Composite Control for Inverted Pendulum System

Yo-Han Kwon, Beom-Soo Kim, Sang-Yup Lee, Myo-Taeg Lim

Abstract: A new composite control method for a carriage balancing single inverted pendulum system is proposed and applied to swing up the pendulum and to stabilize it under the state constraint. The target inverted pendulum system has an extremely limited length of the cart(below $16cm$). The proposed swing-up controller comprises a sliding mode control algorithm and an optimal control algorithm based on two regions: the region near the inverted unstable equilibrium position and the rest of the state space including the downward stable equilibrium position. The sliding mode controller uses a switching control action to converge along the specified path(hyperplane) derived from energy equation from a state around the path to desired state(standing position). An optimal control method is also used to guarantee the stability at unstable equilibrium position. Compared with the reported controllers, it is simpler and easier to implement. Experimental results are given to show the effectiveness of this controller.

Keywords: inverted pendulum, swing-up control, sliding mode control, optimal control, nonlinear control

I. Introduction

An inverted pendulum is one of the most commonly studied systems in the control area. The control objective of the inverted pendulum is to swing up the pendulum hinged on the moving cart of a linear motor from a stable position(vertically down state) to the zero state(vertically upward state) and to keep the vertically upward state in spite of the disturbance[1].

Many swing up control methods have been reported[2][3][4][5][6]. The feed-forward bang-bang control method for the inverted pendulum system is very sensitive to modeling error, noise, and disturbance[7]. The bang-bang control control law with pseudo state feedback proposed in[8] was successfully demonstrated on the rotating-arm inverted pendulum system without a rail length restriction. A linearized technique was applied to the carriage balancing single inverted pendulum(CBSIP) system with long rail length or to the rotating-arm single inverted pendulum system[9][10]. But the nonlinearity of an inverted pendulum system is too strong to linearize. This method is not suitable for controlling the CBSIP system where the rail length is extremely limited. And other swing up control methods were applied to CBSIP system where the rail length is longer than $50cm$ [11][12][13].

In this paper, we design a stable and robust controller for CBSIP systems under the extremely limited travel range of the cart(below $16cm$). The proposed swing-up controller comprises a sliding mode control algorithm and an optimal control algorithm based on the pendulum regions. Two control methods are applied by dividing the system into two regions; the region near the origin of the state space, that is, the inverted and unstable equilibrium position, and the rest including the downward stable equilibrium position, from which the swing-up control starts.

The organization of this paper is as follows. In Section 2, the inverted pendulum control system and its mathematical model are outlined. Using the result of Section 2, an optimal control method to stabilize the pendulum around the zero state on the basis of the linearized model is described in Section 3. A sliding mode controller design method considering the extremely limited rail length is developed in Section 4. In Section 5, we compare the proposed method with the previously reported algorithms by simulation. Then we perform the experiment to verify the effectiveness and robustness of the proposed control algorithm. Finally, the conclusion is given in the last section.

II. Model of the plant

Block diagram of the inverted pendulum control system is illustrated in Fig. 1. The pendulum rotating freely in the vertical plane and the rotary encoder for measuring the angular displacement of the pendulum are installed on the cart. The cart is replaced by a linear motor having a built-in encoder to measure the displacement of the cart. The measured signal flows into the DSP board to calculate the control law which is applied to the linear motor through the three phase inverter.

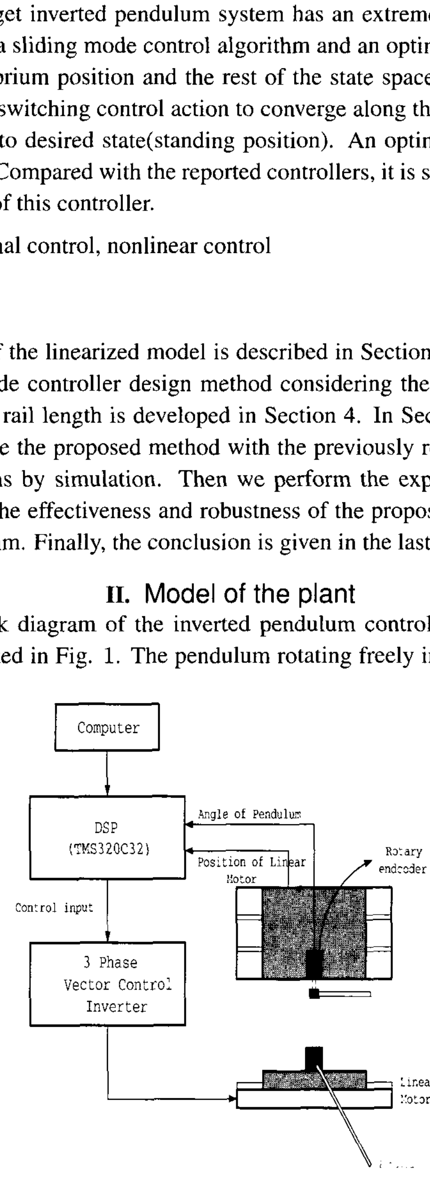

Fig. 1. Experimental setup of the pendulum control system.

The mathematical model of the pendulum plant can be constructed by using the Lagrange equation[10].

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x_i}}\right) - \frac{\partial L}{\partial x_i} + \frac{\partial D}{\partial \dot{x_i}} = Q_q \tag{1}$$

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with

$$x_1 = x,\ x_2 = \theta,\ Q_q = \begin{bmatrix} f \\ 0 \end{bmatrix},\ L = T - V$$

where T is kinetic energy, V is potential energy, x is a position of cart, θ is a pendulum angle, L is Lagrangian, and D is a Rayleigh dissipation function. Kinetic energy of the inverted pendulum is as follows:

$$T = T_{cart} + T_{pendulum} \tag{2}$$

with

$$\begin{aligned} T_{cart} =& \frac{1}{2}m_c\dot{x}^2 \\ T_{pendulum} =& \frac{1}{2}m_1\bar{v}_G^2 + \frac{1}{2}J\dot{\theta}^2 \\ \bar{v}_G^2 =& \dot{r}_x^2 + \dot{r}_y^2 \\ =& \left(\dot{x} + l\dot{\theta}\cos\theta\right)^2 + l^2\dot{\theta}^2\sin^2\theta \end{aligned}$$

where m_c is mass of cart, m_1 is mass of pendulum, and l is the length between the center of mass of the pendulum and the axis.

Thus we can obtain the kinetic energy T as follows:

$$T = \frac{1}{2}m_c\dot{x}^2 + \frac{1}{2}m_1\left(\left(\dot{x} + l\dot{\theta}\cos\theta\right)^2 + l^2\dot{\theta}^2\sin^2\theta\right) + \frac{1}{2}J\dot{\theta}^2 \tag{3}$$

Potential energy of the inverted pendulum is given by (4). Actually the potential energy of the cart is unchanged.

$$V = V_{cart} + V_{pendulum} \tag{4}$$

In (4), V_{cart} is potential energy of the cart and $V_{pendulum}$ is potential energy of the pendulum

$$\begin{aligned} V_{cart} =& 0 \\ V_{pendulum} =& m_1 g\left(l\cos\theta + l_1\right) \end{aligned}$$

where g is the gravity constant. Therefore the potential energy equation becomes as follows:

$$V = m_1 g\left(l\cos\theta + l_1\right) \tag{5}$$

Rayleigh dissipation function is a function for the energy which is spent by friction and given by

$$D = \frac{1}{2}v\dot{x} + \frac{1}{2}C\dot{\theta} \tag{6}$$

where v is a cart friction coefficient and C is a pendulum friction coefficient.

We can express the Lagrangian by using (3) and (5)

$$\begin{aligned} L =& T - V \\ =& \frac{1}{2}m_c\dot{x}^2 + \frac{1}{2}m_1\left(\left(\dot{x} + l\dot{\theta}\cos\theta\right)^2 + l^2\dot{\theta}^2\sin^2\theta\right) \\ &+ \frac{1}{2}J\dot{\theta}^2 - m_1 g\left(l\cos\theta + l_1\right) \end{aligned} \tag{7}$$

From the Lagrangian equation (1), we can obtain the following nonlinear state equation.

$$\dot{\bar{X}} = F(\bar{X}, t) + B(\bar{X})u \tag{8}$$

where

$$F(\bar{X}, t) = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ F_3 \\ F_4 \end{bmatrix}$$

$$B(\bar{X}) = \begin{bmatrix} 0 \\ 0 \\ m_{11} \\ m_{21} \end{bmatrix},\ \bar{X} = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix},$$

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} m_c + m_1 & m_1 l\cos\theta \\ m_1 l\cos\theta & m_1 l^2 + J \end{bmatrix}^{-1}$$

with

$$\begin{aligned} F_3 =& m_{11}\left(m_1 l\sin\theta\dot{\theta}^2 - v\dot{x}\right) + m_{12}\left(m_1 lg\sin\theta - C\dot{\theta}\right) \\ F_4 =& m_{21}\left(m_1 l\sin\theta\dot{\theta}^2 - v\dot{x}\right) + m_{22}\left(m_1 lg\sin\theta - C\dot{\theta}\right) \end{aligned}$$

III. Optimal controller design

In this section, we design an optimal controller to stabilize the pendulum around the zero state by using the linearized model. At the target point ($\theta = 0$) we can obtain the linearized model under the following assumptions.

$$\dot{\theta} = 0,\ \cos\theta \approx 1,\ \sin\theta \approx \theta \tag{9}$$

We can obtain the linearized model (10) by using (8) and (9)

$$\dot{\bar{X}} = A\bar{X} + Bu \tag{10}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & m_1 lgn_{12} & -vn_{11} & -Cn_{12} \\ 0 & m_1 lgn_{22} & -vn_{21} & -Cn_{22} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ n_{11} \\ n_{21} \end{bmatrix},$$

$$\begin{bmatrix} m_c + m_1 & m_1 l \\ m_1 l & m_1 l^2 + J \end{bmatrix}^{-1} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}$$

The quadratic cost function to be minimized is given by

$$J = \int_0^\infty \left(\bar{X}^T Q\bar{X} + u^T Ru\right) dt \tag{11}$$

where $Q \in R^{n\times n}$ is a positive semidefinite matrix and $R \in R^{m\times m}$ is a positive definite matrix.

The well-known optimal control law is given by

$$u = -R^{-1}B^T P\bar{X}(t) \tag{12}$$

with matrix P representing the positive semidefinite stabilizing solution for the following algebraic Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{13}$$

In this paper, we consider the inverted pendulum having state constraint and limited amplitude of input. Assume that the cart is constrained to move in the horizontal range. So we must consider the available rail range to design the controller. The stabilizing optimal control law for the performance index is given by

$$u = \begin{bmatrix} 70.7107 & 203.3644 & 88.8864 & 33.4018 \end{bmatrix}\bar{X} \tag{14}$$

with the following weighting matrix Q and R

$$Q = \begin{bmatrix} 5000 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ R = 1$$

IV. Sliding mode controller design

In the swing up control, we use the full nonlinear dynamic model obtained in Section IIand apply a sliding mode control method to swing up the pendulum. In designing the sliding mode control law, a switching surface is designed and then a reaching control law is designed to drive the states to the designed switching surface.

1. Swing up control

The nonlinear system represented by (8) has a stable equilibrium at $\bar{X}_1 = \begin{bmatrix} x_c & \pi & 0 & 0 \end{bmatrix}^T$, and an unstable equilibrium at $\bar{X}_2 = \begin{bmatrix} x_c & 0 & 0 & 0 \end{bmatrix}^T$. Both of these equilibrium points can be obtained with an arbitrary carriage position x_c, therefore the carriage's position control is possible. Without the control ($u = 0$), the system's natural behavior can be observed by releasing it from the initial state $\bar{X}_1 = \begin{bmatrix} 0 & \theta_0 & 0 & 0 \end{bmatrix}^T$, where $\theta_0 \neq 0°$ and $\theta_0 \neq 180°$. At the $\theta_0 = 10°$ the numerical solution of (8) is computed and shown in Fig. 2. The natural behavior of the system is damped vibration of both the pendulum and the carriage. Two important factors can be observed from the system's natural behavior: (i) due to the translational and the rotational friction, each amplitude of the pendulum and the carriage decreases with the time; and (ii) by decreasing amplitudes, the circular frequency of the vibration increases with the time, which is a nonlinear characteristic. In order to swing-up the pendulum toward the inverted vertical position, its kinetic energy level must be increased. In this system, it is achieved by applying force to the carriage, and this force must be large enough to overcome the friction. The direction and the amplitude of the input force are controlled to increase the pendulum's kinetic energy, despite the difficulties that (a) the force u applies to the pendulum indirectly through the carriage, (b) the vibrations of the pendulum and the carriage are nonlinear, and (c) there is a phase-lag between these two vibrations[11].

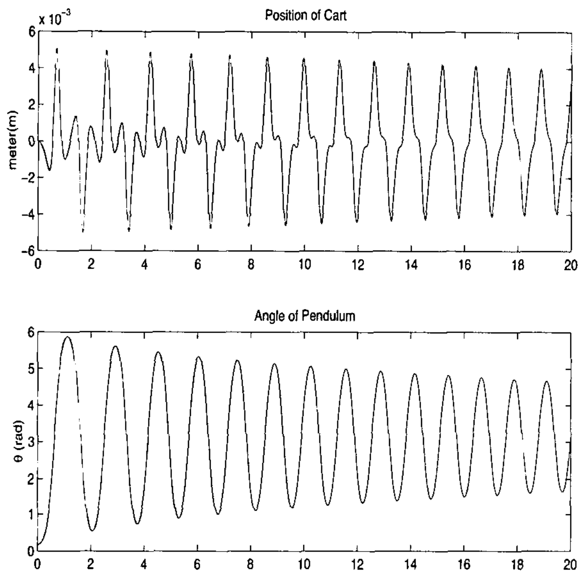

Fig. 2. Natural behavior of CBSIP system.

When the total energy of the pendulum equals to the pendulum's potential energy at the inverted vertical position, the pendulum will be swung up to its upright position. Applying the force to the cart in the direction of Fig. 3 until $l \sin\theta = 0$ according to the nonlinear vibration of the pendulum reduces the calculation efforts and increases the pendulum's energy within short movement of the cart. Thus we can obtain the swing up control law as follows:

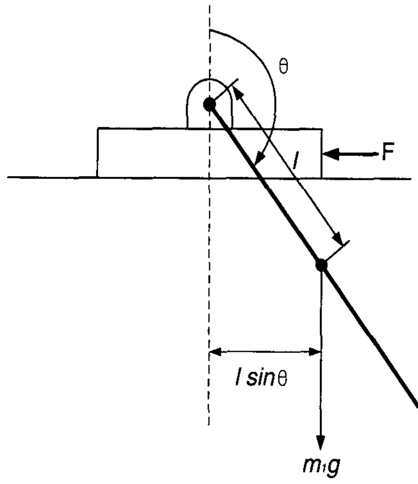

Fig. 3. Carriage Balancing Single Inverted Pendulum System.

$$u_s = -k_s \sin\theta \tag{15}$$

The swing up control (15) requires an assumption that the first initial value of θ is not $\pm n\pi$, $n = 1, 3, 5, \cdots$. The efficient swing up way is to apply the force at the frequency of pendulum's nonlinear vibration to drive the pendulum into resonance in a controlled manner. To change the energy as fast as possible, the magnitude of control signal should be as large as possible. Therefore we can rewrite (15) as follows:

$$u_s = -k_s \mathrm{sgn}(\sin\theta) \tag{16}$$

where

$$\mathrm{sgn}(x) = \begin{cases} 1 & \text{if } x \geq 0, \\ -1 & \text{if } x < 0 \end{cases}$$

By using (16), we can relax the initial condition constraint.

2. Tuning and switching condition

The tuning of k_s depends on the following two factors: the required rail range and the required swing-up time at which the pendulum is close to the inverted vertical state(normally within $\pm 10°$). Generally, increasing k_s leads to decrease swing-up time and to increase the required rail range. The described swing up control method can achieve the goal within the shortest time when k_s is the maximum of bounds of input.

To switch between the swing-up control and the optimal control described in Section III.we consider three important factors[11].

1. The closed-loop system with the optimal control has a restricted stable state region around the origin because the controller design is based on the linearized system.
2. The final state of the swing up control must not force the carriage beyond the available rail range.
3. During the swing up control the pendulum should not be allowed to go over the top.

The ideal swing up controller is that $\dot{\theta}$ equals to zero at $\theta = 0$. But, using an arbitrary value of k_s, we can not guarantee $\dot{\theta} = 0$ at $\theta = 0$. Therefore we use a sliding mode controller so that for an arbitrary k_s an angular velocity ω is close to zero near the target point.

3. Sliding surface design

Sliding mode control, often referred to as variable structure control, is a high-speed switching feedback control that switches between two values based upon some rules to drive the nonlinear plant's state trajectory onto a specified switching surface in the state space.

To design an appropriate switching surface, the plant having a restricted switching surface must respond in a desired manner and the switching control law satisfies a set of sufficient conditions for the existence of sliding mode. The next important aspect of sliding mode control is to guarantee the existence of a sliding mode.

The sliding mode exists when the state trajectory $x(t)$ of the controlled plant satisfies the sliding surface $S(x(t)) = 0$ at every $t \geq t_0$ for some t_0.

The hyperplane for the inverted pendulum system is designed to execute both swing up and stabilization of the pendulum. It is desirable to design the hyperplane for unstable structure with small damping from the energy balance. To design the hyperplane for the pendulum, we use the energy equation of the pendulum given by [12][14]

$$E = \frac{1}{2}J\dot{\theta}^2 + mgl(\cos\theta - 1) \tag{17}$$

where $E = 0$ at the standing position. In [12], the author designed a sliding mode controller with an observer by using the energy equation (17) to suppress the chatter of control input. In [14], the authors used a bang-bang strategy based on the the energy equation (17). When $E = 0$, the pendulum will be stood without the control input. We can obtain the hyperplane of the pendulum by letting $E = 0$ in (17).

$$0 = \sqrt{\frac{J}{mgl}}\dot{\theta} + \sqrt{2(1-\cos\theta)}\mathrm{sign}(\sin\theta) \tag{18}$$

where

$$\mathrm{sign}(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ -1 & \text{if } x < 0. \end{cases}$$

(18) represents the relation between θ and $\dot{\theta}$ which makes the pendulum converge to the origin. Fig. 4 shows this relation. If there exists an input which makes the state track the trajectory in Fig. 4, then the pendulum eventually converges to the origin. Therefore (18) can be directly a hyperplane of sliding mode control and we can rewrite hyperplane equation as follows:

$$s = \sqrt{\frac{J}{mgl}}\dot{\theta} + \sqrt{2(1-\cos\theta)}\mathrm{sign}(\sin\theta) \tag{19}$$

4. Sliding mode controller

Now we consider the sliding mode controller design method considering the extremely limited rail length. The purpose of general sliding mode control is searching for control input which makes all states in the state space converge to the hyperplane. The previously described swing up control law $u_s = -k_s \mathrm{sgn}(\sin\theta)$ makes states approach near to $s = 0$. We design the sliding mode controller by using the following

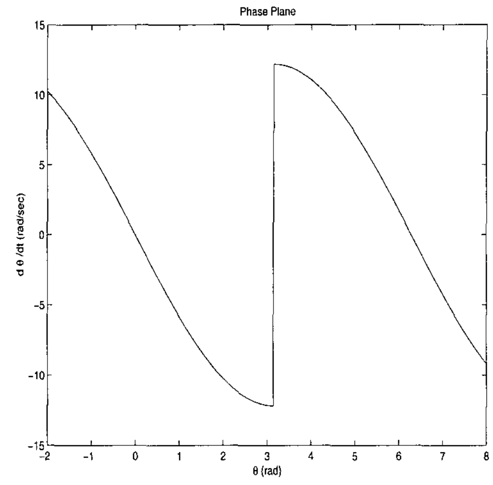

Fig. 4. State trajectory using energy equation.

Lyapunov function

$$\begin{aligned} v &= \frac{1}{2}s^T s \\ \dot{v} &= s^T\dot{s} = s^T\frac{\partial s}{\partial X}\dot{X} = s^T G\dot{X} \end{aligned} \tag{20}$$

In (20), $G = \begin{bmatrix} 0 & |\cos\left(\frac{\theta}{2}\right)| & 0 & \sqrt{\frac{J}{mgl}} \end{bmatrix}$, v is a positive definite matrix, and $\dot{v}$ is a negative definite matrix. It is difficult to obtain a Lyapunov function which satisfies (20) by analytic method. Thus we design a control law by using the following physical concept. The control law in (16) applies a force to the cart according to the nonlinear movement of the cart, which increases the angular velocity by repeating the swing up operation and the states reach the designed sliding surface. The reference angular velocity defined on the sliding surface is easily calculated by referring to the corresponding angle. If the angle belongs to a range of 90° to 270° and exceeds the corresponding reference angular velocity, then the force is applied to the cart from the opposite direction to decrease the angular velocity. When the angular velocity does not reach the reference angular velocity, the control law (16) is applied to increase the angular velocity until the angular velocity reaches the reference angular velocity. In the range of 270° to 90°, we use a contrary control law described in (21).

$$\begin{aligned} \frac{\pi}{2} \leq \theta < \frac{3\pi}{2}: \quad & u_s = -k_s \mathrm{sign}\left(\dot{\theta} - \dot{\theta}_{ref}\right)\mathrm{sgn}\left(\sin\theta\right) \\ \frac{3\pi}{2} \leq \theta < \frac{\pi}{2}: \quad & u_s = k_s \mathrm{sign}\left(\dot{\theta} - \dot{\theta}_{ref}\right)\mathrm{sgn}\left(\sin\theta\right) \end{aligned} \tag{21}$$

The control laws defined in (21) can be rewritten as follows:

$$\begin{aligned} u_s =& -k_s \mathrm{sign}\left(\dot{\theta} - \dot{\theta}_{ref}\right)\mathrm{sgn}\left(\cos\theta \cdot \sin\theta\right) \\ =& -k_s \mathrm{sign}\left(s - 2\sqrt{2(1-\cos\theta)}\mathrm{sign}(\sin\theta)\right) \\ & \times \mathrm{sgn}\left(\cos\theta \cdot \sin\theta\right) \end{aligned} \tag{22}$$

where $\dot{\theta}$ and $\dot{\theta}_{ref}$ are obtained from (19) and (18) respectively. We verified that the s, especially $-0.6 < s < 0.6$, around $s = 0$ converges to the $s = 0$ and the v in (20) is positive definite and the $\dot{v}$ in (20) is negative definite by simulation with the designed control law.

V. Simulation and experimental results

In this section, we compare the proposed method with the Takeshi Kawashima's sliding mode control algorithm[12] and Alan Bradshaw's method [11] by numerical simulation. And then experimental results are described to demonstrate the proposed method.

In Takeshi Kawashima's sliding mode control algorithm [12], the control law u and the hyperplane s are as follows:

$$s = \frac{f_1}{f_2}x + \sqrt{2(1-\cos\theta)}\text{sign}(\sin\theta) + \frac{f_3}{f_2}\frac{dx}{dt} + \frac{f_4}{f_2}\frac{d\theta}{dt} \quad (23)$$

$$u = -\alpha\left(|u_{eq}| + \epsilon\right)\frac{\gamma \cdot s}{|\gamma \cdot s| + \mu} \quad (24)$$

where

$$u_{eq} = G\frac{F(\bar{X},t)}{\gamma}, \quad G = \frac{\partial s}{\partial \bar{X}}, \quad \gamma = GB(\bar{X}) \neq 0,$$
$$\alpha = 4, \quad \epsilon = 0.1, \quad \mu = 0.01$$

and $f = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}$ is a feedback gain of the optimal control derived by using the linearized model at the standing position. Alan Bradshaw's method [11] is as follows:

$$u_s = k_s\omega \quad (25)$$
$$u = f * \bar{X} \quad (26)$$

Fig. 5 and Fig. 6 show the results when the rail length is restricted to 80cm.

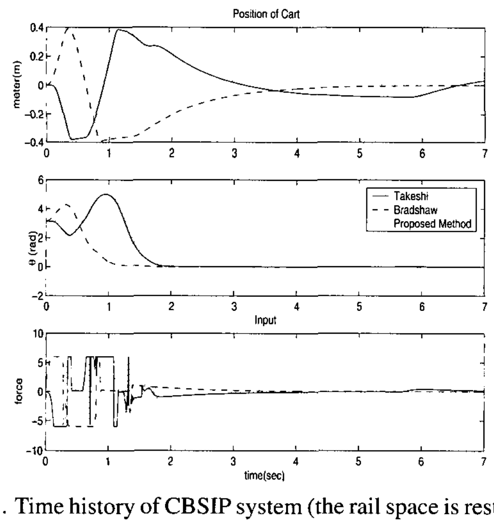

Fig. 5. Time history of CBSIP system (the rail space is restricted to 80cm).

In Fig. 5, Takeshi Kawashima's sliding mode controller uses the full rail length ($80cm$), but the proposed controller uses only the half range of rail and the pendulum converges to the target point. Moreover, in the proposed control method, the displacement of the cart rapidly converges to zero. Fig. 7 and Fig. 8 show the results when the rail length is restricted to $16cm$. In this case, Takeshi Kawashima's method fails to converge. Alan Bradshaw's swing up controller cannot generate energy enough to swing up. On the other hand, using the controller designed in this paper we can see that the pendulum converges to origin within 2.5 second.

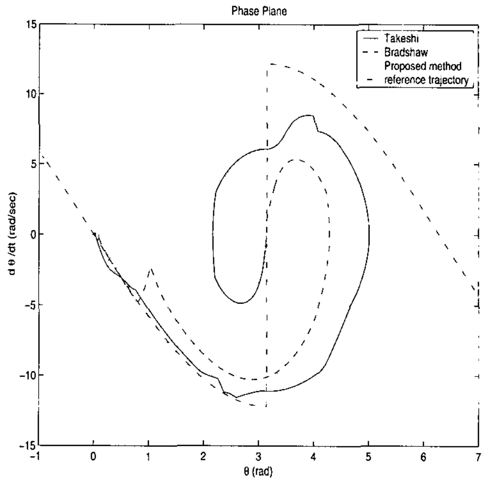

Fig. 6. Phase plane of CBSIP system (the rail space is restricted to 80cm).

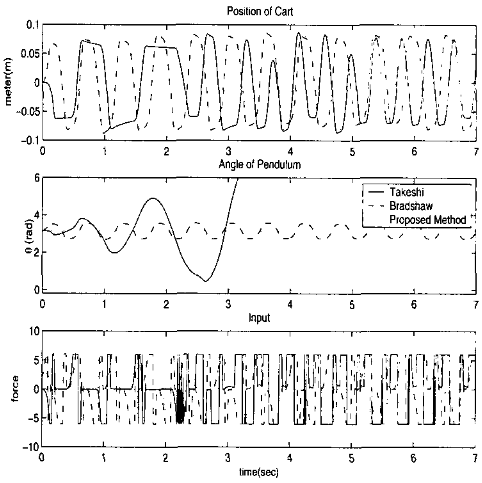

Fig. 7. Time history of CBSIP system (the rail space is restricted to 16cm).

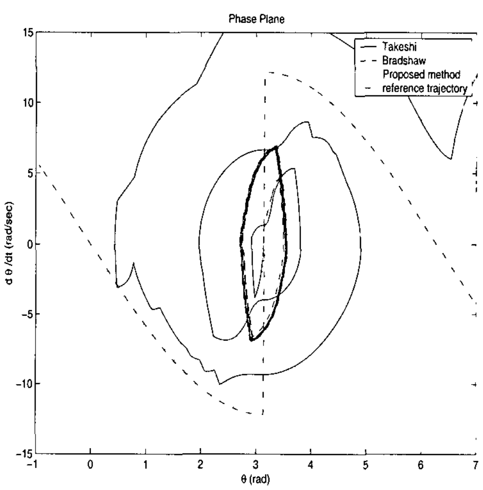

Fig. 8. Phase plane of CBSIP system (the rail range is restricted to 16cm).

Actual experiments have been done by TMS320C32 DSP used as the controller with a sampling time of 500 μsec, a rotary encoder with 2000 pulse/rev, and a liner motor used as a cart. The linear motor specification is described in Table I. The physical parameters of the inverted pendulum system are given in Table. II. The snapshot picture of this inverted pendulum control system is shown in Fig. 9. The pendulum is mounted on the linear motor. It can be observed in Fig. 10 that the pendulum is maintained at the upright equilibrium position within $3.5sec$. The corresponding phase plane trajectory is given in Fig. 11.

Table 1. Linear motor specification

Parameter	Unit	Value
Peak Current	amp RMS	8.18
Continuous Current	amp RMS	@ 25 $^{\circ}C$ 3.7
Resistance	ohm	5.2
Force Constant	N/amp	26.20
Back EMF	volts RMS/m/s	21.59
Thermal Resistance	$^{\circ}C$/W	1.407
Electrical Time Constant	msec	@ 25 $^{\circ}C$ 0.6

Table 2. Parameters of the system

Parameter	Value	Unit
mass of cart m_c	2.5	Kg
maximum cart movable distance from the center of the rail (x_{max})	80	mm
maximum force (M)	4	N
length of the pendulum (l)	400	mm
mass of the pendulum (m_1)	0.088	Kg
diameter of the pendulum	6	mm
moment of the inertia of the pendulum (J)	0.00117	$Kg \cdot m^2$
pendulum friction coefficient (C)	30	$N \cdot m \cdot s$

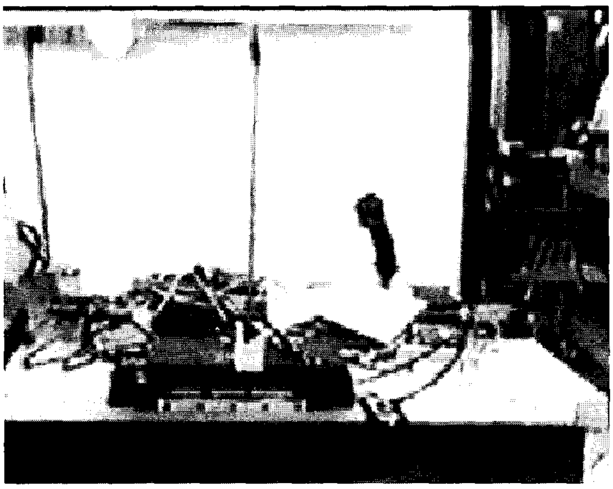

Fig. 9. Experimental setup of the inverted pendulum control system.

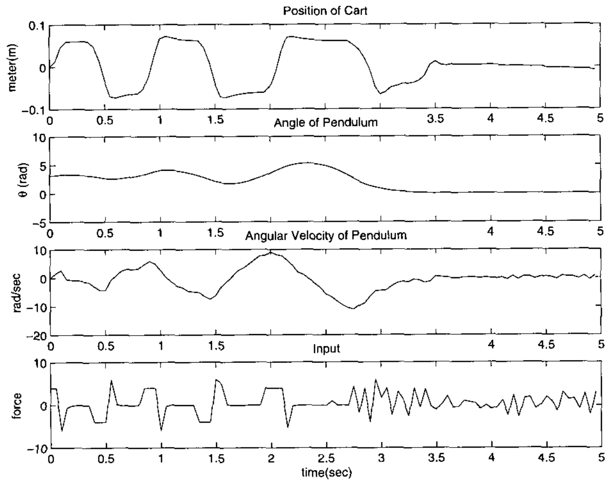

Fig. 10. Dynamic response of the inverted pendulum.

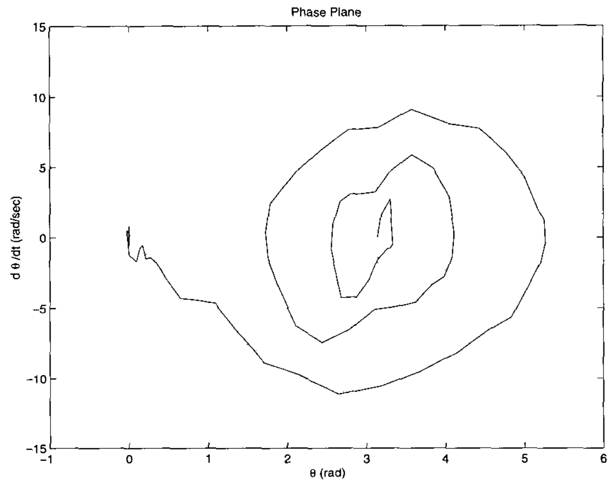

Fig. 11. Phase plane trajectory of the inverted pendulum.

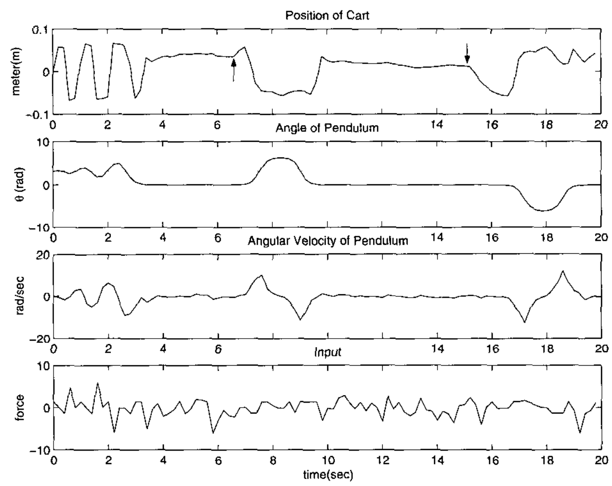

Fig. 12. Dynamic response of the inverted pendulum with external disturbance

In order to verify the robustness of this proposed control method, the pendulum is hit by hand to induce an external disturbance force at the time $6.5sec$ and $15sec$, respectively. The dynamic responses of the inverted pendulum are shown in Fig. 12 for the position of the cart, angle of pendulum, and angular velocity. Fig. 13 shows the phase plane trajectory of the inverted pendulum. In Fig. 12 and 13, it is observed that the linear motor moves quickly in response to the external force applied to the pendulum and the pendulum reaches the specified unstable equilibrium upright position within $3.0 \sim 3.5sec$. The proposed composite control consists a sliding mode con-

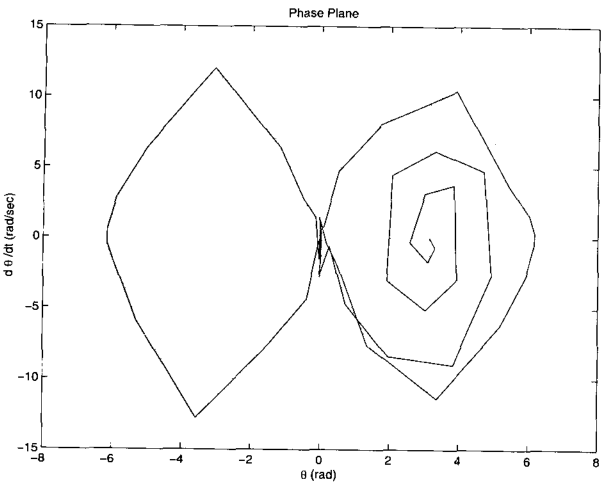

Fig. 13. Phase plane trajectory of the inverted pendulum with external disturbance

troller used to swing up the pendulum and an optimal controller used to stabilize the pendulum around the zero state, where the control switching boundary value is $\theta = \pm 10°$. Thus in Fig. 10 and Fig. 12, the input trajectories around the target point $\theta = 0°$ are obtained by using the optimal control algorithm described in section III and show weak chattering phenomena rather than the results of Takeshi Kawashima's sliding mode control method[12] shown in Fig. 5 and Fig. 7. The experimental result in Fig. 10 is similar to the simulation result in Fig. 7 including the control input.

VI. Conclusion

The main purpose of this paper is to design the swing up controller and stabilizing controller under the environment of extremely limited movable length of cart. To guarantee acceptable performances in spite of the input and state constraint, the whole control algorithm comprises a sliding mode control based on the nonlinear model for swing up the pendulum and an optimal control to stabilize by using the linearized model. The simulation results and the experiment results under the state constraint showed that the control performance was very good. The proposed control method is robust to an external disturbance.

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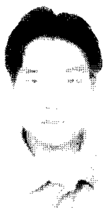

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