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ATTITUDE AND CONFIGURATION CONTROL OF FLEXIBLE MULTI-BODY SPACECRAFT

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ABSTRACT

Multi-body spacecraft attitude and configuration control formulations based on the use of collaborative control theory are considered. The control formulations are based on two-player, nonzero-sum, differential game theory applied using a Nash strategy. It is desired that the control laws allow different components of the multi-body system to perform different tasks. For example, it may be desired that one body points toward a fixed star while another body in the system slews to track another satellite. Although similar to the linear quadratic regulator formulation, the collaborative control formulation contains a number of additional design parameters because the problem is formulated as two control problems coupled together. The use of the freedom of the partitioning of the total problem into two coupled control problems and the selection of the elements of the cross-coupling matrices are specific problems addressed in this paper. Examples are used to show that significant improvement in performance, as measured by realistic criteria, of collaborative control over conventional linear quadratic regulator control can be achieved by using proposed design guidelines.

Keywords: satellite dynamics, attitude control, optimal control

1. INTRODUCTION

Consider a spacecraft, equipped with both a pointing antenna and a tracking antenna that is required to perform a rapid angular maneuvers. If no consideration is given to controlling the attitude of the spacecraft during the large angle maneuver of the tracking antenna, the internal torque which is commanded will induce significant excitation in the rest of the structure and the entire spacecraft will rotate. In such a case an attitude control system is needed that based on proper consideration of the rest of the components of the structure during the application of the command torque to the tracking antenna. A principal objective is to reduce excitations induced in the remainder of the structure.

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A conventional approach for designing a control law for the control of a multi-body structure is to use linear quadratic regulator (LQR) theory with a properly chosen performance index in which the states and controls are, in general, all coupled through the weighting matrices. For spacecraft which have multiple components performing different and sometimes conflicting tasks, it would be advantageous to have "communication" between the control systems which are involved in performing these tasks.

To achieve this communication, Collaborative Control Theory (hereinafter, CCT) which uses control synthesis technique, has been proposed (Fitz-Coy 1990, Fitz-Coy & Cochran 1989). The CCT is formulated using two-player nonzero-sum differential game theory. In this approach two performance indices are defined such that the states and controls are selectively coupled. The two performance indices are minimized by using a Nash strategy (Starr & Ho 1969a, 1969b, Foley & Schmitendorf 1971, Krikelis & Rekasius 1971). In this paper, we provide a new collaborative tracking control law. We also consider enforcement of additional design criteria in CCT by properly changing the structures of the performance indices.

2. COLLABORATIVE TRACKING CONTROL

Collaborative control is based on N-player, nonzero-sum, linear quadratic, differential game theory with strategies $u_i^*, i = 1, 2, ..., N$, which satisfy the following conditions on the cost functions, J_i of the players:

$$J_i(u_1^*, u_2^*, ..., u_N^*) \leq J_i(u_1^*, u_2^*, ..., u_{i-1}^*, u_i, ..., u_N^*), \quad 1 \leq i \leq N.$$

Strategies satisfying the above conditions are known as Nash solution strategies (Starr & Ho 1969a, 1969b, Foley & Schmitendorf 1971, Krikelis & Rekasius 1971). Papavassilopoulos, Medanic, & Cruz (1979) shows the existence of Nash strategies and solutions. The basic premise of this game is that each of the N players has knowledge of the current state of the game, the dynamics involved in the game and the "cost" involved for each player in the game.

Development of the CCT method is based on several assumptions: (1) only two - player games are considered (N = 2), (2) the system is time invariant (This assumption results in algebraic Riccati equations instead of differential Riccati equations), (3) the outputs are completely observable (This assumption guarantees availability of full state feedback). Based on these assumptions, a typical linear dynamic system can be represented in the state space form

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{u}_1(t) + \mathbf{B}_2\mathbf{u}_2(t) \\ \mathbf{y}_1(t) &= \mathbf{C}_1\mathbf{x}(t) \\ \mathbf{y}_2(t) &= \mathbf{C}_2\mathbf{x}(t) \end{aligned} \tag{1}$$

where $\mathbf{u}_1(t)$ and $\mathbf{u}_2(t)$ represent separate control inputs to the plant by players 1 and 2, respectively; $\mathbf{x}(t)$ represents the plant's state (current condition of the game); $\mathbf{y}_1(t)$ and $\mathbf{y}_2(t)$ represent the plant outputs; and $\mathbf{C}_1$ and $\mathbf{C}_2$ are observation matrices. The nonzero-sum differential game is formulated by allowing the controllers (players) to minimize quadratic performance indices J_1 and J_2, defined as

$$\begin{aligned} J_1 &= \int_0^\infty (\mathbf{y}_1^T \mathbf{Q}_1^* \mathbf{y}_1 + \mathbf{u}_1^T \mathbf{R}_{11} \mathbf{u}_1 + \mathbf{u}_2^T \mathbf{R}_{12} \mathbf{u}_2) dt \\ J_2 &= \int_0^\infty (\mathbf{y}_2^T \mathbf{Q}_2^* \mathbf{y}_2 + \mathbf{u}_1^T \mathbf{R}_{21} \mathbf{u}_1 + \mathbf{u}_2^T \mathbf{R}_{22} \mathbf{u}_2) dt \end{aligned} \quad (2)$$

where $\mathbf{Q}_1^*, \mathbf{Q}_2^*, \mathbf{R}_{12}, \mathbf{R}_{21}$, are positive semi-definite matrices, and $\mathbf{R}_{11}$ and $\mathbf{R}_{22}$ are positive definite matrices. The advantage of using output vectors $\mathbf{y}_1$ and $\mathbf{y}_2$ instead of the state vector x is that more attention can be given to minimizing particular combinations of the state variables rather than to minimizing all individual states. Equation (2) can be used to write the following conventional forms of the performance indices

$$\begin{aligned} J_1 &= \int_0^\infty (\mathbf{x}_1^T \mathbf{Q}_1 \mathbf{x}_1 + \mathbf{u}_1^T \mathbf{R}_{11} \mathbf{u}_1 + \mathbf{u}_2^T \mathbf{R}_{12} \mathbf{u}_2) dt \\ J_2 &= \int_0^\infty (\mathbf{x}_2^T \mathbf{Q}_2 \mathbf{x}_2 + \mathbf{u}_1^T \mathbf{R}_{21} \mathbf{u}_1 + \mathbf{u}_2^T \mathbf{R}_{22} \mathbf{u}_2) dt \end{aligned} \quad (3)$$

where $\mathbf{Q}_1 = \mathbf{C}_1^T \mathbf{Q}_1^* \mathbf{C}_1$ and $\mathbf{Q}_2 = \mathbf{C}_2^T \mathbf{Q}_2^* \mathbf{C}_2$. The controls that satisfy the Nash strategy are linear functions of the state variables (Fitz-Coy 1990, Starr & Ho 1969a, Foley & Schmitendorf 1971), that is,

$$\begin{aligned} \mathbf{u}_1^* &= -\mathbf{R}_{11}^{-1} \mathbf{B}_1^T \mathbf{P}_1 \mathbf{x} \\ \mathbf{u}_2^* &= -\mathbf{R}_{22}^{-1} \mathbf{B}_2^T \mathbf{P}_2 \mathbf{x} \end{aligned} \quad (4)$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are solutions to the coupled algebraic Riccati equations (ARE's).

$$\begin{aligned} -\mathbf{P}_1\mathbf{A} - \mathbf{A}^T\mathbf{P}_1 - \mathbf{Q}_1 &+ \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 + \mathbf{P}_1\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_2 \\ &+ \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_1 - \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{R}_{12}\mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_2 = 0 \\ -\mathbf{P}_2\mathbf{A} - \mathbf{A}^T\mathbf{P}_2 - \mathbf{Q}_2 &+ \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_{21}^T\mathbf{P}_2 + \mathbf{P}_2\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 \\ &+ \mathbf{P}_1\mathbf{B}_2\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_2 - \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{R}_{21}\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 = 0 \end{aligned} \quad (5)$$

The coupled ARE's may be solved by using a Newton-Raphson-Kantorovich (NRK) algorithm (Fitz-Coy 1990).

2.1 Track Collaborative Control

For our application we wish to have the system follow a prescribed state trajectory. To accomplish such tracking of desired motion, two new performance indices are introduced.

$$\begin{aligned} J_1 &= (\mathbf{x}_f - \mathbf{r}_f)^T \mathbf{G}_1 (\mathbf{x}_f - \mathbf{r}_f) \\ &\quad + 0.5 \int_0^T [(\mathbf{x} - \mathbf{r})^T \mathbf{Q}_1 (\mathbf{x} - \mathbf{r}) + \mathbf{u}_1^T \mathbf{R}_{11} \mathbf{u}_1 + \mathbf{u}_2^T \mathbf{R}_{12} \mathbf{u}_2] dt \\ J_2 &= (\mathbf{x}_f - \mathbf{r}_f)^T \mathbf{G}_2 (\mathbf{x}_f - \mathbf{r}_f) \\ &\quad + 0.5 \int_0^T [(\mathbf{x} - \mathbf{r})^T \mathbf{Q}_2 (\mathbf{x} - \mathbf{r}) + \mathbf{u}_1^T \mathbf{R}_{21} \mathbf{u}_1 + \mathbf{u}_2^T \mathbf{R}_{22} \mathbf{u}_2] dt \end{aligned} \quad (6)$$

Here, $\mathbf{r}$ is the reference motion and $\mathbf{T}$ is final time. Also, $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{R}_{12}, \mathbf{R}_{21}$, and $\mathbf{G}$ are positive-semi-definite matrices, and $\mathbf{R}_{11}$, and $\mathbf{R}_{22}$, are positive-definite matrices. Control inputs $\mathbf{u}_1$ and $\mathbf{u}_2$ which minimize the performance indices (6) may also be expressed as linear functions of the state $\mathbf{x}$,

$$\begin{aligned} \mathbf{u}_1^* &= -\mathbf{R}_{11}^T\mathbf{B}_1^T(\mathbf{P}_1 x + \mathbf{S}_1) \\ \mathbf{u}_2^* &= -\mathbf{R}_{22}^T\mathbf{B}_2^T(\mathbf{P}_2 x + \mathbf{S}_2) \end{aligned} \tag{7}$$

where the $\mathbf{P}_i$ $(i = 1, 2)$ are solutions of the coupled differential Riccati equations,

$$\begin{aligned} -\mathbf{P}_1\mathbf{A} - \mathbf{A}^T\mathbf{P}_1 - \mathbf{Q}_1 &+ \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 + \mathbf{P}_1 B_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_2 \\ &+ \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_1 - \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{R}_{12}\mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_2 = \dot{\mathbf{P}}_1 \\ -\mathbf{P}_2\mathbf{A} - \mathbf{A}^T\mathbf{P}_2 - \mathbf{Q}_2 &+ \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_{21}^T\mathbf{P}_2 + \mathbf{P}_2\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 \\ &+ \mathbf{P}_1\mathbf{B}_2\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_2 - \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{R}_{21}\mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 = \dot{\mathbf{P}}_2 \end{aligned} \tag{8}$$

The additional functions $\mathbf{S}_i$ $(i = 1, 2)$ are the solutions to the differential equations,

$$\begin{aligned} &(-\mathbf{A}^T + \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T + \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T)\mathbf{S}_1 \\ &+(\mathbf{P}_1\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T - \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{R}_{12}\mathbf{R}_{22}^T\mathbf{B}_2^T)\mathbf{S}_2 + \mathbf{Q}_1 r = \dot{\mathbf{S}}_1 \\ &(-\mathbf{A}^T + \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T + \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T)\mathbf{S}_2 \\ &+(\mathbf{P}_2\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T - \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{R}_{21}\mathbf{R}_{11}^T\mathbf{B}_1^T)\mathbf{S}_1 + \mathbf{Q}_2 r = \dot{\mathbf{S}}_2 \end{aligned} \tag{9}$$

For the infinite-horizon problem, i.e., T$\rightarrow \infty$, the solutions to the coupled differential Riccati equations are constant matrices (Barnett 1975). Thus, Eqs. (8) become coupled ARE. In case of T$\rightarrow \infty$, solutions of Eqs. (9) are not available. However, for very large T (T$\approx \infty$), and for a constant reference track (fixed final state), a constant solution for $\mathbf{S}_i$ is a good approximation (Athans & Felb 1966). Thus, here, Eqs. (9) will be approximated by

$$\begin{aligned} &(-\mathbf{A}^T + \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T + \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T)\mathbf{S}_1 \\ &+(\mathbf{P}_1\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T - \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{R}_{12}\mathbf{R}_{22}^T\mathbf{B}_2^T)\mathbf{S}_2 + \mathbf{Q}_1\mathbf{r} = 0 \\ &(-\mathbf{A}^T + \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T + \mathbf{P}_2\mathbf{B}_2\mathbf{R}_{22}^{-1}\mathbf{B}_2^T)\mathbf{S}_2 \\ &+(\mathbf{P}_2\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{B}_1^T - \mathbf{P}_1\mathbf{B}_1\mathbf{R}_{11}^{-1}\mathbf{R}_{21}\mathbf{R}_{11}^T\mathbf{B}_1^T)\mathbf{S}_1 + \mathbf{Q}_2\mathbf{r} = 0 \end{aligned} \tag{10}$$

The matrices $\mathbf{S}_1$ and $\mathbf{S}_2$ can be calculated easily using the solutions for the $\mathbf{P}_i$, since Eqs. (10) are simple, simultaneous linear equations for $\mathbf{S}_1$ and $\mathbf{S}_2$.

3. COMPARISON OF COLLABORATIVE CONTROL AND LINEAR QUADRATIC REGULATOR CONTROL

3.1 Dynamic Model

Results obtained using CCT and LQR control strategies are compared in this section. Because LQR is well established and widely used, such a comparison should reveal the potential of the CCT control strategy.

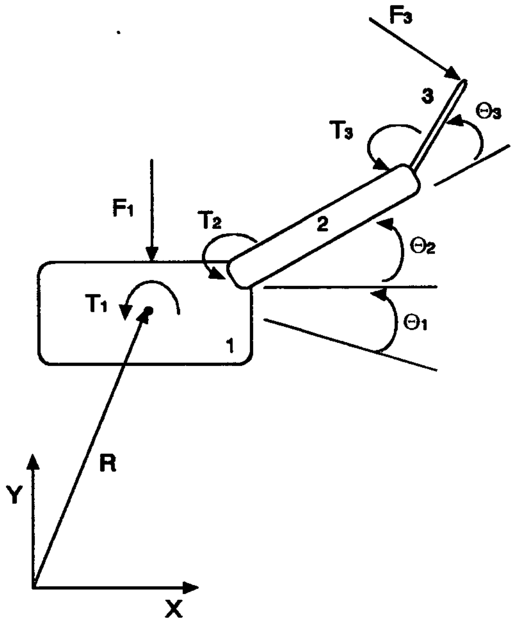

Figure 1. Flexible multi-body system.

A simple spacecraft model is shown in Fig. 1. The model consists of three bodies. Two (body 1 and body 2) are rigid and one (body 3) is flexible. The centers of mass of the labeled bodies, $\mathbf{C}_i, i = 1, 2$, and 3, are located at the geometric centers of the respective bodies. For the purpose of gaining a better insight into the controlled dynamics, only planar motion is considered. The flexible body is modeled as a "pined - free" Euler-Bernoulli beam. The state vector of this system is

$$\mathbf{x} = (\mathbf{Y}, \Theta_1, \Theta_2, \Theta_3, q_i, \dot{\mathbf{Y}}, \omega_1, \omega_2, \omega_3, q_1)^T$$

Here, ω_1, ω_2, and ω_3 are the total angular rates of the respective bodies. The output vector for the LQR is

$$\mathbf{y} = (\mathbf{Y}, \Theta_1, \Theta_2, \Theta_3, \mathbf{w})$$

In the output, $\mathbf{w}$ represents the total transverse displacement of the flexible body and is approximated by $\mathbf{w} = \sum_j^n [\phi_j(\mathrm{x}_3)q_j(t)]$ where $\phi_j(\mathrm{x}_3)$ is j^{th} mode shape function and q_j(t) is generalized coordinates.

The input and output vectors are divided into $\mathbf{u}_1 = (\mathbf{F}_1\mathbf{T}_1\mathbf{F}_3)^T, \mathbf{u}_2 = (\mathbf{T}_2\mathbf{T}_3)^T, \mathbf{y}_1 = (\mathbf{Y}\Theta_1 w)^T$, and $\mathbf{y}_2 = (\Theta_2\Theta_3)^T$ respectively. The **B** and **C** matrices were divided accordingly into $\mathbf{B}_1$ and $\mathbf{B}_2$, and $\mathbf{C}_1$ and $\mathbf{C}_2$, respectively. The linearized equations are given in the appendix.

In the Newton-Raphson-Kantorovich iteration method, the selection of the initial guess is important for fast convergence. Since CCT strategy is comparable to LQR strategy (Fitz-Coy 1990), the solution of the algebraic Riccati equation for LQR strategy should be a good initial guess for

Table 1. Weight matrices.

$$\mathbf{Q}^* = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \times 10^6 \quad \mathbf{R} = \begin{bmatrix} 150 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 \\ 0 & 0 & 60 & 0 & 0 \\ 0 & 0 & 0 & 120 & 0 \\ 0 & 0 & 0 & 0 & 900 \end{bmatrix}$$

$$\mathbf{Q}_1^* = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 5 \end{bmatrix} \times 10^6 \quad \mathbf{Q}_2^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times 10^5$$

$$\mathbf{R}_{11} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix} \quad \mathbf{R}_{22} = \begin{bmatrix} 120 & 0 \\ 0 & 120 \end{bmatrix}$$

$$\mathbf{R}_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{R}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

the solution to the corresponding coupled algebraic Riccati equation for the CCT strategy (Fitz-Coy 1990, Fitz-Coy & Cochran 1989). With the ARE solutions in hand, feedback gains are expressed as

$$\begin{aligned} \mathbf{K}_1 &= \mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 \\ \mathbf{K}_2 &= \mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_2 \end{aligned} \tag{11}$$

Then, the control laws are expressed as

$$\begin{aligned} \mathbf{u}_1^* &= \mathbf{K}_1\mathbf{x} - \mathbf{R}_{11}^{-1}\mathbf{B}_1^T\mathbf{P}_1 \\ \mathbf{u}_2^* &= \mathbf{K}_2\mathbf{x} - \mathbf{R}_{22}^{-1}\mathbf{B}_2^T\mathbf{P}_2 \end{aligned} \tag{12}$$

3.2 Weighting Matrices and Performance Criteria

In the optimal control problem, the selection of weighting matrices plays a critical role in defining the "best" control law. Usually, for the multi - variable optimal control, there is no easy way to obtain the weighting matrices for the "best" optimal solution. Table 1 contains the weighting matrices used for this comparison that were used to get an idea of how LQR and CCT control differ.

The maneuver to be controlled is the simultaneous rotation of Body 2 by 5 degrees relative to body 1 and body 3 by 5 degrees relative to body 2. The attitude of body 1 is to be maintained as close to being fixed as possible with the given controller. Control is via three torques and two forces. Torque $\mathbf{T}_1$ and force $\mathbf{F}_1$ act on body 1; torques $\mathbf{T}_2$ and $\mathbf{T}_3$ act between bodies 1 and 2, and bodies 2 and 3, respectively. Force F_3 acts on the tip of the flexible body 3. The final time is open. Specified performance criteria are $|\Theta_1| < 0.1°$, and Θ_2 and Θ_3 both within $0.1°$ of $5°$ in 10 sec.

Simulation results for both LQR and CCT closed - loop systems are shown in Figs. 3 - 7. Both the LQR controller and the CCT controller were "optimized" by carefully selecting the weighting matrices. To compare the LQR and CCT results fairly, the magnitudes of the control inputs were equalized. This is similar to the use of a unit step input for competitive controller tests in classical control theory. Figures 3, 4, and 7 indicate that when the CCT controller is used, the amount of

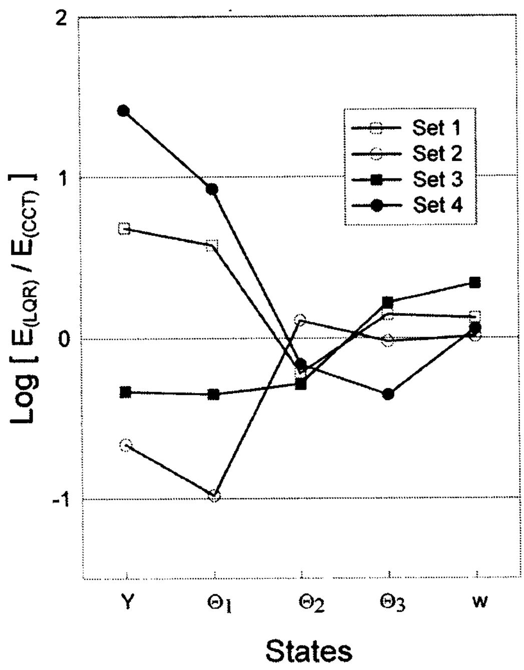

Figure 2. Comparison of performance error with different sets.

Table 2. Strategy set.

Strategy	combination
set 1	$\Theta_2, \Theta_3, \mathbf{w} - \mathbf{Y}, \Theta_1$
set 2	$\mathbf{Y}, \Theta_1, \Theta_3, \mathbf{w} - \Theta_2$
set 3	$\mathbf{Y}, \Theta_1, \Theta_2, \Theta_1, \mathbf{w}$
set 4	$\mathbf{Y}, \Theta_1, \mathbf{w} - \Theta_2, \Theta_3$

induced rotation of body 1 and the excitation of the flexible deformation of body 3 are much smaller in magnitude than when the LQR controller is used. That is, if the individual control systems collaborate while performing their respective tasks, then induced motions are reduced.

4. EFFECT OF INFORMATION AVAILABLE ON PERFORMANCE

4.1 Selection of Strategy Sets for the "Best" Performance

One of the unique features of CCT is that it provides many choices of strategy sets. Here a "strategy set" consists of two sub-sets of controls (player 1 and player 2). To present the general idea of how to perform a strategy set selection, several example strategy sets are used in performing the same task.

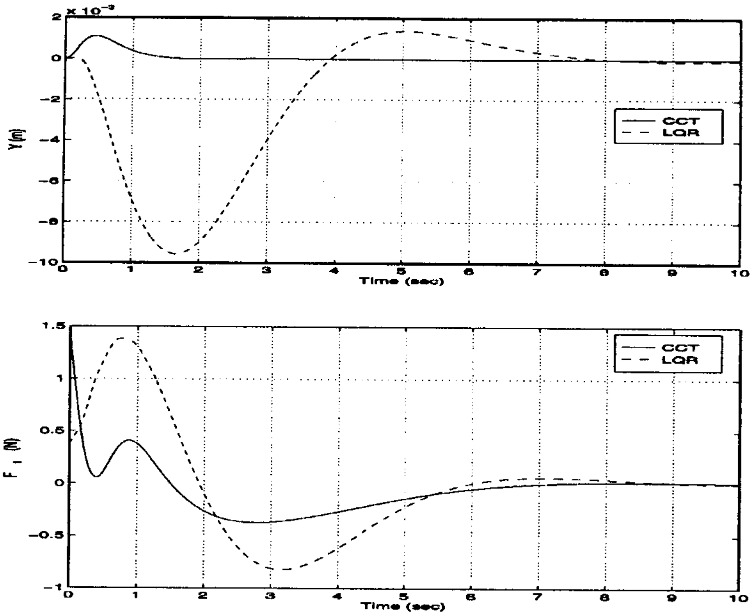

Figure 3. Body one's position and control force.

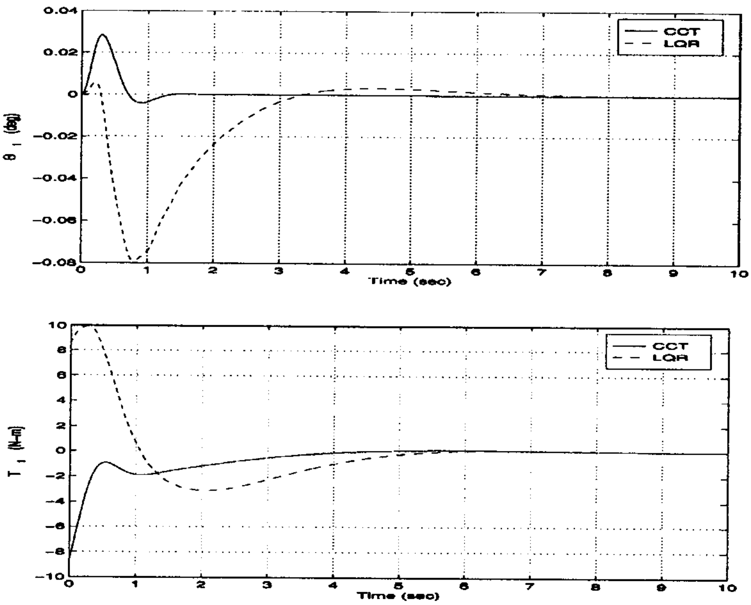

Figure 4. Body one's attitude and reaction wheel command.

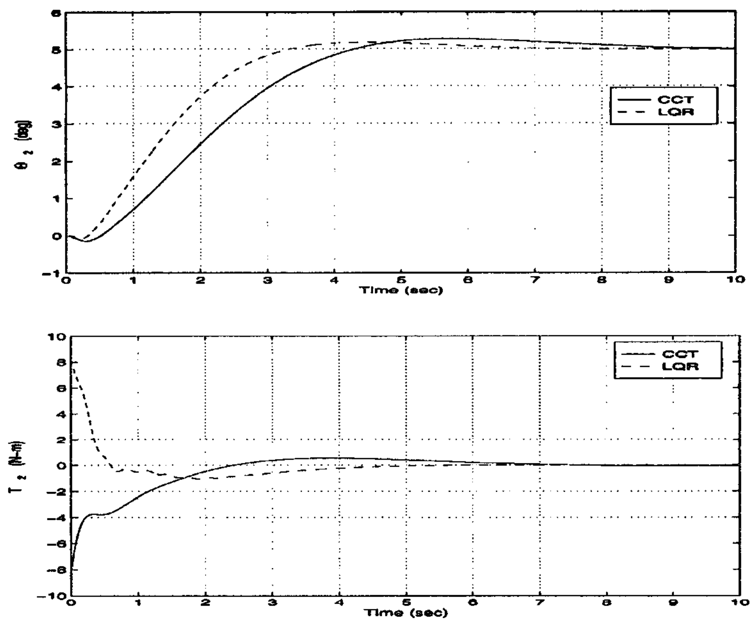

Figure 5. Body two's angular displacement and torque.

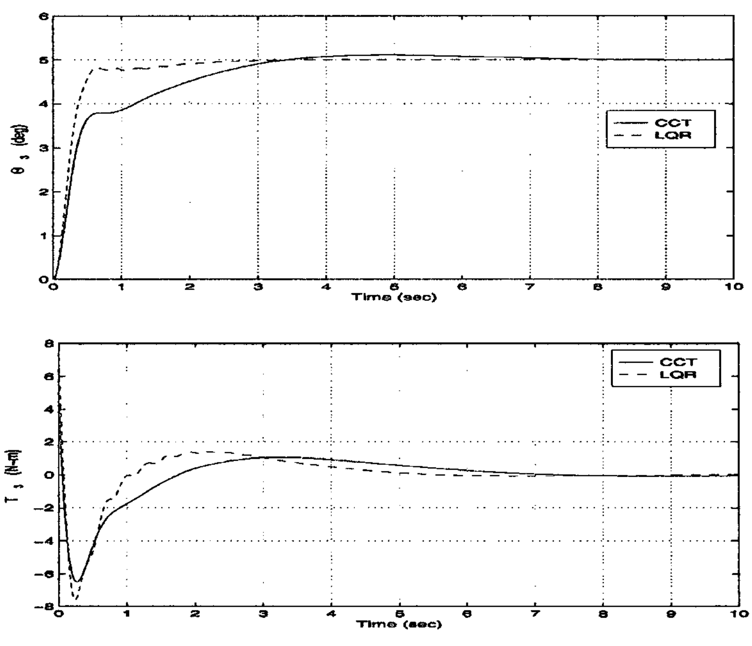

Figure 6. Body three's angular displacement and torque.

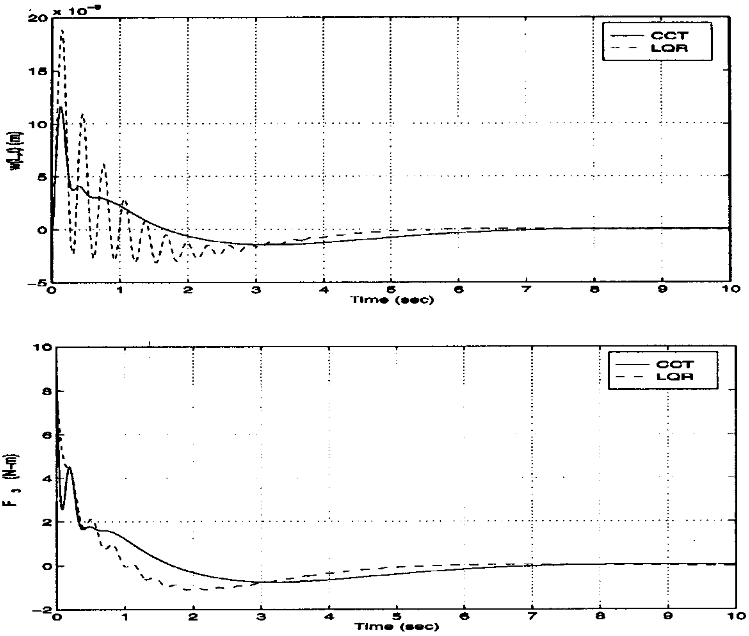

Figure 7. Body three's transverse displacement and control force.

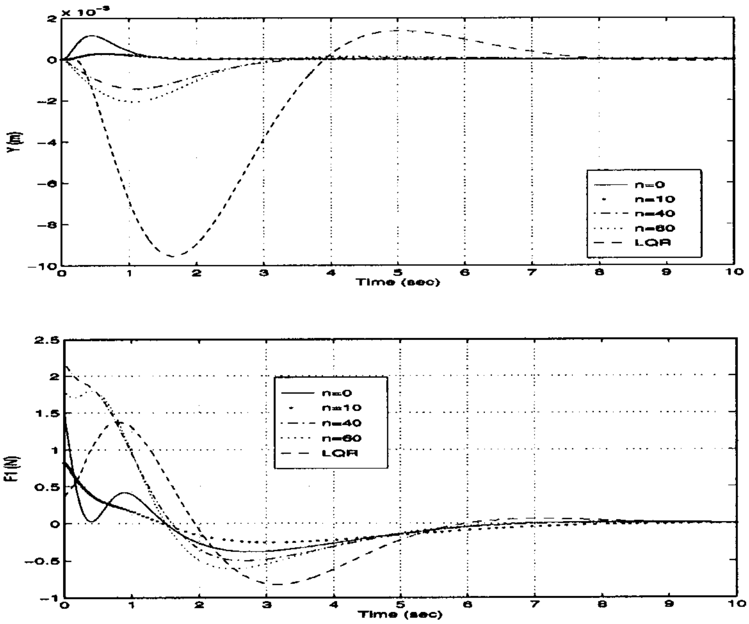

Figure 8. Body one's position and control force.

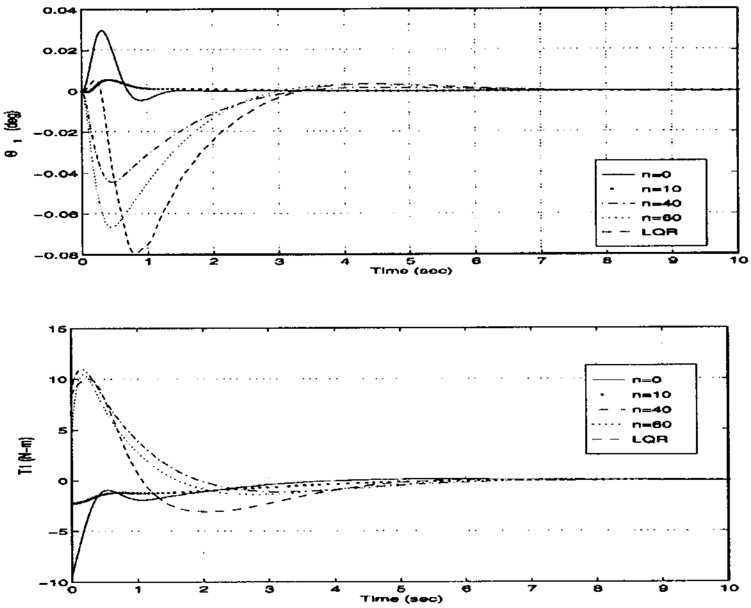

Figure 9. Body one's attitude and reaction wheel command.

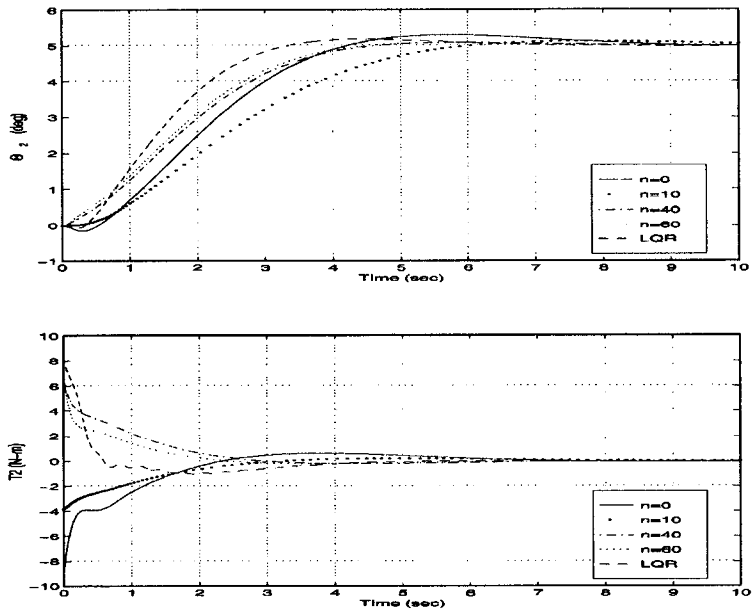

Figure 10. Body two's angular displacement and torque.

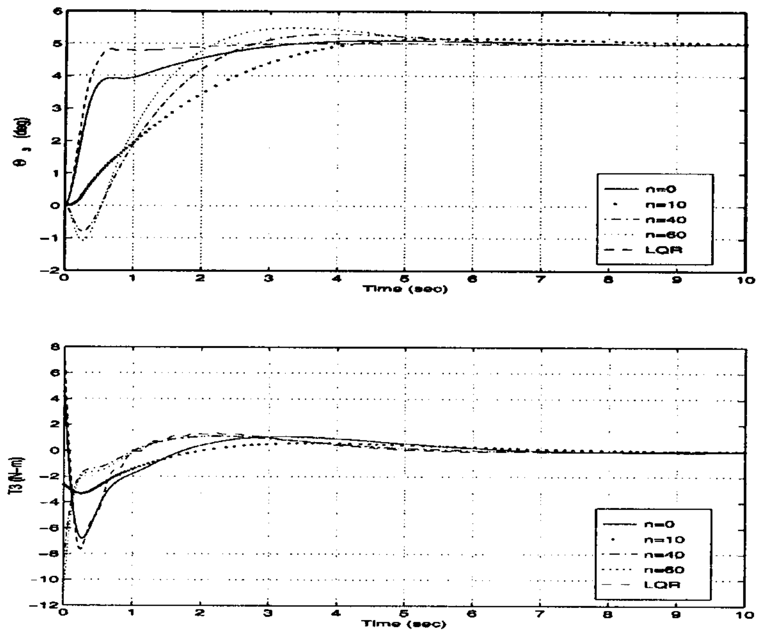

Figure 11. Body three's angular displacement and torque.

To compare overall performances of strategy sets effectively, a particular performance error is used. The performance error (E) is expressed as $E = \int_0^\infty |\mathbf{X} - \mathbf{X}_f| dt$. Figure 2 shows a comparison of the performance error for 4 different strategy sets (see Table 2). For a convenient comparison, log[E(LQR)/E(CC)] is used. The control goals are $\mathbf{Y}_f = 0, \Theta_{1f} = 0, \Theta_{2f} = 5°, \Theta_{3f} = 5°$, and $\mathbf{w}_f = 0$.

By considering Fig. 2, it is seen that the strategy set $(\mathbf{Y}, \Theta_1, \mathbf{w} - \Theta_2, \Theta_3)$ should be selected as the "best" strategy set from among all the others. It can be seen that : (1) CCT control law mainly improved the performance of the set which has only induced motions ("perturbed set"). (2) The set which has a specific motion to perform ("perturbing set") is not significantly influenced by the collaboration. Based on the above conclusions, the strategy set should be separated into a perturbing sub-set and a perturbed sub-set for a conflicting task.

4.2 Optimal Performance Control

Another interesting and unique feature of CCT control is a strong sensitivity of performance to the weight. The performance of the LQR controller is not significantly changed when the off - diagonal term of the weighting matrices are changed. On the other hand, the performance of the CCT controller is very sensitive to changes of the cross-coupling weight matrices $\mathbf{R}_{12}$ and $\mathbf{R}_{21}$. To obtain the some insight regarding the relationship between cross-coupling weight and performances, for sets of cross coupling weight matrices were used in Eq. 13. These were defined as

$$\mathbf{R}_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times n, \quad \mathbf{R}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times n. \quad (n = 0, 10, 40, 60) \qquad (13)$$

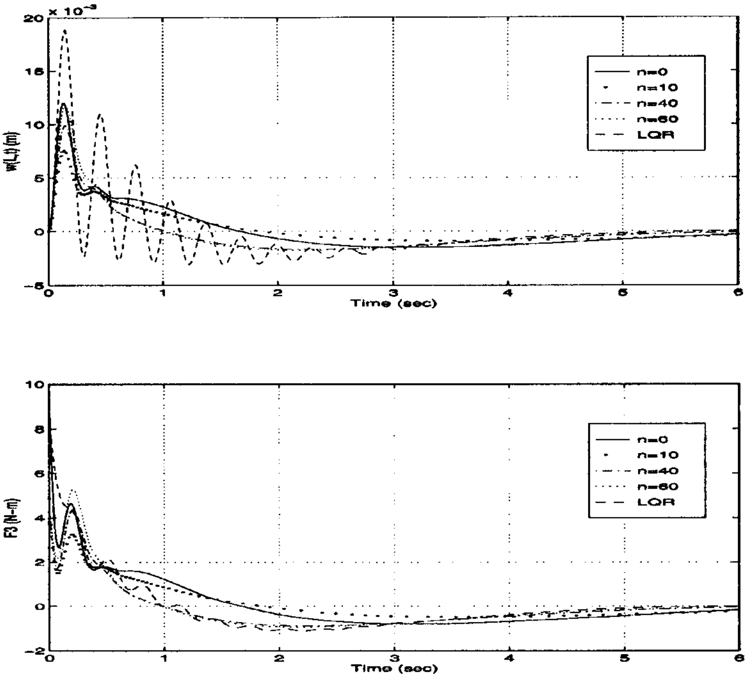

Figure 12. Body three's transverse displacement and control force.

Figures 8 - 12 show that the time responses of outputs and inputs are dependent on the cross-coupling weight matrices. In case of $n = 10$, the performances are best among the others. It is noticed that input patterns are changed with cross-coupling weight. This pattern change produce a different performance and the pattern change cannot be achieved without the cross-coupling weight. It should be mentioned that in this paper, we investigate only the multiple of identity matrix for the cross-coupling weight as an example. More detail investigation can be done for the best result.

5. CONCLUSION

Since the CCT is formulated by two-player nonzero-sum linear quadratic differential game theory, communication between the players plays an important role in achieving "optimal" control. In CCT, communication is accomplished by using two performance indices. The cross-coupling weight matrices and the selection of a strategy set are the principal factors that determine how the communication takes place. The proper selection of a strategy set results in a protocol for effective communication. The cross-coupling weights determine the weight given by one player to the other player expenditure of control effort.

In designing a control system using CCT, one can select a strategy set to match performance to specific control objectives. Once the strategy set is selected, the designer can then find the "best" performance control law by investigating the effect of different cross-coupling weight matrices. The options to select strategy sets and the cross - coupling weight matrices gives a control designer the flexibility to exercise a variety of control system designs that can be used to meet specific control system requirements.

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APPENDIX

Kinematic Equations

Linearized equations for translational motion of each body are expressed as (see Figure A1.)

$$\begin{aligned} &m_1\ddot{\mathbf{R}}_1 = \mathbf{F}_{12} + \mathbf{F}_{lc} \\ &m_2\left[\ddot{\mathbf{R}}_1 + \dot{\omega}_1 \times \mathbf{r}_1 + \dot{\omega}_2 \times \frac{1}{2}\mathbf{r}_2\right] = \mathbf{F}_{21} + \mathbf{F}_{23} \\ &\int_0^L (\ddot{\mathbf{R}}_1 + \dot{\omega}_1 \times \mathbf{r}_1 + \dot{\omega}_2 \times \mathbf{r}_2 + \dot{\omega}_3 \times \mathbf{X}_3 + \ddot{\mathbf{w}}_3)dm_3 = \mathbf{F}_{32} + \mathbf{F}_{3c}) \end{aligned} \tag{A1}$$

Here, m_j is the mass of body j; ω_j is the angular velocity of body j; $\mathbf{F}_{21}, \mathbf{F}_{23}$, and $\mathbf{F}_{32}$ are constraint forces; $\mathbf{F}_{1c}$ and $\mathbf{F}_{3c}$ are control forces on bodies 1 and 3 respectively; and $\mathbf{C}_1$ and $\mathbf{C}_2$ are the center of mass of bodies 1 and 2. $\mathbf{R}_1, \mathbf{r}_1, \mathbf{r}_2, \mathbf{x}_3, \mathbf{r}_3$, and $\mathbf{w}_3$ are position vectors indicated in Fig. A1.

The rotational equations of this system are described as

$$\begin{aligned} &\mathbf{I}_1\dot{\omega}_1 = \mathbf{T}_1 - \mathbf{T}_2 + [\mathbf{r}_1 \times \mathbf{F}_{12}] \\ &\mathbf{I}_2\dot{\omega}_2 = \mathbf{T}_2 - \mathbf{T}_3 + \left[\frac{1}{2}(-\mathbf{r}_2) \times \mathbf{F}_{21} + \frac{1}{2}\mathbf{r}_2 \times \mathbf{F}_{23}\right] \\ &\int_0^L \rho_3 \times (\ddot{\mathbf{R}}_1 + \dot{\omega}_1 \times \mathbf{r}_1 + \dot{\omega}_2 \times \mathbf{r}_2 + \dot{\omega}_3 \times \mathbf{x}_3 + \ddot{\mathbf{w}}_3)dm = \mathbf{T}_3 + \int_0^L \rho_3 \times d\mathbf{F} \end{aligned} \tag{A2}$$

where $\rho_3 = \mathbf{X}_3 + \mathbf{w}_3$ and $d\mathbf{F} = \mathbf{F}_{32}\delta(\mathbf{X}_3)d\mathbf{X}_3 + \mathbf{F}_{3c}\delta(\mathbf{X}_3 - \mathbf{L})d\mathbf{X}_3 + \mathbf{f}_3^{elastic}d\mathbf{x}_3$. The δ is a

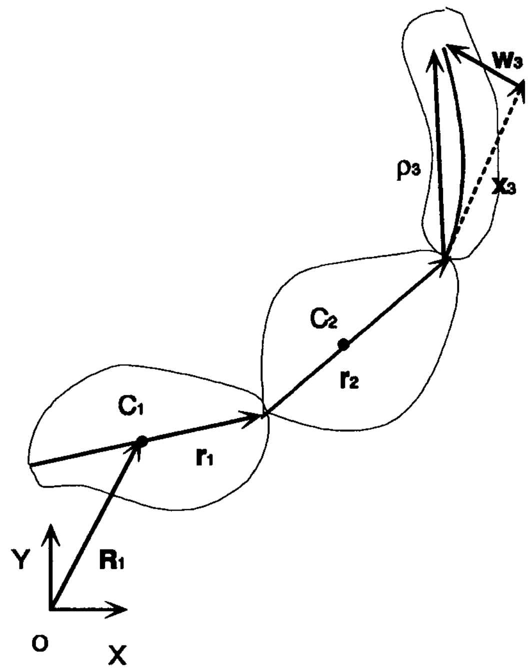

Figure A1. Flexible 3-body model.

delta function and $\mathbf{f}_3^{elastic}$ is a elastic force. The equations for the transverse vibration of body 3 are

$$\int_0^L \phi_k dm_3 \cdot (\ddot{\mathbf{R}}_1 + \dot{\omega}_1 \times \mathbf{r}_1 + \dot{\omega}_2 \times \mathbf{r}_2) + \int_0^L \phi_k \cdot (\ddot{\omega}_3 \times \mathbf{x}_3 + \ddot{\mathbf{w}}_3) dm_3 = \int_0^L \phi_k \cdot d\mathbf{F}. \quad k = 1, 2 \quad \text{(A3)}$$

Linearized State-Space Form of the Equations

From Eqs. (A1)-(A3), the following linearized equations of motion in matrix form are obtained.

$$\mathbf{M}_s \dot{\mathbf{x}}_2 + \mathbf{K}_s \mathbf{x}_1 = \mathbf{D}_s \mathbf{u} \quad \text{(A4)}$$

where $\mathbf{x}_2 = [\mathrm{Y}\omega_1\omega_2\omega_3 q_i]^T$, $\mathbf{x}_1 = [\mathrm{Y}\Theta_1\Theta_2\Theta_3 q_i]^T$ and $\mathbf{u} = [\mathbf{F}_{1c}\mathbf{T}_1\mathbf{T}_2\mathbf{T}_3\mathbf{F}_{3c}]^T$. The matrices $\mathbf{M}_s, \mathbf{K}_s$, and $\mathbf{D}_s$ are mass, stiffness, and control influence matrices, respectively.

The state-space representation of the system is defined as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \quad \text{(A5)}$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are expressed as

$$\mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{A}_s \\ -\mathbf{M}_s^{-1}\mathbf{K}_s & \mathbf{O} \end{bmatrix}, \quad \mathbf{A}_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{(A6)}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{O} \\ \mathbf{M}_s\mathbf{D}_s \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_i(\mathbf{L}_3) & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A7}$$

and $\mathbf{O}$ is the appropriate null matrix. The state-space representation for the CCT is defined as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{u}_1(t) + \mathbf{B}_2\mathbf{u}_2(t) \\ \mathbf{y}_1(t) &= \mathbf{C}_1\mathbf{x}(t) \\ \mathbf{y}_2(t) &= \mathbf{C}_2\mathbf{x}(t) \end{aligned} \tag{A8}$$

In the case in which $\mathbf{u}_1 = (\mathbf{F}_1\mathbf{T}_1\mathbf{F}_3)^T$ and $\mathbf{u}_2 = (\mathbf{T}_2\mathbf{T}_3)^T$, the $\mathbf{B}$ and $\mathbf{C}$ are divided accordingly into $\mathbf{B}_1$ and $\mathbf{B}_2$, and $\mathbf{C}_1$, and $\mathbf{C}_2$, respectively.

In this paper, the length of bodies 1, 2 and 3 are 5, 5, and 2 m and masses of bodies 1, 2, and 3 are 100, 25, and 6 kg, respectively. For the elastic deformation model, the 1st mode is used. In Table A1, $\mathbf{M}_s$, and $\mathbf{D}_s$ matrices used this paper are given.

Table A1. $\mathbf{M}_s, \mathbf{K}_s$, and $\mathbf{D}_s$ matrices.

$$\mathbf{M}_s = \begin{bmatrix} 131.00 & 77.50 & 92.50 & 6.00 & -0.91 \\ 77.50 & 402.08 & 231.25 & 15.00 & -2.27 \\ 92.50 & 231.25 & 358.33 & 30.00 & -4.54 \\ 6.00 & 15.00 & 30.00 & 8.00 & 0.00 \\ -0.91 & -2.2679 & -4.54 & 0.00 & 1.00 \end{bmatrix}$$

$$\mathbf{K}_s = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 383.08 \end{bmatrix}$$

$$\mathbf{D}_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 2.5 \\ 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0.8165 \end{bmatrix}$$