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신뢰성 향상을 위한 실험설계 및 분석

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Experimental Design and Analysis for Reliability Impovement

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Abstract

Experimental design has become one of the primary tools for achieving quality of manufactured products. However, this important tool has not been extensively used in achieving reliability. In this paper, we describe how experimental design can be used to achieve reliability.

1. Introduction

Industry rediscovered statistically designed experiments as an efficient tool for improving quality of products and processes. Therefore, many companies are striving to introduce new initiatives of proactive strategies so that they can meet and exceed the quality and reliability of their competitors. In response to this effort, experimental design in combination with reliability can be an example of an active program.

Reliability could be just one aspect of quality. In an editorial in IEEE Transactions on Reliability, Evans (1985) says that the concept of quality must expand to include not only the initial performance characteristics but also those characteristics that influence length of life.

In this paper, we consider how to improve the reliability of products and processes through the use of designed experiments. In Section 2, examples of experiments with failure time data are introduced. Censored data are introduced in Section 2. A common approach to analyzing lifetime data when there is censoring and when there are independent variables is using the Weibull or lognormal regression model. Therefore, in Section 3, regression models that can deal with independent variables and a response variable are discussed. This approach can be used to determine which factors affect the lifetime of a unit. Likelihood approaches are used to estimate the parameters of the models. In Section 4, the experimental data shown in Section 2 are analyzed using the approach in Section 3. Finally in Section 5 we reemphasize the importance of experimental designs in reliability analysis and comment on another way of dealing with censored data.

2. Examples of Experiments with Failure Time Data

Experimental designs have been used in improving the quality of manufactured products. When there are many factors to consider, fractional factorial designs are generally used for economic reasons. However, experimental designs have not been as widely used in reliability as in quality. One of the literatures dealing with reliability in the experimental designs is Hellstrand(1989), where it deals with 2^3 full factorial design without replicates. It deals with Heat, Osculation and Cage as factors and Lifetime as a response variable(Table 1). This data will be analyzed in Section 4.

<Table 1> Rolling Ball Bearing Lifetimes

Run	Heat (X_1)	Osculation (X_2)	Cage (X_3)	Lifetime (t)
1	-1	-1	-1	17
2	1	-1	-1	26
3	-1	1	-1	25
4	1	1	-1	85
5	-1	-1	1	19
6	1	-1	1	16
7	-1	1	1	21
8	1	1	1	128

Bullington et al (1993) provide an example of a 12-run Plackett-Burman design to improve the reliability of industrial thermostats (Table 2). Eleven factors(A-K) were studied in which 10 thermostats were manufactured at each of the 12 run settings (Table 3). This data will also be analyzed in Section 4.

<Table 2> 12-Run Plackett-Burman Design for the Thermostat Experiment

Run	A	B	C	D	E	F	G	H	I	J	K
1	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	+	+	+	+	+	+
3	-	-	+	+	+	-	-	-	+	+	+
4	-	+	-	+	+	-	+	+	-	-	+
5	-	+	+	-	+	+	-	+	-	+	-
6	-	+	+	+	-	+	+	-	+	-	-
7	+	-	+	+	-	-	+	+	-	+	-
8	+	-	+	-	+	+	+	-	-	-	+
9	+	-	-	+	+	+	-	+	+	-	-
10	+	+	+	-	-	-	-	+	+	-	+
11	+	+	-	+	-	+	-	-	-	+	+
12	+	+	-	-	+	-	+	-	+	+	-

<Table 3> Data from the Thermostat Experiment

Run	Ordered Lifetime Data (unit : k-cycle)									
1	957	2846	.	.	.	.	.	.	.	.
2	206	284	296	305	313	343	364	420	422	543
3	63	113	129	138	149	153	217	272	311	402
4	76	104	113	234	270	364	398	481	517	611
5	92	126	245	250	390	390	479	487	533	573
6	490	971	1642	6768	.	.	.	.	.	.
7	232	326	326	351	372	446	459	590	597	732
8	56	71	92	104	126	156	161	167	216	263
9	142	142	238	247	310	318	420	482	663	672
10	259	266	306	337	347	368	372	426	451	510
11	381	420	.	.	.	.	.	.	.	.
12	56	62	92	104	113	121	164	232	258	731

. means right censored at 7342 k-cycle

In the above experiments, no explicit noise factors have been used. However, noise variation is represented by the use of replicates for each control run. In the next experiment noise factors are to be considered. In an experiment for improving the reliability of drill bits used in fabricating multi-layer printed circuit boards, 11 control factors as well as 5 noise factors are considered as in Table 4. The experiment employed a cross array consisting of a 16-run control array and an 8-run noise array. The number of holes drilled before it breaks is defined as the failure time of a drill bit. The cross array and the data (Hamada, 1993) are given in Table 5. Note that the testing was stopped after 3000 holes were drilled. If a drill bit did not break before 3000 holes, its failure time was right censored at 3000.

<Table 4> Factors and Their Levels for the Drill Bit Experiment

<table>
<tr><td rowspan="2">Control factor</td><td colspan="4">Level</td></tr>
<tr><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>A. carbide cobalt (%)</td><td>A1</td><td>A2</td><td>A3</td><td>A4</td></tr>
<tr><td rowspan="2"></td><td colspan="4">Level</td></tr>
<tr><td colspan="2">-</td><td colspan="2">+</td></tr>
<tr><td>B. body length (in.)</td><td colspan="2">Minimum</td><td colspan="2">Minimum + 30%</td></tr>
<tr><td>C. web thickness (% diameter)</td><td colspan="2">C1</td><td colspan="2">C2</td></tr>
<tr><td>D. web taper</td><td colspan="2">D1</td><td colspan="2">D2</td></tr>
<tr><td>E. moment of inertia (in.)</td><td colspan="2">Standard</td><td colspan="2">Standard+X</td></tr>
<tr><td>F. tsdial rake</td><td colspan="2">F1</td><td colspan="2">F2</td></tr>
<tr><td>G. helix angle</td><td colspan="2">G1</td><td colspan="2">G2</td></tr>
<tr><td>H. axial rake</td><td colspan="2">H1</td><td colspan="2">H2</td></tr>
<tr><td>I. flute length (in.)</td><td colspan="2">Minimum</td><td colspan="2">Minimum+50%</td></tr>
<tr><td>J. point angle</td><td colspan="2">J1</td><td colspan="2">J2</td></tr>
<tr><td>L. point style</td><td colspan="2">Standard</td><td colspan="2">Strong</td></tr>
<tr><td rowspan="2">Noise factor</td><td colspan="4">Level</td></tr>
<tr><td colspan="2">-</td><td colspan="2">+</td></tr>
<tr><td>M. feed rate (in./min)</td><td colspan="2">10</td><td colspan="2">20</td></tr>
<tr><td>N. backup material</td><td colspan="2">Hard board</td><td colspan="2">phenolic</td></tr>
<tr><td>O. pcb material</td><td colspan="2">Epoxy</td><td colspan="2">Polyamide</td></tr>
<tr><td>P. number of layers</td><td colspan="2">8</td><td colspan="2">12</td></tr>
<tr><td>Q. four 2-oz layers</td><td colspan="2">No</td><td colspan="2">Yes</td></tr>
</table>

<Table 5> Cross Array and Failure Times in the Drill Bit Experiment

Control factors											Noise factors							
										M	-	-	-	-	+	+	+	+
										N	-	-	+	+	-	-	+	+
										O	-	-	+	+	+	+	-	-
A	D	B	C	F	G	H	I	E	J	L	P -	+	-	+	-	+	-	+
											Q -	+	+	-	+	-	-	+
0	-	-	-	-	-	-	-	-	-	-	1280	44	150	20	60	2	65	25
0	-	-	-	-	+	+	+	+	+	+	2680	125	120	2	165	100	795	307
0	+	+	+	+	-	-	-	-	+	+	2670	480	762	130	1422	280	670	130
0	+	+	+	+	+	+	+	+	-	-	2655	90	7	27	3	15	90	480
1	-	-	+	+	-	-	+	+	-	+	3000	440	480	10	1260	5	1720	3000
1	-	-	+	+	+	+	-	-	+	-	2586	6	370	45	2190	36	1030	16
1	+	+	-	-	-	-	+	+	+	-	3000	2580	20	320	425	85	950	3000
1	+	+	-	-	+	+	-	-	-	+	800	45	260	250	1650	470	1250	70
2	-	+	-	+	-	+	-	+	-	-	3000	190	140	2	100	3	450	840
2	-	+	-	+	+	-	+	-	+	+	3000	638	440	145	690	140	1180	1080
2	+	-	+	-	-	+	-	+	+	+	3000	970	180	220	415	70	2630	3000
2	+	-	+	-	+	-	+	-	-	-	3000	180	870	310	2820	240	2190	1100
3	-	+	+	-	-	+	+	-	-	+	3000	612	1611	625	1720	195	1881	2780
3	-	+	+	-	+	-	-	+	+	-	3000	1145	1060	198	1340	95	2509	345
3	+	-	-	+	-	+	+	-	+	-	3000	3000	794	40	160	50	495	3000
3	+	-	-	+	+	-	-	+	-	+	680	140	809	275	1130	145	2025	125

3. Regression Models for Failure Time Data

Since failure time t can only take nonnegative values, a standard linear regression modeling of t is not appropriate. The most obvious choice is the log transformation y=ln t. The following two failure time distributions are commonly used for reliability analysis: Lognormal density function

$$f(t)=\frac{1}{\sqrt{2\pi\sigma^2}}\,t^{-1}\exp\left(\frac{-(\ln t-\mu)^2}{2\sigma^2}\right), \qquad t>0$$

Weibull density function

$$f(t)=\frac{\phi}{\lambda}\left(\frac{t}{\lambda}\right)^{\phi-1}\exp\left[-\left(\frac{t}{\lambda}\right)^{\phi}\right], \qquad t>0$$

where λ the scale parameter and ϕ is the shape parameter

It is known that if t has a lognormal distribution, then y=ln t has a normal distribution N(μ, σ^2) and that if it has a Weibull distribution, then y=ln t has an extreme value distribution whose density is

$$f(y)=\frac{1}{\sigma}\exp\left[\frac{y-\mu}{\sigma}-\exp\left(\frac{y-\mu}{\sigma}\right)\right]$$

where $\mu=-\ln\lambda$ and $\sigma=\phi^{-1}$ become the location and scale parameters, respectively. Now a linear regression model can be applied to the transformed value of y as follows:

$$\begin{aligned} y_i &= \ln(t_i)=x_i^T\beta+\sigma\varepsilon_i \\ &=\beta_0+\beta_1 x_{i1}+\cdots+\beta_p x_{ip}+\sigma\varepsilon_i \quad i=1,2,\cdots,n \end{aligned}$$

where the { t_i } are the failure times, $x_i^T=(1,x_{i1,}\cdots,x_{ip})$ are the corresponding vectors of covariates, $\beta=(\beta_0,\beta_1,\ldots,\beta_p)^T$ are the vector of regression parameters, and σ is the scale parameter. The errors { ε_i } have independent standard extreme value distributions if the failure times follow a Weibull distribution and have independent N(0, 1) distributions if the failure times follow a lognormal distribution. The covariates correspond to an intercept, main effects, and possibly some interactions in reliability improvement experiments.

In reliability experiments, it is common to have censored data even though accelerated life testing is performed. In this case, if we can assume that life times follow some types of distribution such as a lognormal or Weibull distribution, we can handle censored data using likelihood approach to find some significant factors and interactions. In this paper, we will show the likelihood only for the situation in which right censored data are observed. But the likelihood approach can also be used for estimation in other cases. For the right censored case, by collecting the contributions from the failure and right-censored data, the likelihood can be shown to be

$$L(\beta,\sigma^2)=\prod_{i\in FAIL}(2\pi\sigma^2)^{-1/2}\exp\left(-\frac{1}{2}\right)[(y_i-x_i^T\beta)/\sigma]^2\times\prod_{i\in CEN}$$
$$\{1-\phi[(y_i-x_i^T\beta)/\sigma]$$

for the lognormal regression model and

$$L(\beta, \sigma^2) = \prod_{i \in FAIL} (1/\sigma) \exp[(y_i - x_i^T\beta)/\sigma] - \exp[(y_i - x_i^T\beta)/\sigma] \prod_{i \in CEN} \exp\{-\exp[(y_i - x_i^T\beta)/\sigma]\}$$

for the Weibull regression model, where FAIL and CEN denote the sets of observed failure times and censoring times, respectively.

Once the likelihood is obtained, maximum likelihood estimates can be obtained by maximizing the likelihood with respect to the parameters. Then they need to be compared with their standard errors to test the significance of the estimates. Standard errors can be obtained from the Fishers information matrix I or from the observed information matrix I_0 defined respectively as

$$I(\hat{\theta}) = E\left(\frac{-\delta^2 L(\hat{\theta})}{\delta\theta_i \delta\theta_j}\right) \Big|_{\theta=\hat{\theta}}$$

and

$$I_0(\hat{\theta}) = \frac{-\delta^2 L(\hat{\theta})}{\delta\theta_i \delta\theta_j} \Big|_{\theta=\hat{\theta}}$$

which are evaluated at the MLE $\hat{\theta}$.

Likelihood ratio tests provide an alternative method for assessing significance of the i^{th} parameter in θ:

$$\Gamma = -2 \ln \frac{L(\hat{\theta}_{(-i)})}{L(\hat{\theta})}$$

where $\theta_{(-i)}$ in the numerator are re the remaining parameters of the model from which the i^{th} covariate is dropped. Under the null hypothesis $\theta_i = 0$ Γ is distributed asymptotically as χ^2 with one degree of freedom.

The exact conditions for the existence of MLEs can be found in Silvapulle and Burridge (1986). In the reliability context, Hamada and Tse (1992) concluded that the estimability problem tends to occur if the fitted model has nearly the same number of parameters as the number of observations and especially if there is a single replicate. However, even if the MLEs may be infinite the likelihood is well defined and the likelihood ratio tests can be used (Clarkson and Jennrich, 1991).

4. Analysis of Data from Reliability Experiments

In this section, reliability data appeared in Section 2 are examined using the LIFEREG procedure of the SAS System. We do not use the signal-to-noise ratio based on the loss-function approach due to the presence of censored data. Instead, response-model approach is used since in reliability data analysis certain types of distributions such as lognormal or Weibull distributions for the failure times can be usually assumed.

4.1. Rolling Ball Bearing Experiment

A 2^3full factorial experiment without replicates has been performed to improve the reliability of the rolling ball bearing. Three factors were studied in which 8 ball bearings were manufactured at each of the 8 run settings. It is assumed that a Weibull distribution would be appropriate for the failure times. Hence, the Weibull regression model would be

$$y=\ln(t)=\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3+\sigma\varepsilon$$

the LIFEREG procedure of the SAS System was run to give the MLEs, likelihood ratio statistics and p values for the three main effects as in Table 6. The main effects Heat(X1) and Osculation(X2) are found to be significant at the significance level of 0.05.

<Table 6> MLEs, Liklihood Ratio(LR)s and p-values from the Rolling Ball Bearing Experimente

Effect	MLE	LR	p-value
intercept	3.608	1022.76	0.0001
Heat(X_1)	0.441	7.26	0.007**
Osculation(X_2)	0.445	7.29	0.007**
Cage(X_3)	0.099	0.80	0.372
σ	0.303		

4.2. Thermostat Experiment

A 12-run Plackett-Burman experimental design was run to improve the reliability of industrial thermostats. Eleven factors were studied in which 10 thermostats were manufactured at each of the 12 run settings. It is assumed that a lognormal distribution would be appropriate for the failure times. The lognormal regression model is appropriate for this data. The model would be

$$y = \ln(t) = \beta_0 + \beta_1 A + \beta_2 B + \beta_3 C + \beta_4 D + \beta_5 E + \beta_6 F + \beta_7 G + \beta_8 H + \beta_9 I + \beta_{10} J + \beta_{11} K + \sigma\varepsilon$$

The MLEs, likelihood ratio statistics and p values for the 11 main effects are given as in Table 7.

<Table 7> MLEs, Likelihood Ratio(LR)s and p-values from the Thermostat Experiment

Effect	MLE	LR	p-value
Intercept	6.354	7412.96	0.0001
A	-0.312	18.29	0.0001
B	0.221	9.18	0.0025
C	-0.319	19.06	0.0001
D	0.285	15.24	0.0001
E	-1.023	192.30*	0.0001
F	0.231	10.00	0.0016
G	-0.390	28.36	0.0001
H	-0.557	57.01*	0.0001
I	-0.332	20.65	0.0001
J	-0.277	14.42	0.0001
K	-0.352	23.30	0.0001
σ	0.764		

Effects E and H are found to be most significant. Since Plackett-Burman design has been run it is known that the main effects other than E and H have a 1/3 or -1/3 partial aliasing coefficient with EH. A narrow range of the other main effects strongly suggests that they are due to the presence of an EH interaction. Dropping the B main effect (the least significant from model) and adding EH to the model, the model would be

$$y = \ln(t) = \beta_0 + \beta_1 A + \beta_2 (EH) + \beta_3 C + \beta_4 D + \beta_5 E + \beta_6 F + \beta_7 G + \beta_8 H + \beta_9 I + \beta_{10} J + \beta_{11} K + \sigma\varepsilon$$

with the results as shown in Table8. Addition of EH to the model with the deletion of the B main effect confirms that E, H and EH are the only significant effects.

<Table 8> MLEs, Likelihood Ratio(LR)s and p-values from the Thermostat Experiment

Effect	MLE	LR	p-value
Intercept	6.354	7412.96	0.0001
A	-0.091	0.74	0.389
B	0.663	9.18	0.003**
C	-0.098	0.88	0.346
D	0.064	0.42	0.518
E	-0.023	192.30	0.0001**
F	0.010	0.01	0.922
G	-0.169	2.59	0.108
H	-0.557	57.01	0.0001**
I	-0.112	1.13	0.287
J	-0.056	0.28	0.596
K	-0.131	1.54	0.215
σ	0.764		

4.3. Drill Bit Experiment

11 control factors and 5 noise factors are considered in order to improve the reliability of drill bits. The experiment employed a cross array consisting of a 16-run control array and an 8-run noise array. Note that the testing was stopped after 3000 holes were drilled.

A Weibull regression model is entertained for this data set. In addition to the other control main effects, the AJ interaction can be estimated. For the noise array, only the main effects are considered. In addition, 55(=11 × 5) control-by-noise interactions can also be estimated. the LIFEREG procedure of the SAS System was run to give the MLEs, likelihood ratio statistics and p values for all those parameters as in Table 9. For the SAS program, see appendix. Therefore, we can tell that all of control main effects but B, E and G are significant and that some interactions between control and noise factors are also significant.

<Table 9> MLEs, Likelihood Ratio(LR)s and p-values from the Drill Bit Experiment

Factor	Effect	MLE	LR	p-value	p-value
	Intercept	6.208	9253.49	0.0001	**
Main Control Factor	A	0.746	59.60	0.0001	**
	B	0.134	2.411	0.121	
	C	0.198	5.09	0.024	**
	D	0.235	7.13	0.008	**
	E	-0.071	0.68	0.408	
	F	-0.222	6.65	0.010	**
	G	-0.134	2.34	0.126	
	H	-0.180	4.33	0.037	**
	I	0.314	13.01	0.0003	**
	J	0.226	10.93	0.001	**
	L	0.164	5.37	0.021	**
Main Noise Factor	M	-0.041	0.28	0.5943	
	N	-0.137	3.11	0.078	
	O	-0.854	169.64	0.0001	**
	P	-0.801	109.17	0.0001	**
	Q	0.153	3.95	0.047	**
Control*Contol	A*J	-0.148	2.32	0.128	
Control*Noise	A*M	-0.150	1.86	0.173	
	A*N	0.140	1.62	0.203	
	A*O	-0.089	0.85	0.355	
	A*P	0.236	4.40	0.036	**
	A*Q	0.194	3.09	3.079	
	B*M	-0.045	0.28	0.597	
	B*N	-0.082	0.92	0.337	
	B*O	-0.054	0.40	0.528	
	B*P	0.166	3.65	0.056	
	B*Q	-0.091	1.09	0.296	
	C*M	0.062	0.50	0.480	
	C*N	0.034	0.15	0.696	
	C*O	0.063	0.51	0.475	
	C*P	-0.129	2.08	0.149	
	C*Q	-0.093	1.08	0.298	
	D*M	0.049	0.31	0.575	
	D*N	0.024	0.07	0.788	
	D*O	0.058	0.43	0.513	
	D*P	0.331	13.63	0.0002	**
	D*Q	-0.086	0.92	0.339	
	E*M	0.081	0.86	0.354	
	E*N	0.056	0.41	0.522	
	E*O	-0.407	22.21	0.0001	**
	E*P	0.089	1.00	0.317	
	E*Q	0.055	0.38	0.536	

	F*M	0.008	0.01	0.930	
	F*N	0.027	0.10	0.755	
	F*O	-0.138	2.58	0.108	
	F*P	-0.076	0.75	0.388	
	F*Q	0.094	1.14	0.286	
	G*M	0.215	5.82	0.016	**
	G*N	-0.001	0.00	0.993	
	G*O	0.359	16.83	0.0001	**
	G*P	-0.157	2.96	0.085	
	G*Q	-0.322	12.52	0.0004	**
	H*M	0.044	0.26	0.609	
	H*N	0.002	0.00	0.985	
	H*O	-0.142	2.68	0.102	
	H*P	0.017	0.04	0.850	
	H*Q	-0.094	1.14	0.284	
	I*M	0.026	0.09	0.763	
	I*N	-0.006	0.00	0.950	
	I*O	-0.363	17.33	0.0001	**
	I*P	0.235	6.99	0.008	
	I*Q	0.122	1.88	0.171	
	J*M	-0.075	0.88	0.349	
	J*N	-0.133	2.80	0.094	
	J*O	-0.028	0.17	0.676	
	J*P	0.129	2.65	0.104	
	J*Q	0.016	0.04	0.844	
	L*M	0.180	4.96	0.026	
	L*N	0.052	0.42	0.519	
	L*O	0.162	5.60	0.018	
	L*P	0.002	0.00	0.978	
	L*Q	-0.027	0.11	0.7535	
Scale	σ	0.620			

5. Conclusion

Design of experiment has been used to improve quality. But it can also be used to maintain or improve reliability as shown in the above examples. In design and analysis of experiments for reliability, lifetimes may not be normally distributed. In this case, the usual F test for the analysis of variance is not valid. In addition, we may have a censored data in reliability experiment. This type of incomplete information must be accounted for when we analyze such data. A Weibull or lognormal regression is a model that can take into account nonnormality and consoring. In this paper, Weibull and lognormal regression models have been used to analyze data sets from the reliability experiments. This approach has been found to be useful in identifying significant factors in reliability data.

In this paper, we have used only Weibull or lognormal regression model in order to identify significant factors. However, since we assume that the distribution of lifetimes is Weibull or lognormal, we can also get appropriate measures for mean and standard deviation in each run even when we have censored data and, after getting signal-to-noise ratio from each run, we can identify significant factors. Therefore, it would be of interest to compare the results from the two procedures.

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Appendix : A SAS program for Analysis of the Drill Bit Experiment

```
data one;
input a d b c f g h i e j l  t1-t8 @@;
a=(2/3)*a-1;
cards;
0 -1 -1 -1 -1 -1 -1 -1 -1 -1 1  1280 44 150  20 60  2  65  25
0 -1 -1 -1 1  1  1  1  1  1  1  2680  125  120  2 165 100  795  307
0 1  1  1  1 -1 -1 -1 -1  1  1 2670  480  762  130 1422 280 670  130
0  1  1  1  1  1  1  1  1 -1 -1  2655  90  7  27  3  15  90  480
1 -1 -1  1  1 -1 -1  1  1 -1  1 3000 440  480  10 1260  5  1720  3000
1 -1 -1  1  1  1  1 -1 -1  1 -1 2586 6 370  45  2190  36  1030  16
1  1  1 -1 -1 -1 -1  1  1  1 -1 3000 2580  20 320 425  85  950  3000
1  1  1 -1 -1  1  1 -1 -1 -1  1 800  45 260 250 1650  470  1250  70
2 -1  1 -1  1 -1  1 -1  1 -1 -1 3000 190 140  2 100  3  450  840
2 -1  1 -1  1  1 -1  1 -1  1  1 3000 638 440 145 690  140  1180  1080
2  1 -1  1 -1 -1  1 -1  1  1  1 3000 970 180 220 415  70  2630  3000
2  1 -1  1 -1  1 -1  1 -1 -1 -1 3000 180 870 310 2820 240  2190  1100
3 -1  1 1 -1 -1  1  1 -1 -1 1  3000 612 1611 625 1720 195  1881  2780
3 -1  1 1 -1  1 -1 -1  1  1 -1  3000 1145 1060 198 1340 95 2509  345
3  1 -1 -1 1 -1  1  1 -1  1 -1 3000 3000 794 40 160  50  495  3000
3  1 -1 -1 1 1 -1 -1  1 -1  1 680  140 809 275  1130  145  2025  125
;
data one1;  set one;  m=-1; n=-1; o=-1; p=-1; q=-1;  t=t1; run;
data one2;  set one;  m=-1; n=-1; o=-1; p=1;  q=1;   t=t2; run;
data one3;  set one;  m=-1; n=1; o=1; p=-1;  q=1;    t=t3; run;
data one4;  set one;  m=-1; n=1; o=1; p=1; q=-1;     t=t4; run;
data one5;  set one;  m=1; n=-1; o=1; p=-1; q=1;     t=t5; run;
data one6;  set one;  m=1; n=-1; o=1; p=1; q=-1;     t=t6; run;
data one7;  set one;  m=1; n=1; o=-1; p=-1; q=-1;    t=t7; run;
data one8;  set one;  m=1; n=1; o=-1; p=1; q=1;      t=t8; run;

data two;
set one1 one2 one3 one4 one5 one6 one7 one8;  drop t1-t8;
if t=3000 then censor=1; else censor=0;
aj=a*j;
am=a*m; an=a*n; ao=a*o; ap=a*p; aq=a*q;
bm=b*m; bn=b*n; bo=b*o; bp=b*p; bq=b*q;
cm=c*m; cn=c*n; co=c*o; cp=c*p; cq=c*q;
```

```
dm=d*m; dn=d*n; do=d*o; dp=d*p; dq=d*q;
em=e*m; en=e*n; eo=e*o; ep=e*p; eq=e*q;
fm=f*m; fn=f*n; fo=f*o; fp=f*p; fq=f*q;
gm=g*m; gn=g*n; go=g*o; gp=g*p; gq=g*q;
hm=h*m; hn=h*n; ho=h*o; hp=h*p; hq=h*q;
im=i*m; in=i*n; io=i*o; ip=i*p; iq=i*q;
jm=j*m; jn=j*n; jo=j*o; jp=j*p; jq=j*q;
lm=l*m; ln=l*n; lo=l*o; lp=l*p; lq=l*q;
run;

proc lifereg data=two;
model t*censor(1)=  a b c d e f g h i j l  m n o p q
am an ao ap aq   bm bn bo bp bq   cm cn co cp cq   dm dn do dp dq
em en eo ep eq   fm fn fo fp fq    gm gn go gp gq   hm hn ho hp hq
im  in io  ip  iq   jm jn jo jp jq     lm  ln lo  lp  lq ;
run;
```