapse, $(D_g>1.0)$, for different designs of the model bridge as functions of PGA levels.

Table 2. 1993 Commodity Flow Survey for the United States

Mode	Value (million dollars)	Tons (thousands)	Ton miles (millions)	Value per ton (dollars)
Total	6,123,832	12,157,105	3,627,919	$503.70
Truck	4,403,495	6,385,915	869,536	$689.60

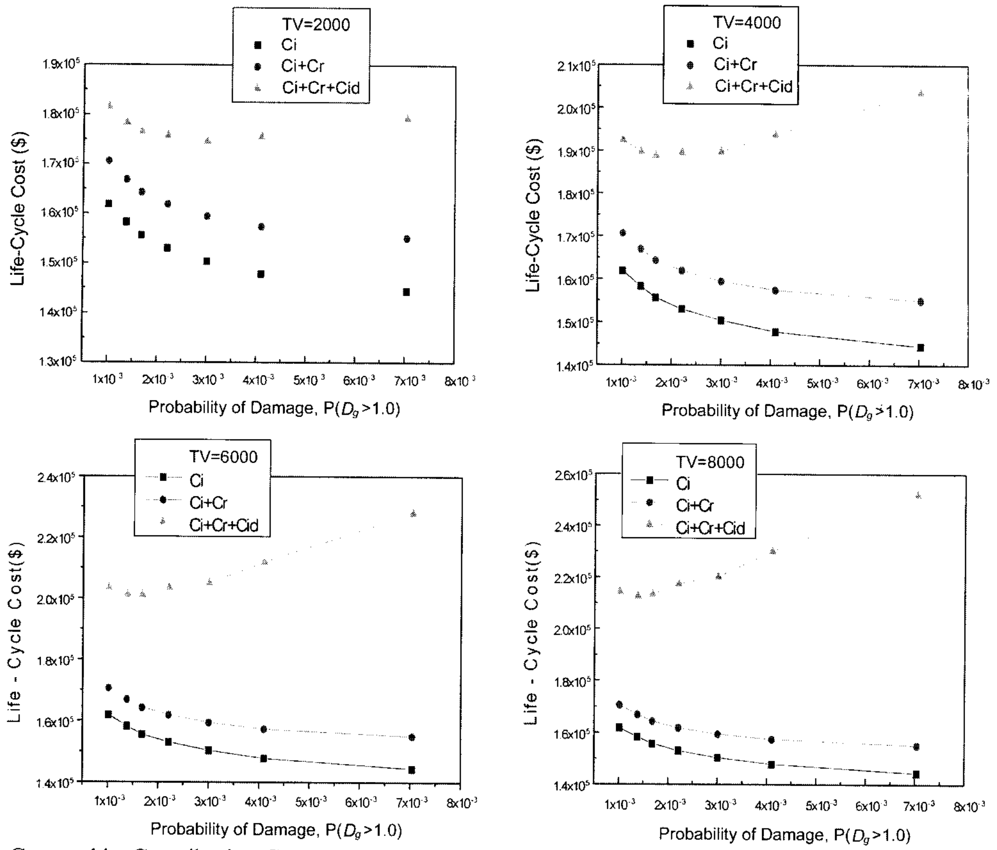

Fig. 5. Life-Cycle Cost and its Contributing Components for Different Traffic Volumes.

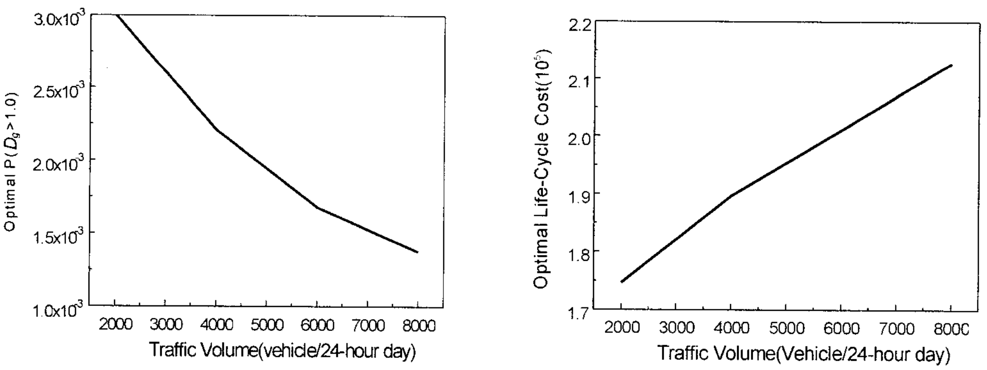

Fig. 6. Optimal Target Damage Probability and Life-Cycle Cost v.s. Traffic Volume.

traffic volume, TV, the three curves in each figure show the incremental contributions of the cost components to the total life-cycle cost. Clearly, the indirect economic loss from potential earthquakes overshadows the cost of physical repair.

For a given traffic volume, one can also observe from Fig. 5 the optimal target collapse probability that corresponds to the lowest mean life-cycle cost for the bridge. Also, it can be seen that this optimal design point changes with the traffic volume; this is more clearly shown in Fig. 6. Observe also that high traffic volume will involve higher indirect economic loss and thus would require a lower damage probability for a bridge. This observation suggests that it will need stronger design for a bridge that is expected to carry heavy traffic.

Therefore, with increasing traffic volume, the expected indirect losses will become more important, while the expected repair cost will remain essentially unchanged; i.e., the life-cycle losses of bridges will depend largely on the functional losses of the bridge, more than on the losses from damage or collapse of the bridge itself. With the use of the procedure suggested in this paper, one can determine the optimal aseismic target reliability for bridges.

5.2 Concluding Remarks

In this paper, the cost-effectiveness in design is introduced to determine the target reliability of concrete bridges under seismic actions. The method is based on minimizing the mean life-cycle cost of a bridge. The importance of a bridge in a transportation network is taken into account. The following main conclusions can be observed:

1. The mean life-cycle cost of a bridge will be a function of the target reliability (or probability of damage) underlying its design.
2. For a class of R/C bridges, an optimal target reliability may be determined on the basis of the minimum mean life-cycle cost.
3. For bridges with high traffic volumes, the indirect losses caused by the loss of function of a bridge will be the dominant contributor to the total life-cycle cost, whereas the repair cost from physical damage to the bridge may not change much with traffic volume.
4. As expected, the optimal target reliability will depend on the traffic volume (a measure of the importance of a bridge); higher traffic volume will require higher target reliability and thus stronger design.
5. Based on the results for the example bridge analyzed in this paper, the optimal target failure probabilities for the corresponding class of R/C bridges are between 1.0×10^{-3} to 3.0×10^{-3} within the traffic volumes considered.

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