Dynamic Process Capability Indices

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Abstract

Process capability indices as an important kind of indices are intended to provide single-number assessments of the inherent process capability to meet specification limits on quality characteristic(s) of interest. In this paper the condition for the application of process capability indices is analyzed. On the basis of process capability indices, dynamic process capability indices as a new kind of indices to show the current process capability are discussed and the condition for the application of dynamic process capability indices is exhibited. Comparison between process capability index and dynamic process capability index and comparison between D_p and D_{pk} are made and the conclusions provide the approach for process control. According to the requirement of process capability indices provided by customer, quality control based on process capability indices dynamic process capability indices is discussed.

Key words: Process capability indices, Dynamic process capability indices, Statistical process control.

1. Introduction

For the near zero-nonconformity processes, the fraction non-conforming is so low that it becomes decreased into the gratitude of ppm(10^{-6}), even ppb(10^{-9}) from 10^{-2} in the past, i.e. a large amount of items are conforming, and a non-conforming item appears occasionally. Monitoring process capability is an alternative approach for monitoring the fraction non-conforming. As we know, process capability indices signed as PCIs are intended to provide single-number assessments of the inherent process capability to meet specification limits on quality characteristic(s) of interest for the process being operated in a state of statistical control. In practice, there are some processes temporarily uncertain to their state in control or not and some processes being operated in a state out of control, for example, the startup of a new production system or a passing phase from old production level to new level and so on.

However we need to show their process capability for customers or practitioners. Then a new kind of indices called dynamic process capability indices has been proposed to provide single-number assessments of the current process capability to meet specification limits. In this paper, dynamic process capability indices signed as DPCIs will be analyzed compared with process capability indices and quality control based on process capability indices and dynamic process capability indices will be discussed.

2. Process Capability Indices

Manufacturing processes' product characteristics are often measured in order to determine that these processes conform to the specification limits. Consider the common case in which lower and upper specification limits (*LSL, USL*) are evenly laid in either side of the nominal target and the measured characteristics have a normal distribution with mean, μ, and standard deviation, σ. A measurement is conforming if it falls within the specification limits. There are various indices that are used to describe the process capability relative to the specification limits. The two most common ones are C_p and C_{pk}.

C_p is defined as follows, in which $T = USL - LSL$.

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{T}{6\sigma} \tag{1}$$

However, the location or the centering of the process is not considered. It would be possible to have any percentage of values outside the specification limits with a high C_p value. For this reason, it is important to consider the scaled distance between the nominal target of specification limits and the mean of the measured characteristics, μ. C_{pk} is defined as follows, in which $K = \frac{\varepsilon}{T/2}$, where P_U is the rate of measurements falling outside *USL*.

$$C_{pk} = (1 - K)C_p \tag{2}$$

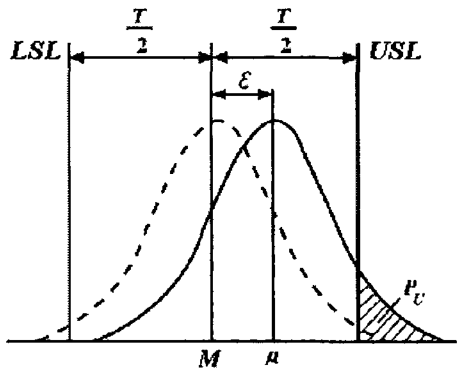

Figure 1. Nominal target of specification limits, *M*, different from μ

When only one side specification limit of product characteristics is considered, the upper process capability index C_{pU} or the lower process capability index C_{pL} is defined as follows,

$$C_{pU} = \frac{USL - \mu}{3\sigma} \tag{3}$$

$$C_{pL} = \frac{\mu - LSL}{3\sigma} \tag{4}$$

3. Condition for the application of process capability indices

The function of quality control system is to provide a statistical signal when assignable causes of variation are present. Though continuous determined efforts, the systematic elimination of assignable causes of excessive variation brings the process into a state of statistical control. Once the process is operating in statistical control, its performance is predictable and its capability to meet the specifications can be assessed.

Process capability indices are determined by the total variation that comes from common causes, that is, the minimum variation that can be achieved after all assignable causes have been eliminated. Process capability indices represent the performance of the process itself, as demonstrated when the process is being operated in a state of statistical control. As such, the process must first be brought into statistical control before its capability indices can be assessed. Thus, the assessment of process capability indices begins after special causes have been identified, analyzed, corrected and prevented from recurring and the ongoing control charts reflect a process that has remained in statistical control. In general, the distribution of the process output is compared with the engineering specifications, to see whether these specifications can consistently be met.

4. Dynamic Process Capability Indices

The prerequisite of process capability indices is process being operated in statistical control. But in practice, the current process capability indices also need to be estimated. Just for this purpose, dynamic process capability indices have been proposed and defined as the specification width divided by the current process standard deviation, irrespective of process centering, expressed as D_p

$$D_p = \frac{USL - LSL}{6\hat{\sigma}_D} = \frac{T}{6\hat{\sigma}_D} \quad (5)$$

where $\hat{\sigma}_D$ is the estimation of current process standard deviation.

D_{pk} is dynamic capability process index that accounts for process centering as follow,

$$D_{pk} = (1 - K)D_p \quad (6)$$

where $K = \frac{|\hat{\mu}_D - M|}{T/2}$ and $\hat{\mu}_D$ is the estimation of current process mean.

D_{pU} is dynamic capability process index that only accounts for the upper specification limit as follow,

$$D_{pU} = \frac{USL - \hat{\mu}_D}{3\hat{\sigma}_D} \quad (7)$$

D_{pL} is dynamic capability process index that only accounts for the lower specification limit as follow,

$$D_{pL} = \frac{\hat{\mu}_D - LSL}{3\hat{\sigma}_D} \tag{8}$$

5. Numeration of process capability indices

If mean μ and standard deviation σ of the measured characteristics are known, process capability indices can be obtained directly, which reveal process capability to meet the specifications when the process is being operated in statistical control. If mean μ and standard deviation σ of the measured characteristics are unknown, the corresponding process must be brought into the state of statistical control at first, then mean μ and standard deviation σ of the measured characteristics can be obtained according to the estimators of process standard variation and process mean in statistical control. Indeed process standard variation in control is intended to provide single-number estimate of total variation that comes from common causes when all assignable causes have been eliminated. Process being operated in a state of statistical control is the fundamental of process capability indices.

When process is judged to be operated in control according to the criteria of statistical control, the estimator of process mean can be obtained from the average of sample average, $\bar{\bar{x}}$, as shown in Table 1, and the estimator of process standard deviation can be obtained from $\bar{R}/d_2$ or $\bar{s}/c_4$ where $\bar{R}$ and $\bar{s}$ are the average of sample range and sample standard deviation respectively as shown in Table 1, d_2 and c_4 are divisors influenced by the number of observations per sample n and can be obtained from ISO 8258:1991 directly, for example, d_2=2.326 and c_4= 0.940 when n = 5. In Table 1, m is the number of sample. As we know, if

Table 1 Basic statistical analysis of samples

Sample No.	Sample Observations				Sample Average $\bar{x}_t = \frac{1}{n}\sum_{i=1}^{n} x_{t,i}$	Sample Range $R_t = x_{\max} - x_{\min}$	Sample Standard Deviation $s_t = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{t,i} - \bar{x}_t)^2}$
1	$x_{1,1}$	$x_{1,2}$	⋯	$x_{1,n}$	$\bar{x}_1$	R_1	s_1
2	$x_{2,1}$	$x_{2,2}$	⋯	$x_{2,n}$	$\bar{x}_2$	R_2	s_2
⋯	⋯	⋯	⋯	⋯	⋯	⋯	⋯
m	$x_{m,1}$	$x_{m,2}$	⋯	$x_{m,n}$	$\bar{x}_m$	R_m	s_m
Total					$\bar{\bar{x}} = \frac{1}{m}\sum_{t=1}^{m}\bar{x}_t$	$\bar{R} = \frac{1}{m}\sum_{t=1}^{m} R_t$	$\bar{s} = \frac{1}{m}\sum_{t=1}^{m} s_t$

we want to evaluate the process state, at least 25 samples should be collected.

6. Numeration of dynamic process capability indices

Process capability indices are tools to provide single-number assessments of the inherent process capability to meet specification limits on quality characteristic(s) of interest. Dynamic process capability indices have been proposed to match with process capability indices to exhibit the current process capability and its calculation is similar. The estimator of current process mean is obtained from the average of sample average, $\bar{\bar{x}}$, and the estimator of current process standard deviation from $\bar{R}/d_2$ or $\bar{s}/c_4$ as shown in Table 2 where m_D is the number of sample collected without any restriction and symbol * means that sample observations are influenced by assignable causes.

When process is operated in a state of statistical control according to the criteria of statistical control, $\bar{\bar{x}}$ and $\bar{R}/d_2$ or $\bar{s}/c_4$ can be used as the estimators of process mean and process standard variation to show the normal process capability without assignable causes. When we are unable to judge the process in control or out of control, $\bar{\bar{x}}$ and $\bar{R}/d_2$ or $\bar{s}/c_4$ are only the estimators of current process mean and current process standard variation to show the current process capability and it is possible for the current process to exist some assignable causes not to be eliminated. This is the main difference between process capability indices and dynamic process capability indices.

Table 2 Basic statistical analysis of samples

Sample No.	Sample Observations				Sample Average $\bar{x}_t = \frac{1}{n}\sum_{i=1}^{n} x_{t,i}$	Sample Range $R_t = x_{\max} - x_{\min}$	Sample Standard Deviation $s_t = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{t,i} - \bar{x}_t)^2}$
1	$x_{1,1}$	$x_{1,2}$	$\cdots$	$x_{1,n}$	$\bar{x}_1$	R_1	s_1
2	$x_{2,1}$	$x_{2,2}$	$\cdots$	$x_{2,n}$	$\bar{x}_2$	R_2	s_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
K	$x^*_{k,1}$	$x^*_{k,2}$	$\cdots$	$x^*_{k,n}$	$\bar{x}^*_k$	R^*_k	s^*_k
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
m_D	$x_{m_D,1}$	$x_{m_D,2}$	$\cdots$	$x_{m_D,n}$	$\bar{x}_{m_D}$	R_{m_D}	s_{m_D}
Total					$\bar{\bar{x}} = \frac{1}{m_D}\sum_{t=1}^{m_D} \bar{x}_t$	$\bar{R} = \frac{1}{m_D}\sum_{t=1}^{m_D} R_t$	$\bar{s} = \frac{1}{m_D}\sum_{t=1}^{m_D} s_t$

Note: Symbol * means that sample observations are influenced by assignable causes.

7. Condition for the application of dynamic process capability indices

However, we have to know the current process capability no matter whether the process is in control or out of control. Some situations are described as follows,

- ☐ If a new product is being trial-manufactured and the process is being influenced by many assignable causes and not in statistical control, the designers need to know the current process capability.
- ☐ During the procedure of assignable causes being identified, analyzed, corrected and prevented from recurring, it is required to provide the current process capability for customer.
- ☐ Small lot production is quite popular. If the total number of samples is less than 25, that is the basic requirement for sample number in order to make a judgment whether a process in control or not, the process state is unable to be determined and process capability indices are unable to be assessed.
- ☐ Monitoring a process with control chart is usually divided into two stages. The first is the stage of control chart for analysis and the second is that of control chart for control. In the first one, the aim is to bring the process into the state of statistical control. The aim of the second stage is to keep the process in statistical control. In the first stage, process capability indices cannot be assessed. Only in the second stage, the process capability in statistical control can be estimated, but in some cases the real-time process capability also needs to be provided. No matter which stage of the process is located, dynamic process capability indices can provide the real-time process capability in addition to process capability indices.
- ☐ If statistical tools such as control charts are not adopted to monitor a process, it is impossible to know whether the process in a state of statistical control or not.

Dynamic process capability indices are tools to provide the current process capability to meet the specifications regardless of its operational state.

8. Quality control based on Process Capability Indices and Dynamic Process Capability Indices

Monitoring process capability is an alternative approach of monitoring the nonconforming fraction. For the near zero-nonconformity processes, the nonconforming fraction is so low that it becomes decreased into the gratitude of ppm(10^{-6}), even ppb(10^{-9}) from 10^{-2} in the past. As we know, even the most well-controlled processes experience some shifts of the process mean due to changes in equipment, operators, environmental conditions and incoming materials and so on. Such shifts can be as much as 1.5 standard deviations. C_{pk} provides a measure to account for such shifts, that is, the location or the centering of the processes. If shifts don't happen, $C_p = C_{pk} = 2$ means that 99.9999998 percent of all parts produced would be nonconformity free, that is, the nonconforming fraction is 2 parts per billion. If shifts as much as 1.5 standard deviations do normally occur,

from $$C_p = \frac{T}{6\sigma} = 2$$

we have $$T = 12\sigma$$

then $$K = \frac{\varepsilon}{T/2} = \frac{1.5\sigma}{12\sigma/2} = 0.25$$

$$C_{pk} = (1-K)C_p = (1-0.25)\times 2 = 1.5$$

$C_p = 2$, $C_{pk} = 1.5$ means that 99.99966 percent of all parts produced would be nonconformity free, that is, the nonconforming fraction is 3.4 parts per million. Monitoring process capability is an alternative and better approach of monitoring the fraction non-conforming.

Process capability indices provide the standard values that indicate the inherent process capability to meet specification limits when process is being operated in a state of statistical control, the state of only common causes without assignable causes. From the comparison of process capability index and dynamic process capability index, we can conclude,

(1) If $D_p < C_p$, the current process capability is lower than the inherent process capability. It means that the current process is not being operated in the state of statistical control and assignable causes have induced the occurrence of excessive variation. The assignable causes need to be identified, analyzed, corrected and prevented from recurring in order to raise the current process capability to the level of the inherent process capability.

(2) If $D_p \approx C_p$, the current process capability is approximate to the inherent process capability. It means that the current process is being operated in the state of statistical control.

(3) If $D_p > C_p$, the current process capability is higher than the inherent process capability. It means that common causes of variation of the current process have been decreased and management decision has been made to allocate resources to eliminate or correct the common causes so that the process has been improved. Thus control chart for analysis should be used to confirm the process in statistical control and then the inherent process capability will be improved to the new level.

From the comparison of D_p and D_{pk}, we can conclude,

(1) If D_{pk} is approximate to D_p, it means the shift of the process mean is little.

(2) If D_{pk} is much lower than D_p, it means that the process mean shifts away from the nominal target. Some decisions should be made to correct the equipment, operators and environmental conditions to adjust the process mean back to the nominal target of specification limits.

It can be concluded that comparison between process capability index and dynamic process capability index and comparison between D_p and D_{pk} provide the evidence for making decision of process control. The aim of implementing quality control is to realize customer satisfaction. Some requirements of customer may be simplified as the measured characteristics' process capability indices. If the requirement to process capability indices is provided by customer, then the required mean μ and standard deviation σ of the measured characteristics can be induced and quality control should be operated under customer's requirement. Process capability indices can provide single-number assessments of the inherent process capability to meet specification limits for the process in a state of statistical control and it also can be proposed by customer and used as the benchmark of process control. For example, if customers give their requirement to process capability indices on quality characteristics of interest as follow,

$C_p = 1.5 \qquad C_{pk} = 1$

it means

$$C_p = \frac{T}{6\sigma} = 1.5$$

Then the requirement to standard deviation σ can be found,

$$T = 9\sigma$$

$$\sigma = \frac{T}{9}$$

And $C_{pk} = (1-K)C_p = (1-\frac{\varepsilon}{T/2})C_p = (1-\frac{|\mu - M|}{T/2})C_p$

we have

$$|\mu - M| = 1.5\sigma$$

$$\mu = M \pm 1.5\sigma$$

So the requirement to the mean μ and standard deviation σ of the measured characteristics is known. According to μ and σ, quality control can be operated directly.

If we consider process capability indices of quality characteristics supplied by customers as nominal values, comparison with dynamic process capability indices can provide the current process operational state.

(1) $D_p < C_p$ means the deviation of current process is much larger than customer's requirement. Some strict quality inspection plans have to be used for picking out the item not specified customer's deviation requirement. Certain approaches

for quality improvement should be adopted to decrease the deviation of process.

(2) $D_p \geq C_p$ means the deviation of current process can satisfy the customer's requirement. If $D_{pk} < C_{pk}$, some strict quality inspection plans should be used for picking out the item not specified customer's mean requirement. Certain management decision must be made to decrease the shift of process mean. If $D_{pk} \geq C_{pk}$, it means the mean of current process can satisfy the customer's requirement. The current process can provide items required by customer.

Above all, no matter process capability indices used as the single-number assessments of the inherent process capability or as nominal values supplied by customer, comparison between process capability index and dynamic process capability index and comparison between D_p and D_{pk} will discover the essential of current process and point out the approach of quality improvement.

9. Conclusion

Monitoring process capability is an alternative approach of monitoring the fraction non-conforming especially for the near zero-nonconformity processes. In this paper, process capability indices and dynamic process capability indices are discussed and conditions for the application of each kind of indices are analyzed. Dynamic process capability indices are compared with process capability index to provide technical support for judging the state of process and give approach to process control. Dp is compared with Dpk to analyze the process mean shift. At last, according to the process capability indices supplied by customer, quality improvement based on process capability indices and dynamic process capability indices is discussed.

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