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ESTIMATION OF A GENERAL ALONG-TRACK ACCELERATION IN THE KOMPSAT-1 ORBIT

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ABSTRACT

General along-track acceleration was estimated in the KOMPSAT-1 orbit determination process. Several sets of the atmospheric drag and solar radiation pressure coefficients were also derived with the different spacecraft area. State vectors in the orbit determination with the different spacecraft area were compared in the time frame. The orbit prediction using the estimated coefficients was performed and compared with the orbit determination results. The orbit prediction with the different general acceleration values was also carried out for the comparison

Keywords: orbit determination, orbit prediction, satellite orbit, parameter estimation, GPS, KOMPSAT-1

1. INTRODUCTION

Korea Multi-Purpose Satellite-1 (KOMPSAT-1) has been successfully operated since the launch of the spacecraft on December 21, 1999. Mission Analysis and Planning Subsystem (MAPS) is used for the spacecraft flight dynamics support and mission operations planning (Won et al. 1999). The orbit determination using GPS navigation solutions from on-board the KOMPSAT-1 is performed everyday for the flight dynamics support (Lee et al. 2000a). The orbital perturbations such as the asphericity of the Earth gravity, luni-solar gravity, atmospheric drag, and solar radiation pressure are mathematically modeled in the orbit determination (OD) and orbit prediction (OP) program. Currently, only position and velocity vectors at a given epoch are estimated in the OD. Another parameters such as atmospheric drag and solar radiation pressure coefficients are not estimated in the normal operational OD process at the KOMPSAT mission control. The estimation of the atmospheric drag and solar radiation pressure coefficients of the KOMPSAT-1 was performed by Lee et al. (2001). However, the atmospheric drag undergoes large un-modeled fluctuations in the low Earth orbit. Such un-modeled fluctuations and the effects of other uncertainties in the mathematical model lead to a discrepancy between the model forces and the actual forces (Tapley 1972). The un-modeled atmospheric fluctuations generally act upon the along-track direction.

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In this paper, a general along-track acceleration that is not modeled in the orbital perturbation is also estimated in the batch weighted least square orbit determination process. Several sets of the coefficients such as the atmospheric drag, solar radiation pressure, and the general along-track acceleration are estimated with the different spacecraft cross-sectional area. State vectors in the OD with the different spacecraft area are compared in the time frame. The OP using the estimated coefficients is performed and compared with the OD results. The OP with the different general acceleration values is also carried out for the comparison.

2. FORCE MODELING

The equation of motion for the KOMPSAT-1 in the geocentric inertial coordinate system is of the form

$$\ddot{\bar{r}} = -\frac{\mu_e}{|\bar{r}^3|}\bar{r} + \ddot{\bar{r}}_{geo} + \ddot{\bar{r}}_{sm} + \ddot{\bar{r}}_{drag} + \ddot{\bar{r}}_{srp} + \ddot{\bar{r}}_{GA} \tag{1}$$

where, $\bar{r}$ is the position vector of the spacecraft, $\mu_e = GM$, where G is the gravitational constant and M is the mass of the Earth, $\ddot{\bar{r}}_{geo}, \ddot{\bar{r}}_{sm}, \ddot{\bar{r}}_{drag}, \ddot{\bar{r}}_{srp}$, and $\ddot{\bar{r}}_{GA}$ are the accelerations due to the Earth geopotential, luni-solar gravity, atmospheric drag, solar radiation pressure, and general acceleration, respectively.

The acceleration due to the atmospheric drag is given by

$$\ddot{\bar{r}}_{drag} = -\frac{1}{2}C_D\frac{A_s}{m_s}\rho_D v_r \bar{v}_r \tag{2}$$

where, C_D is the drag coefficient of the satellite, A_s is the cross-sectional area of the satellite, m_s is the mass of the satellite, ρ_D is the density of the atmosphere at the satellite position, $\bar{v}_r$ is the velocity vector of the satellite relative to the atmosphere, and v_r is the magnitude of the velocity vector.

The acceleration due to the solar radiation pressure is formulated as

$$\ddot{\bar{r}}_{srp} = -v\, C_R\frac{A_s}{m_s}P_S\, \hat{r}_s \tag{3}$$

where, v is the eclipse factor, C_R is the solar radiation pressure coefficient which is depending upon the reflective characteristics of the satellite, A_S is the cross-sectional area of the satellite, m_s is the mass of the satellite, P_s is the solar radiation pressure in the vicinity of the Earth, and $\hat{r}_s$ is the geocentric unit vector pointing to the Sun.

The general acceleration due to the un-modeled perturbation is formulated as (McCarthy et al. 1993)

$$\ddot{\bar{r}}_{GA} = \alpha\hat{u} \tag{4}$$

where, $\hat{u}$ is a unit vector defining the direction of the acceleration and α is the solved-for parameter. The direction of the general acceleration consists of the radial, along-track, and cross-track direction, respectively.

$$\hat{u}_r = \frac{\bar{r}}{|\bar{r}|}, \quad \hat{u}_a = \frac{\bar{v}}{|\bar{v}|}, \quad \hat{u}_c = \frac{\bar{r}\times\bar{v}}{|\bar{r}\times\bar{v}|} \tag{5}$$

Table 1. Estimated C_D, C_R, and G_A in the orbit determination processes ($m_s = 448$ kg).

GPS data interval	02/12 - 02/14 (1st OD)			02/14 - 02/15 (2nd OD)			02/15 - 02/16 (3rd OD)		
Estimation Parameters	C_D	C_R	$G_A(\mathrm{m/s^2})$	C_D	C_R	$G_A(\mathrm{m/s^2})$	C_D	C_R	$G_A(\mathrm{m/s^2})$
Case1 ($A_s = 6.5\mathrm{m}^2$)	2.65	1.70	-1.557E-8	3.41	1.93	1.160E-8	2.79	1.92	-2.643E-9
Case2 ($A_s = 7.5\mathrm{m}^2$)	**2.33**	**1.48**	**-1.467E-8**	2.97	1.68	1.681E-8	2.45	1.67	-2.075E-9
Case3 ($A_s = 8.5\mathrm{m}^2$)	2.07	1.31	-1.407E-8	2.63	1.49	1.224E-8	**2.17**	**1.48**	**-1.725E-9**
Case4 ($A_s = 9.5\mathrm{m}^2$)	1.86	1.17	-1.373E-8	**2.35**	**1.33**	**1.239E-8**	1.95	1.32	-1.500E-9

Table 2. The 1st orbit determination results using two-day GPS navigation solutions (Epoch: 2001/02/13 00:00:29.0 UTC).

Case number	Case 1	Case 2	Case 3	Case 4
Iterations	Converge After 5	Converge After 5	Converge After 5	Converge After 5
a (km)	7058.306722	7058.306718	7058.306716	7058.306714
$e(-)$	.002384347234	.002384337844	.002384341446	.002384339937
i (deg)	98.153161850	98.153161843	98.153161366	98.153161364
Ω (deg)	305.729639689	305.729639698	305.729639790	305.729639793
ω (deg)	66.692812109	66.692692844	66.692555147	66.692509659
M (deg)	166.791886316	166.792005043	166.792142163	166.792187445
$\omega + M$(deg)	233.484698425	233.484697887	233.484697310	233.484697104
RMS position (m)	3.349	3.354	3.357	3.360
RMS velocity (m/s)	0.003222	0.003226	0.003229	0.003230

In this study, only the along-track acceleration is considered as solved-for parameter because along-track position determination is very important in the Earth imaging mission such as the KOMPSAT-1. The along-track acceleration $\hat{u}_a$ in Eq.(5) is considered as G_A from here.

3. ESTIMATION OF THE KOMPSAT-1 C_D, C_R, AND G_A

Estimation of the C_D, C_R, and G_A are not performed in the normal orbit determination process of the KOMPSAT-1 for the operational convenience. So, the fixed values of the atmospheric drag coefficient of $C_D = 2.2$, the solar radiation pressure coefficient of $C_R = 1.5$, and the general along-track acceleration of $G_A = 0$ are used. However, the OD program in KOMPSAT-1 MAPS has the capability to estimate the three coefficients based on the fixed cross-sectional area(A_S) and mass of the satellite(m_s).

KOMPSAT-1 GPS navigation solutions during 2001/02/13~2001/02/16 are used for estimating the three coefficients. Two-day GPS navigation solutions are used in the OD as a sliding time bases. Totals of 12 OD are performed with the different cross-sectional area of the KOMPSAT-1.

Table 1 shows the estimated coefficients in the 12 OD processes. The estimated values of C_D, C_R, and G_A are varied with the GPS data for a fixed value of the spacecraft area. The bold typed C_D and C_R are close to the nominal values. The estimated general accelerations are in the

Table 3. Statistics of the position and velocity differences between the orbit determination and GPS navigation solutions.

Position	Mean (m)	Std (m)	Velocity	Mean (m/s)	Std (m/s)
x	4.359	35.717	vx	-0.0236	2.234
y	3.393	31.461	vy	-0.0141	1.707
z	0.764	27.836	vz	0.0038	1.028
r	33.092	44.454	v	1.156	2.762

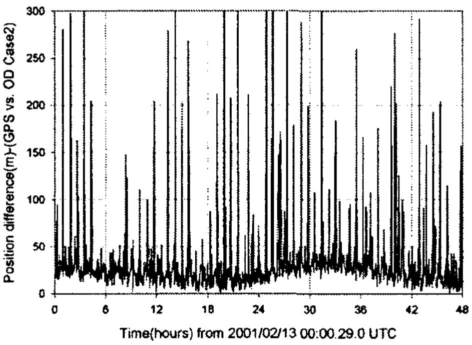

Figure 1. Position difference between GPS data and OD results.

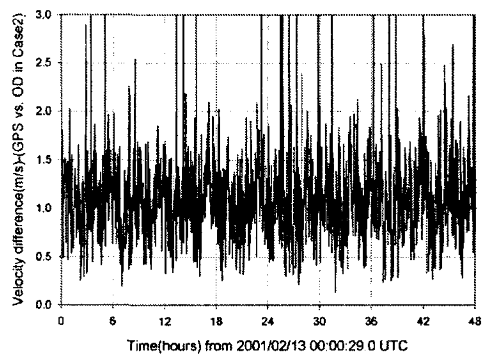

Figure 2. Velocity difference between GPS data and OD results.

order of $10^{-8}\mathrm{m/s}^2$. The estimated values of the 1st OD in Case 2, the 2nd OD in Case 4, and the 3rd OD in Case 3 are close to the nominal values of $C_D = 2.2$ and $C_R = 1.5$. The cross-sectional area of $7.5\mathrm{m}^2, 8.5\mathrm{m}^2$, and $9.5\mathrm{m}^2$ are somewhat larger than that of expected. However, the estimated position and velocity vectors in the OD processes are almost same as shown in Table 2.

Figure 1 and 2 shows the position and velocity differences between the GPS navigation solutions and the 1st OD in Case 2 results. The individual quality of the GPS navigation solutions is improved since the termination of the Selective Availability (Lee et al. 2000b). However, there are many spurious bad points in the GPS navigation solutions. And the quality of the GPS velocity solution is not good compare to the position solutions as indicated in Montenbruck et al. (1996). Table 3 shows the statistics of the differences between the GPS navigation solutions and the 1st OD in Case 2 results as shown in Figure 1 and Figure 2.

Figure 3 shows the OD position differences between Case 1 and Case 4. The time span of one OD is 48 hours and shifts of 24 hours for the next orbit determination are applied. The position differences in the OD processes are below 50cm.

Figure 4 presents the OD position difference between the overlapping periods during 2001/02/14 ~2001/02/15 in Case 3 of Table 1. In Case 3, the first 24 hours of the 2nd OD is overlapped with the last 24 hours of the 1st OD, and the last 24 hours of the 2nd OD is overlapped with the first 24 hours of the 3rd OD. The position differences below 12m during the overlapping periods are shown in the figure. The result shows the consistency of the consecutive orbit determination.

Figure 5 shows the position difference between the OD and OP for Case 3. The OD results at

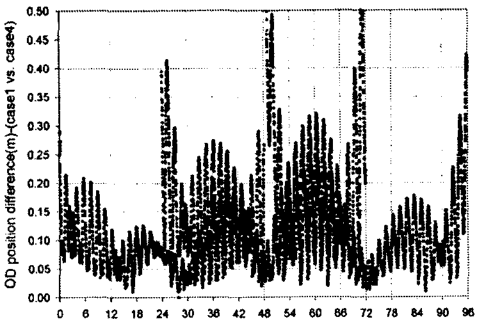

Figure 3. OD position difference in Case1 and Case 4.

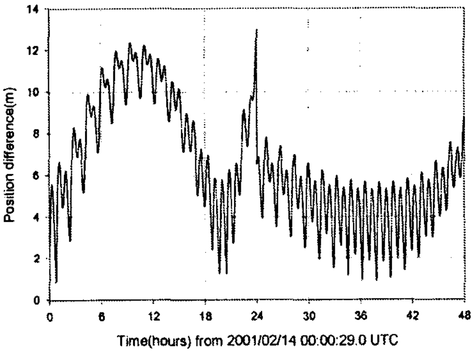

Figure 4. OD position difference in Case 3.

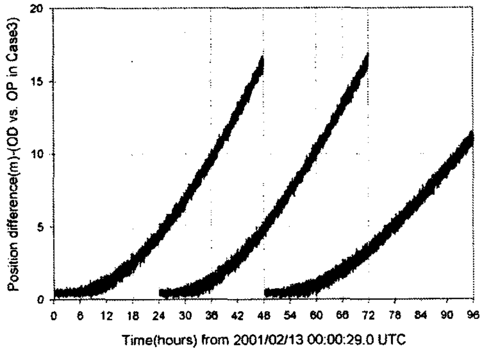

Figure 5. Position differences in OD and OP for Case 3.

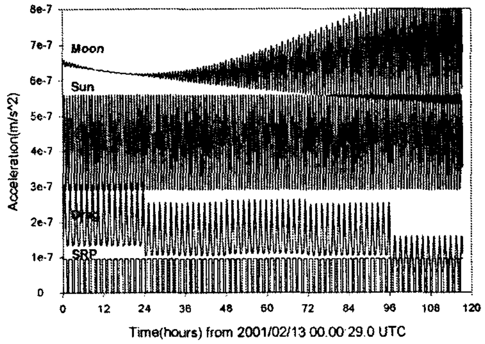

Figure 6. Contributions of the perturbations.

the start time are used for the 48-hours OP. The estimated C_D, C_R, and G_A in Case 3 are also used for OP. The position differences in Figure 5 are below 17m for the 48-hours OP and it shows the estimated coefficients are well matched for orbit propagation.

Figure 6 presents the contributions of the perturbations on the KOMPSAT-1 orbit. The OP is performed for 5 days using the 1st OD results in Case 3. The contribution of the Earth geopotential harmonics is about an order of $10^{-2}\mathrm{m/s^2}$ and this is not displayed with another $10^{-7}\mathrm{m/s^2}$ order terms. Also, the general acceleration in the along-track direction GA is about an order of $10^{-8}\mathrm{m/s^2}$. This order of magnitude was used for the acceleration errors in the orbit determination analysis (Kang et al. 1997).

In the nominal KOMPSAT-1 mean altitude of 685 km, the gravitational effect of the moon is slightly bigger than that of Sun. And the lunar effect is varied with the phase of the moon. The eclipse effect of the solar radiation pressure is also shown in the figure. Diurnal and latitudinal effects of the atmospheric drag in Jacchia 71 model are also shown. Currently, the atmospheric drag is largest because it is around the solar maximum in 11-year sunspot cycle (Phillips 2001).

Figure 7 shows the differences between the positions in OP using the previous two-day OD result and the positions in next two-day OD. The input orbital elements in OP come from the 1st OD

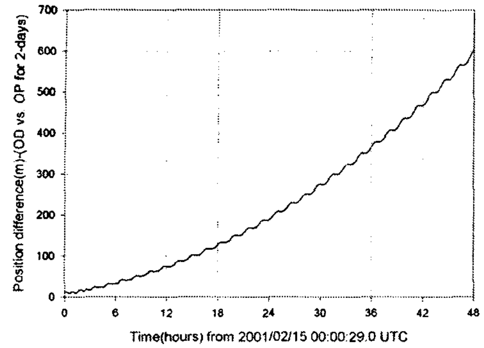

Figure 7. Position differences between OD results and OP based on the previous OD.

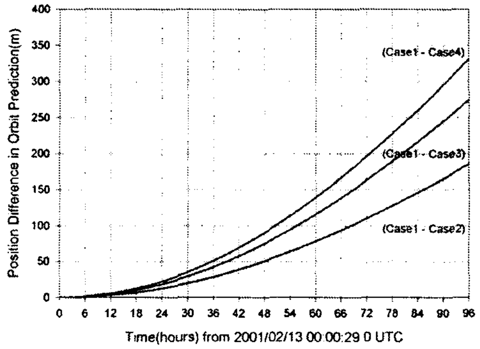

Figure 8. Position differences in the orbit prediction.

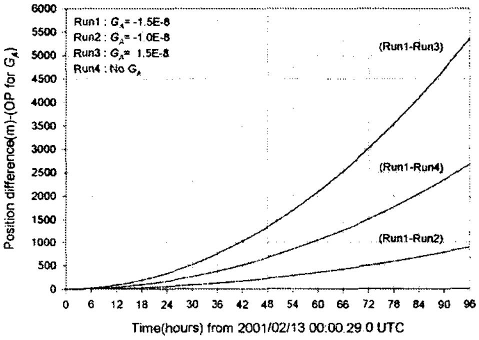

Figure 9. Position differences in OP with different general accelerations.

results in Case 3. So, the OP results are not affected by the next OD measurement data. About 600m for 48 hours position differences are shown. It means that the estimated coefficients in the previous OD are not good enough for predicting the next day orbit. It is because the estimated coefficients do not show any consistency with the next day values as in Table 1. The position difference is affected by the general along-track acceleration from the 1st OD result. Lee et al. (2001) performed similar analysis without general along-track acceleration. Only the atmospheric drag and solar radiation pressure coefficients were included at that time. The position difference in Lee et al. (2001) was smaller than that of the figure 7.

Figure 8 represents the position differences in OP based on the estimated state vectors and coefficients in Table 1 and Table 2. Maximum position differences of about 350m for 96 hours are shown. The difference in this study is bigger than that of Lee et al. (2001) because the effect of the general acceleration is accumulated in OP.

Figure 9 presents the position differences in OP by different general acceleration values. The same epoch state vectors, C_D, and C_R values in Case 2 OD are used in four orbit predictions. Relatively very large position differences are shown. It is because the general acceleration is accumulated in the four-day orbit prediction even though the value itself is an order of $10^{-8}\mathrm{m/s}^2$. The result shows that the careful use of the general acceleration in orbit prediction is requested.

4. CONCLUSIONS

A general along-track acceleration of the KOMPSAT-1 orbit was estimated in the orbit determination process. Estimation of the atmospheric drag and solar radiation pressure coefficients were also performed with different cross-sectional area. The GPS navigation solutions from on-board the spacecraft were used for orbit determination. The position differences between the orbit determinations for the different cross-sectional area were in the order of ten centimeters. The position differences between the overlapping periods in two consecutive orbit determinations with day-shifted were about ten-meter order of magnitude. The orbit propagation with the estimated states and coefficients was performed and compared with the orbit determination results. The position differences were below 20 meters for 48 hours, and it showed that the estimated coefficients were well matched with representing orbit determination.

The orbit prediction for the future orbit based on the previous orbit determination was also performed with the three coefficients. The results were compared with the next day orbit determination. About 600 meters of position differences for 48 hours were shown. This showed that the coefficients in the previous orbit determination were not useful for the precision orbit prediction for the next day. The estimated coefficients did not show any consistencies with the next day values. The orbit propagation with the same orbit state but different general along-track accelerations showed big differences about a few kilometers for 96 hours even though the accelerations were an order of $10^{-8}\mathrm{m/s^2}$. The result indicates that the careful application of the general acceleration is required. Only the general acceleration in along-track direction was considered in this study. Estimation of the general accelerations in the radial and cross-track directions is reserves for the next study.

REFERENCES

Kang, Z., Schwintzer, P., Reigber, Ch., & Zhu, S. Y. 1997, Adv. Space Res., 19, 1667

Lee, B.-S., Lee, J.-S., Kim, J.-H., Kim, H.-D., Kim E.-K., & Choi, H.-J. 2001, Estimation of the Atmospheric Drag and Solar Radiation Pressure Coefficients in the KOMPSAT-1 Orbit Determination, Proc. of KSAS Spring Annual Meeting 2001, pp.579-582

Lee, B.-S., Lee, J.-S., Lee H.-J., & Lee, S.-P. 2000b, Improvement of the KOMPSAT-1 GPS Navigation Solutions After Termination of the Selective Availability, The 7th GNSS Workshop-International Symposium on GPS/GNSS, pp.36-39

Lee, B.-S., Lee, J.-S., Lee, H.-J., Lee, S.-P., Kim, J.-A., & Choi, H.-J. 2000a, Orbit Determination for the KOMPSAT-1 Using GPS Navigation Solutions, Proc. of KSAS Spring Annual Meeting 2000, pp.131-134

McCarthy, J. J., Rowton, S., Moore, D., Pavlis, D. E., Luthcke, S. B., & Tsaoussi, L. S. 1993, Systems Description-GEODYN-II (Greenbelt: Hughes STX Systems), pp.126-125

Montenbruck, O., Gill, E., & Fraile-Ordonez, J. M. 1996, Orbit Determination of the MIRS Space Station Using MOMSNAV GPS Measurements, Proc. of 11th International Astrodynamics Symposium, 96-c-53

Phillips, T. 2001, The Sun Does a Flip, http://science.msfc.nasa.gov/headlines/y2001/ast15feb_1.htm

Tapley, B. D. 1972, in Recent Advances in Dynamical Astronomy, Statistical Orbit Determination Theory, eds. B. D. Tapley & V. Szebehely (Drodrecht: Reidel), pp.396-425

Won, C.-H., Lee, J.-S., Lee, B.-S., & Eun, J.-W. 1999, ETRI J., 21, 29