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A Study on the RPC Model Generation from the Physical Sensor Model

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Abstract

The rational polynomial coefficients (RPC) model is a generalized sensor model that is used as an alternative solution for the physical sensor model for IKONOS of the Space Imaging. As the number of sensors increases along with greater complexity, and the standard sensor model is needed, the applicability of the RPC model is increasing. The RPC model has the advantages in being able to substitute for all sensor models, such as the projective, the linear pushbroom and the SAR.

This report aimed to generate a RPC model from the physical sensor model of the KOMPSAT-1(Korean Multi-Purpose Satellite) and aerial photography. The KOMPSAT-1 collects 510~730 nm panchromatic imagery with a ground sample distance (GSD) of 6.6 m and a swath width of 17 km by pushbroom scanning. The least square solution was used to estimate the RPC. In addition, data normalization and regularization were applied to improve the accuracy and minimize noise.This study found that the RPC model is suitable for both KOMPSAT-1 and aerial photography.

Keywords : KOMPSAT-1, RPC model

1. Introduction

To acquire a 3-D position from 2-D imagery, sensor models are required to determine the functional relationships between the image space and the ground space. There are many types of sensors, and the imagery from each sensor has to be processed by a different physical sensor model for a geometric correction. With increasing availability of the new sensor types, it is necessary to develop a general sensor model that can be applied to all sensor models. In addition, recent satellite sensor models are not open to the public. (e.g. IKONOS)

A RPC model that is generic and has an expressive form for various sensor models, has been used as an alternative to the physical sensor model. The RPC model forms the coordinates of the image point at ratios of the third degree polynomials in the coordinates of the world point. A set of images is given to determine the set of polynomial coefficients in the RPC model in order to minimize error. In this paper, an algorithm that solves the RPC model estimation problem by applying a least square method process is proposed. The RPC we generated from a physical sensor model of KOMPSAT-1 and aerial photography. In addition, the accuracy of the test was evaluated based on the 2-D image coordinates.

2. Rational Polynomial Coefficients Model

The RFM uses a ratio of two polynomial functions to compute the image coordinate. The RPC form between the image coordinates (x, y) and the world coordinates (X, Y, Z) can be presented below.

$$x=\frac{p_1(X,Y,Z)}{p_3(X,Y,Z)}=\frac{\sum_{i=0}^{3}\sum_{j=0}^{i}\sum_{k=0}^{j}a_m X^{i-j}Y^{j-k}Z^k}{\sum_{i=0}^{3}\sum_{j=0}^{i}\sum_{k=0}^{j}c_m X^{i-j}Y^{j-k}Z^k} \quad (1)$$

$$y=\frac{p_2(X,Y,Z)}{p_3(X,Y,Z)}=\frac{\sum_{i=0}^{3}\sum_{j=0}^{i}\sum_{k=0}^{j}b_m X^{i-j}Y^{j-k}Z^k}{\sum_{i=0}^{3}\sum_{j=0}^{i}\sum_{k=0}^{j}c_m X^{i-j}Y^{j-k}Z^k} \quad (2)$$

where a_m, b_m, c_m = polynomial coefficients ($c_0 = 1$)

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$$m=\frac{i(i+1)(i+2)}{6}+\frac{j(j+1)}{2}+k$$

The power of each world coordinate and the total power of all world coordinates are limited to 3. Therefore, each polynomial has 20 terms: i.e.

$$\begin{aligned} p_1 = {} & a_0 + a_1 Z + a_2 Y + a_3 X + a_4 ZY + a_5 ZX + a_6 YX \\ & + a_7 Z^2 + a_8 Y^2 + a_9 X^2 + a_{10} ZXY + a_{11} Z^2 Y \\ & + a_{12} Z^2 X + a_{13} Y^2 Z + a_{14} Y^2 X + a_{15} ZX^2 + a_{16} YX^2 \\ & + a_{17} Z^3 + a_{18} Y^3 + a_{19} Z^3 \end{aligned} \quad (3)$$

Equations (1), and (2) can be rewritten as follows;

$$x=\frac{(1\ Z\ Y\ X\ \cdots\ Y^3\ X^3)\cdot(a_0\ a_1\ \cdots a_{19})^T}{(1\ Z\ Y\ X\ \cdots\ Y^3\ X^3)\cdot(1\ c_1\ \cdots c_{19})^T} \quad (4)$$

$$y=\frac{(1\ Z\ Y\ X\ \cdots\ Y^3\ X^3)\cdot(b_0\ b_1\ \cdots b_{19})^T}{(1\ Z\ Y\ X\ \cdots\ Y^3\ X^3)\cdot(1\ c_1\ \cdots c_{19})^T} \quad (5)$$

The distortions caused by the optical projection can generally be represented by the ratios of first order terms, and the corrections such as the curvature of the earth, atmospheric refraction, can be approximated by the second order terms. Other unknown distortions can be modeled with the third order terms.

3. Solution to RPC Model

A given set of image to world correspondences $(x, y)\rightarrow(X, Y, Z)$ was assumed. The corresponding points were extracted from the physical sensor model. The task was to compute the polynomial coefficients from the correspondence points. To accomplish this, a linear method based on the linear least-square method was used.

3.1 Extraction of Corresponding Points

The basic scheme of this algorithm is to build a virtual space reflecting the physical sensor model and obtain the RPC that fits to the virtual space. The image to the world corresponding points was extracted to build the virtual space.

Initially, m by m grid points on the image coordinates need to be determined. The world grid points corresponding to each image grid point can then computed. The physical sensor model was used to compute the point position of the world coordinates. In addition, about each image point, n world points with different elevations were obtained. N elevation layers were distributed uniformly.

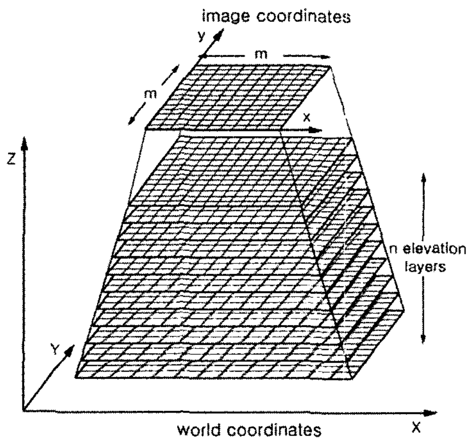

Fig. 1. Extraction of the corresponding points.

3.2 Linear Estimation of The RPC Model

A pair of equations was obtained from equations (4) and (5) by cross multiplication.

$$(1\ ZYX\cdots\ Y^3X^3)\cdot(a_0\ a_1\cdots\ a_{19})^T$$
$$-x(1\ ZYX\cdots\ Y^3X^3)\cdot(1c_1\cdots\ c_{19})^T=0 \quad (6)$$
$$(1\ ZYX\cdots\ Y^3X^3)\cdot(b_0\ b_1\cdots\ b_{19})^T$$
$$-y(1\ ZYX\cdots\ Y^3X^3)\cdot(1c_1\cdots\ c_{19})^T=0 \quad (7)$$

The above equations are non-linear in X, Y, Z, but they are linear in the coefficients of the polynomials. Since each correspondence gives a pair of equations, and there are a total of 58 unknown parameters. Therefore, at least 29 correspondences are needed to solve for the polynomial coefficients. An equation with more than 29 points is solved by least-square techniques.

An error equation of n correspondence points can be written as

$$\begin{bmatrix} v_{x1} \\ \vdots \\ v_{xn} \\ v_{y1} \\ \vdots \\ v_{yn} \end{bmatrix} = \begin{bmatrix} 1 & \cdots & X_1^3 & 0 & \cdots & 0 & -x_1Z_1 & \cdots & -x_1X_1^3 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & X_n^3 & 0 & \cdots & 0 & -x_nZ_n & \cdots & -x_nX_n^3 \\ 0 & \cdots & 0 & 1 & \cdots & X_1^3 & -y_1Z_1 & \cdots & -y_1X_1^3 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & \cdots & X_n^3 & -y_nZ_n & \cdots & -y_nX_n^3 \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_{19} \\ b_0 \\ \vdots \\ b_{19} \\ c_1 \\ \vdots \\ c_{19} \end{bmatrix} - \begin{bmatrix} x_1 \\ \vdots \\ x_n \\ y_1 \\ \vdots \\ y_n \end{bmatrix} \quad (8)$$

where v = error

Equation (8) can be rewritten briefly as

$$V=AX-L \quad (9)$$

The normal equation of the above equation is

$$A^TAX-A^TL=0 \quad (10)$$

Therefore, the coefficients matrix, I, can be solved as follows :

$$X=(A^TA)^{-1}A^TL \quad (11)$$

3.3 Data Normalization

In order to improve the numerical stability of the computation, the two image coordinates and three world coordinates are each offset and scaled to fit the range -1.0~+1.0. It is essential to normalize the data with scale factors and offsets prior to running the linear algorithm in order to compute the parameters.

$$x'=\frac{x-x_o}{x_x},\quad y'=\frac{y-y_o}{y_x} \quad (12)$$

$$X'=\frac{X-X_o}{X_x},\quad Y'=\frac{Y-Y_o}{Y_x},\quad Z'=\frac{Z-Z_o}{Z_x} \quad (13)$$

where

x_o, y_0= the offset value for the image coordinates

x_s, y_s= the scale value for the image coordinates

X_o, T_o, Z_o= the offset value for the world coordinates

X_s, T_s, Z_s= the scale value for the world coordinates

Without prenormalization, each term of the equations has a wide range. Therefore, the equation matrix will be poorly conditioned.

3.4 Regularization

In practice, the input data are not distributed evenly. Consequently, the matrix A of the equation (9) can become ill conditioned and the matrix A^TA in the equation (10) can become singular. Therefore, the regularization was applied using the Tikhonov method.

This method is the most widely used technique for regularizing the discrete ill-posed problems. It yields a unique solution by adding a constraint condition. The solution to this least square problem can be obtained by solving the normal equations.

$$(A^TA+\lambda^2I)X-A^TL=0 \quad (14)$$

where I = the identity matrix

λ = the regularization parameter

Parameter λ can be determined experimentally.

4. Test Data

In this study, the panchromatic image of the KOMPSAT-1 EOC(Electro-Optical Camera) and aerial photography was used for the test.

Table1. Basic information of the KOMPSAT-1 data

Sensor	EOC(Electro-Optical Camera)
Ground Resolution	6.6m(Nadir)
Swath	17km
Altitude	685km
Band Swath	0.51μm ~0.73μm(panchromatic)
Scanner System	Pushbroom
Focal Length	1045mm
Scene Center Time	2001/03/18 01:55:29.745
Image Pixel Size	2592×2798 pixel

4.1 KOMPSAT-1 Data

The test image of KOMPSAT-1 is a 17km×17km full scene of southern Seoul, Korea as shown in Figure 2. Basic information about the scene is listed below.

The physical sensor model of KOMPSAT-1 is pushbroom model, like as the equation (15), (16). The different equations are applied to each line.

$$x_i=-f\frac{m_{11}(X_i-\overline{X}_{oi})+n_{12}(Y_i-\overline{Y}_{oi})+m_{13}(Z_i-\overline{Z}_{oi})}{m_{31}(X_i-\overline{X}_{oi})+n_{32}(Y_i-\overline{Y}_{oi})+m_{33}(Z_i-\overline{Z}_{oi})} \quad (15)$$

$$O=-f\frac{m_{21}(X_i-\overline{X}_{oi})+m_{22}(Y_i-\overline{Y}_{oi})+m_{23}(Z_i-\overline{Z}_{oi})}{m_{31}(X_i-\overline{X}_{oi})+m_{32}(Y_i-\overline{Y}_{oi})+m_{33}(Z_i-\overline{Z}_{oi})} \quad (16)$$

where,

$m_{11}=\cos\phi_i\cos\kappa_i$, $m_{12}=\cos\omega_i\sin\kappa_i+\sin\omega_i\sin\phi_i\cos\kappa_i$,

$m_{13}=\sin\omega_i\sin\kappa_i-\cos\omega_i\sin\phi_i\cos\kappa_i$, $m_{21}=-\cos\phi_i\sin\kappa_i$,

$m_{22}=\cos\omega_i\cos\kappa_i-\sin\omega_i\sin\phi_i\sin\kappa_i$,

$m_{23}=\sin\omega_i\cos\kappa_i+\cos\omega_i\sin\phi_i\cos\kappa_i$

$m_{31}=\sin\phi_i$, $m_{32}=-\sin\omega_i\cos\phi_i$, $m_{33}=\cos\omega_i\cos\phi_i$

$\kappa_{oi}=\kappa_o+K_1y+K_7y^2$, $\phi_{oi}=\phi_o+K_2y+K_8y^2$,

$\omega_{oi}=\omega_o+K_3y+K_9y^2$, $\overline{X}_{oi}=X_o+K_4y+K_{10}y^2$,

$\overline{Y}_{oi}=Y_o+K_5y+K_{11}y^2$, $\overline{Z}_{oi}=Z_o+K_6y+K_{12}y^2$

In order to extract the corresponding points in this

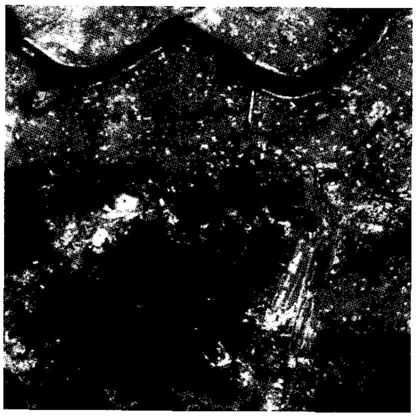

Fig. 2. KOMPSAT-1 EOC scene of Seoul.

data, we acquired 144(12×12) image grid points equally in the entire scene. In addition, arbitrary 41 evaluation layers(3840000m~3860000m from equator) were applied uniformly to each image grid point. Therefore, total 5904 points were used as the corresponding points. Furthermore, 100 check points were extracted randomly at the target virtual space to estimate the accuracy.

4.2 Airphoto Data

Additional data using this test was obtained from aerial photographs of Sungnam, Korea. A summary of the information and scene are shown below.

Table 2. Basic information of the airphoto

Scale	1/5,000
Altitude	885m
Focal Length	153.59mm
Time of Acquisition	1999/12/11 13:00
Image Pixel Size	11908×11908 pixel

$$x_i = -f\frac{m_{11}(X_i - X_o) + m_{12}(Y_i - Y_o) + m_{13}(Z_i - Z_o)}{m_{31}(X_i - X_o) + m_{32}(Y_i - Y_o) + m_{33}(Z_i - Z_o)} \quad (17)$$

$$y_i = -f\frac{m_{21}(X_i - X_o) + m_{22}(Y_i - Y_o) + m_{23}(Z_i - Z_o)}{m_{31}(X_i - X_o) + m_{32}(Y_i - Y_o) + m_{33}(Z_i - Z_o)} \quad (18)$$

The physical sensor model of airphoto is the collinearity condition, and the external orientation parameters of the test data are listed below.

Table 3. External orientation parameters of the airphoto

X_o	333007.9356m	ω	-1.7042
Y_o	4137591.5042m	ϕ	1.4658
Z_o	885.2473m	κ	1.2278

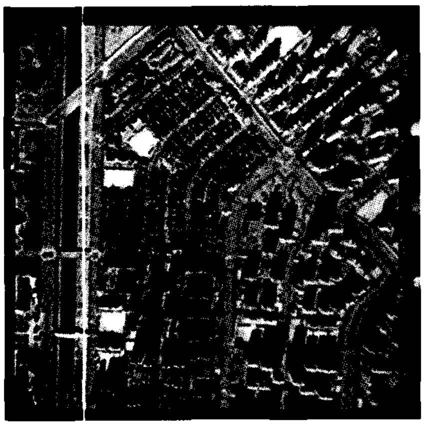

Fig. 3. Aerial photograph of Sungnam.

In this data, a total 4464 corresponding points were extracted. 144(12×12) image grid points were extracted and 31 evaluation layers(-50m~250m from datum) were applied equally. One hundred check points for an accuracy evaluation were also obtained.

5. Results and Evaluation

λ was determined to be 0.002 from the results of many tests for various regularization parameters. Therefore, the following results were calculated by λ=0.002.

The RPCs generated ranged from -0.01 to +2.0 and trended to decrease with the higher order terms. However, there was only a weak correlation.

In order to evaluate the fitting accuracy of the RPC model, the error was calculated based on the image coordinates when each point was applied for both the physical sensor model and the RPC model. In addition, the root mean square error(RMSE) and the maximum error of the corresponding points and check points are listed at Table 3 and Table 4, respectively.

Table 3. Error of corresponding points

		KOMPSAT-1	Airphoto
RMSE (pixel)	x	4.79×10^{-5}	4.58×10^{-5}
	y	2.68×10^{-5}	1.03×10^{-5}
Max. Error (pixel)	x	1.92×10^{-4}	1.92×10^{-4}
	y	1.16×10^{-4}	1.95×10^{-4}

Table 4. Error of check points

		KOMPSAT-1	Airphoto
RMSE (pixel)	x	4.02×10^{-5}	3.99×10^{-5}
	y	2.30×10^{-5}	3.55×10^{-5}
Max. Error (pixel)	x	1.18×10^{-4}	1.20×10^{-4}
	y	4.74×10^{-5}	7.23×10^{-5}

As shown in the above tables, the error between the physical sensor model and the RPC model are negligible, for both the KOMPSAT-1 and the aerial photographs. Furthermore, there were no remarkable differences between the KOMPSAT-1 and the aerial photographs. However, there was a little difference between the row and column error of the KOMSAT-1 images.

6. Conclusion

In this study, a least-squares solution to the generation of the RPC model from the physical sensor model was proposed. In addition, a regularization method was applied to solve the singular problem.

The tests showed that the RPC model was able to approximate the physical sensor model of the KOMPSAT-1 pushbroom and aerial photographic data. Therefore, it is expected that the RPC model can be used as an alternative sensor model for the diverse physical sensor model. It is meaningful from the point of view that recent satellite sensors are becoming increasingly complex and diverse.

A future study will attempt to obtain a more correct range of elevation layers and the number of layer will be defined. Furthermore, the result can be improved by minimizing the coefficients of the nonlinear terms in order to generate a more realistic model on the points outside the data set.

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