# Beyond robust design: an example of synergy between statistics and advanced engineering design

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## Abstract

Higher efficiency and effectiveness of Research & Development phases can be attained using advanced statistical methodologies. In this work statistical methodologies are combined with a deterministic approach to engineering design. In order to show the potentiality of such integration, a simple but effective example is presented. It concerns the problem of optimising the performances of a paper helicopter. The design of this simple device is not new in quality engineering literature and has been mainly used for educational purposes. Taking full advantage of fundamental engineering knowledge, an aerodynamic model is originally formulated in order to describe the flight of the helicopter.

Screening experiments were necessary to get first estimates of model parameters. Subsequently, deterministic evaluations based on this model were necessary to set up further experimental phases needed to search for a better design. Thanks to this integration of statistical and deterministic phases, a significant performance improvement is obtained. Moreover, the engineering knowledge turns out to be developed since an explanation of the "why" of better performances, although approximate, is achieved. The final design solution is robust in a broader sense, being both validated by experimental evidence and closely examined by engineering knowledge.

**Key words:** Robust Design, Quality Engineering

## 1. Introduction

In engineering design problems whereas no specific model is currently available, two approaches are normally practised:

- exploitation of deterministic models used in similar problems;
- utilisation of experimentation based on statistical methodologies.

Both are currently practised in industry and in research contexts. However, the integration of experimental results with deterministic modelling is often difficult to attain since they come from two totally separate processes. The benefits of the above mentioned processes are well known. In fact while statistical methodologies can produce optimal solutions in short time, explaining "how" the phenomenon occurs, solutions based on deterministic models can be used to simulate the mechanics of the phenomenon, explaining "why" it occurs (Box, 1976).

However, the risks of a poor integration of engineering and statistical competencies are twofold. Firstly, the inappropriate use of statistical tools leads to invalid or unrealistic conclusions (Nair, 1992); secondly, the correct use of Statistics without a thorough knowledge of the phenomenology under study can lead to useless conclusions (compared with engineering knowledge) (Box & Liu, 1999).

In this work the results of a study aimed at combining, in an original way, an empirical approach and a deterministic one are proposed. The example concerns a simple device: the paper helicopter. In particular the design objective is the optimisation of flight time performance. The paper helicopter is one of the most famous educational devices used to illustrate the application of statistical methodologies to experimentation (Box, 1992; Box, 1993; Santy & Einwalter, 1998).

In fact the simplicity of prototype making and operating and the rapidity with which experimental runs can be performed have made the paper helicopter a valuable tool, still often proposed in recent literature (Lanzotti, 1995; Box & Liu, 1999).

Motivated by these facts and fostered by engineering knowledge, we proceeded not only to employ robust design methodologies but also to build and formulate a completely original deterministic model aimed at understanding the aerodynamic phenomenon hidden behind the apparently "simple" helicopter fall.

By means of this model a numerical simulation of the physical phenomenon under study is proposed, in a kind of virtual lab (Phadke, 1989; Koehler & Owen, 1996). The logical flow of experimental and numerical phases discussed in the following sections is presented in Figure 1.

The phases can be briefly summarised as:

- screening phase and experiments on real prototype aimed at selecting the significant parameters (section 2);
- design and formulation of the deterministic model and computer experiments on virtual prototype for design improvement (section 3, 4);
- experimental phase on real prototype aimed at validating the deterministic model and verify design improvement (section 5).

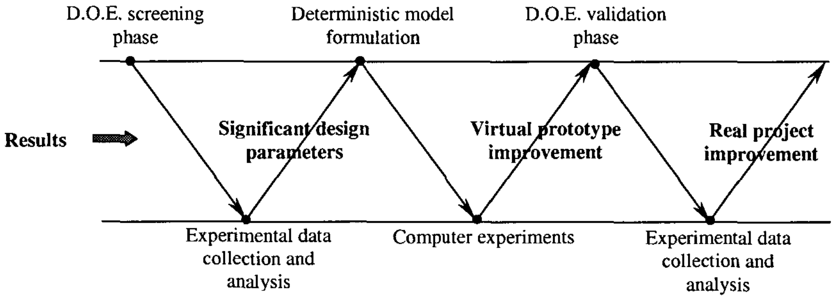


**Figure 1.** Alternation of statistical and deterministic phases and relative results.

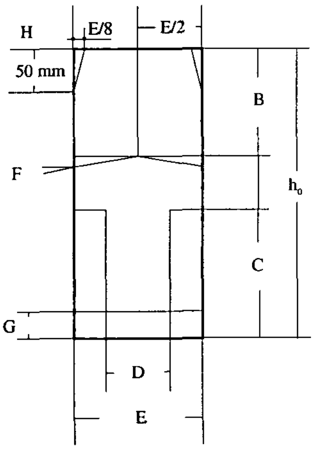
## 2. First experimental phase: screening

In order to have a starting reference, the performances of the paper helicopter prototype originally proposed by Neu in (Mander et al., 1967) are assumed as benchmark. The experiment consists in releasing the prototype in a free fall. The method of releasing is standardised. The experimental runs are performed in lab, fixing a release height of 4.30 m. In (Lanzotti, 1995) the observed flight times, defined as duration from the release instant to the impact to ground, are presented: average time $\bar{t}$= 4.58 sec; $s^2$= 0.06 sec$^2$.

Eight design parameters are selected for the screening phase and two levels are chosen for each of them. The constructional scheme is illustrated in the left-hand side of Figure 2; $h_0$ is a fixed parameter equal to the height of an A4 format sheet. On the right-hand side of the figure, design parameters description and levels are also presented.

Being interested, in this phase, only to recognise which of the selected design parameters significantly affect the performance, a fractional factorial design $2^{8-4}_{IV}$ is adopted. The design array is obtained using the confounding pattern: $E = +BCD$ $F = +ACD$ $G = +ABC$ $H = +ABD$.

The experimental design and the observations are presented in Table 1. Five replications are performed for each run, by ensuring a complete randomisation during the execution of the experiments. The rather small experimental variability (pooled variance $s^2_{pool}$= 0.04) gives confidence of the accuracy of data collection. The estimated main effects and Pareto chart, presented in Figure 3, show that the parameters B, E and H are the most important. The analysis of residuals evaluated by the simplest ANOVA model (Figure 4) and the correct randomisation assure the applicability of the analysis of variance (Table 2), which confirms the significance of the effects of B, E and H at the chosen significance level 0.99.



| | Parameter description | level "-" | level "+" |
|---|---|---|---|
| **A** | Paper density (g/m)$^2$ | 60 | 80 |
| **B** | Blade length (mm) | 108 | 130 |
| **C** | Body length (mm) | 93 | 121 |
| **D** | Body width ratio | 0.33 | 0.43 |
| **E** | Width (mm) | 65 | 98 |
| **F** | Geometrical setting angle (degrees) | 0 | 10 |
| **G** | Fold | No | Yes |
| **H** | Angle of twist | No | Yes |

**Figure 2.** Constructional scheme of the paper helicopter and description of design parameters and levels adopted for the screening phase.

**Table 1.** Experimental results of the screening phase.

| Run | A | B | C | D | E | F | G | H | O$t_{ij}$ (sec)bservations | | | | | $\overline{t}$ | $s^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | - | - | - | - | - | - | - | - | 4.65 | 4.35 | 4.25 | 4.40 | 4.20 | 4.37 | 0.03 |
| 2 | + | - | - | - | - | + | + | + | 3.45 | 3.65 | 3.60 | 3.75 | 3.50 | 3.59 | 0.01 |
| 3 | - | + | - | - | + | - | + | + | 2.90 | 3.25 | 3.00 | 2.85 | 3.40 | 3.08 | 0.06 |
| 4 | + | + | - | - | + | + | - | - | 3.75 | 4.20 | 3.80 | 3.65 | 4.45 | 3.97 | 0.12 |
| 5 | - | - | + | - | + | + | + | - | 3.35 | 3.75 | 3.60 | 3.60 | 3.40 | 3.54 | 0.03 |
| 6 | + | - | + | - | + | - | - | + | 3.75 | 3.75 | 3.80 | 3.35 | 3.35 | 3.60 | 0.05 |
| 7 | - | + | + | - | - | + | - | + | 4.50 | 4.20 | 4.45 | 4.35 | 4.40 | 4.38 | 0.01 |
| 8 | + | + | + | - | - | - | + | - | 4.90 | 4.85 | 5.05 | 5.20 | 5.05 | 5.01 | 0.02 |
| 9 | - | - | - | + | + | + | - | + | 3.75 | 3.35 | 3.30 | 3.45 | 3.35 | 3.44 | 0.03 |
| 10 | + | - | - | + | + | - | + | - | 3.30 | 3.65 | 3.65 | 3.50 | 3.45 | 3.51 | 0.02 |
| 11 | - | + | - | + | - | + | + | - | 5.35 | 5.20 | 5.15 | 5.15 | 5.15 | 5.20 | 0.01 |
| 12 | + | + | - | + | - | - | - | + | 3.60 | 4.10 | 3.95 | 4.15 | 3.55 | 3.87 | 0.08 |
| 13 | - | - | + | + | - | - | + | + | 3.65 | 3.60 | 3.40 | 3.55 | 3.35 | 3.51 | 0.02 |
| 14 | + | - | + | + | - | + | - | - | 3.85 | 4.30 | 3.90 | 4.30 | 4.30 | 4.13 | 0.05 |
| 15 | - | + | + | + | + | - | - | - | 4.35 | 3.65 | 4.05 | 3.90 | 3.75 | 3.94 | 0.08 |
| 16 | + | + | + | + | + | + | + | + | 4.15 | 3.80 | 4.00 | 3.90 | 3.85 | 3.94 | 0.02 |

| Parameter | Main effect |
|---|---|
| Paper density | *A* = 0.02 ± 0.04 |
| Blade length | *B* = 0.46 ± 0.04 |
| Body length | *C* = 0.13 ± 0.04 |
| Body width ratio | *D* = 0.00 ± 0.04 |
| Width | *E* = -0.63 ± 0.04 |
| Geometrical setting angle | *F* = 0.16 ± 0.04 |
| Fold | *G* = -0.04 ± 0.04 |
| Angle of twist | *H* = -0.53 ± 0.04 |

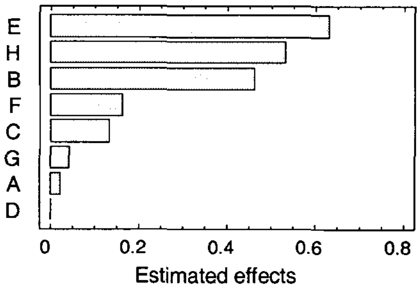


**Figure 3.** Estimated main effects and Pareto chart.

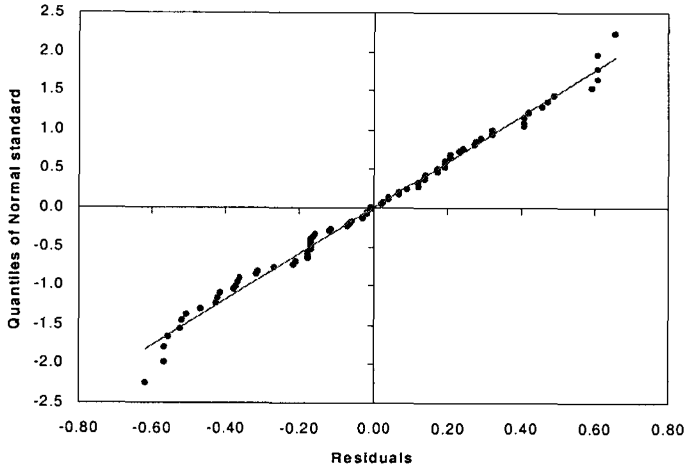


**Figure 4.** Normal Probability Plot of residuals.

**Table 2.** Analysis of variance for the screening phase

| Source of variation | SS | df | MS | F ratio | p-value |
|---|---|---|---|---|---|
| *A* | 0.008 | 1 | 0.008 | 0.068 | 0.795 |
| *B* | 4.278 | 1 | 4.278 | 36.552 | 0.000 |
| *C* | 0.325 | 1 | 0.325 | 2.778 | 0.100 |
| *D* | 0.000 | 1 | 0.000 | 0.000 | 1.000 |
| *E* | 7.938 | 1 | 7.938 | 67.822 | 0.000 |
| *F* | 0.528 | 1 | 0.528 | 4.512 | 0.037 |
| *G* | 0.032 | 1 | 0.032 | 0.273 | 0.603 |
| *H* | 5.671 | 1 | 5.671 | 45.454 | 0.000 |
| Residual | 8.310 | 71 | 0.117 | | |
| Total | 27.091 | 79 | | | |

The first experimental phase allows to select the design parameters more influent on the performance and the optimal levels of all the parameters in order to improve the flight time. Moreover, the first experimental phase gives a clear indication of the way of modifying parameters in order to improve the project. An outline scheme is presented in Table 3. Regarding the significant continuous parameters, B and E, it is possible to forecast an improvement of the performance, by increasing (▲) or decreasing (▼) the level of the corresponding design parameter with respect to the level previously experimented. Regarding the significant binary type parameter, H, we obtained the best level to be left unaltered (■) in further experimental phases.

## 3. Deterministic model design, formulation and calibration

The experimental observation of the paper helicopter free fall permits to notice the coexistence of two phases of the flight: an incipient phase (*transient*) in which the helicopter starts spinning, followed by a second phase (*stable condition*) characterised by approximately constant velocities.

Similar phenomena of real technical interest, connected for instance to the fall of helicopters as consequence of a breakdown, are studied in the Aerodynamics of Helicopters (Gessow & Myers, 1967), but they cannot be directly exploited for the phenomenon under study. In the case of the paper helicopter the initial angular velocity is zero and the rotary motion starts due to the particular antisymmetric shape (Figure 5); hence the angular velocity is one of the unknown time functions to be determined.

Three hypotheses are formulated for the model development:

i) the motion of the paper helicopter, released in free fall, is a rotary/translatory motion which develops about a spin barycentric axis, integral with the body, which stands vertical during the motion ($z$-axis);

ii) the blades can rigidly and frictionless swing about their bending axis which constitutes an ideal constraint;

iii) the parts of the helicopter are indeformable.

**Table 3.** Significant parameters, optimal levels and relative decision aimed at design improvement.

| | Parameter description | level | Value | Decision |
|---|---|---|---|---|
| B | Blade length | +1 | 130 mm | ▲ |
| E | Width | -1 | 65 mm | ▼ |
| H | Angle of twist | -1 | no | ■ |

From the Figure 5.a we can observe that, in a generic instant $t$ and under the hypotheses i), ii) and iii), the exact configuration of the helicopter is known if the position of the centre of gravity along the $z$ axis ($z_\Gamma$), the angle which the blade forms with the horizontal plane (*flapping angle* $\psi$) and the angular velocity ($\Omega$) about the $z$-axis are known. Hence, the rotary/translatory motion is completely set if we know the three time functions $z_\Gamma(t)$, $\psi(t)$ and $\Omega(t)$ (Figure 5.b and Figure 6).

An initial value problem consisting of a system of differential equations and a set of appropriate initial conditions is built in order to calculate the unknown functions. Three differential equations proceed from the physical laws governing the rigid rotary/translatory motion and from the application of aerodynamic theories (*blade element theory*).

The first differential equation derives from the second law of dynamics applied to the barycentre $\Gamma$ of the helicopter:

$$\frac{W}{g}\ddot{z}_\Gamma(t) = W - T \qquad (1)$$

where:

$W$ is the helicopter weight,

$g$ is the gravity acceleration,

$\ddot{z}_\Gamma$ is the vertical component of barycentre acceleration,

$T$ is the vertical component of the aerodynamic force acting on the blades and induced by the rotary/translatory motion,

$T = T(z_\Gamma, \psi, \Omega, t, \underline{\beta_a})$ where $\underline{\beta_a}$ is a set of aerodynamic coefficients.

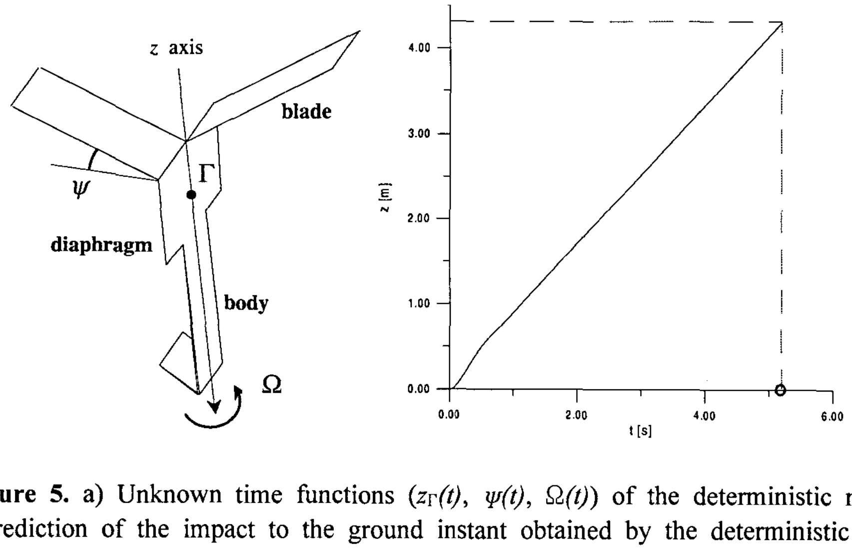


**Figure 5.** a) Unknown time functions ($z_\Gamma(t)$, $\psi(t)$, $\Omega(t)$) of the deterministic model. b) Prediction of the impact to the ground instant obtained by the deterministic model.

The second differential equation describes the rotary motion about the z-axis:

$$I_v \, \dot{\Omega} = M_a - M_c - M_b \tag{2}$$

where:

$I_v$ is the moment of inertia to the rotation about z-axis,

$M_a$ is the moment of aerodynamic force,

$M_c$ is the resisting moment due to centrifugal force,

$M_b$ is the resisting moment due to viscous drag of the diaphragm and body.

The third differential equation describes the rotary motion of the blade about its bending axis:

$$I_b \ddot{\psi} = M_p \tag{3}$$

where:

$I_b$ is the moment of inertia to the blade rotation about its bending axis,

$\ddot{\psi}$ is the angular acceleration in the rotary motion,

$M_p$ is the moment of the forces acting on the blade.

Initial conditions reproduce the real release conditions of the paper helicopter in the experimental phase; the helicopter is released from a position chosen as origin of the fixed frame of reference, with initial vertical velocity equal to zero, zero angular velocity and with the blades standing in horizontal position:

$$\begin{cases} z_\Gamma(0) = 0; \quad \dot{z}_\Gamma(0) = 0 \\ \Omega(0) = 0 \\ \psi(0) = 0; \quad \dot{\psi}(0) = 0 \end{cases} \tag{4}$$

The complete formulation of this initial value problem is presented in (Barone, 1999) and has been solved developing an algorithm in FORTRAN language. The algorithm required a suitable calibration, consisting of the fine-tuning of the aerodynamic coefficients and calculation parameters specific of the development environment, in order to adapt the results of the deterministic model to the experimental results of the screening phase. At the end of this phase, for instance, the prediction of the flight time of the deterministic model matches the mean value experimentally observed with the best parameter combination of the screening phase (Run 11, Table 1). A graph of the vertical position of centre of gravity as function of time, $z_\Gamma(t)$, is plotted in Figure 5.b. It enables to visualise the value of $t$ (5.2 sec) corresponding to $z_\Gamma$ = 4.3 m, which is very close to impact instant.

In addition, the graphs of the functions $\Omega(t)$ and $\psi(t)$ are presented in Figure 6. From these graphs, it is possible to observe that the model explains the transient (lasting

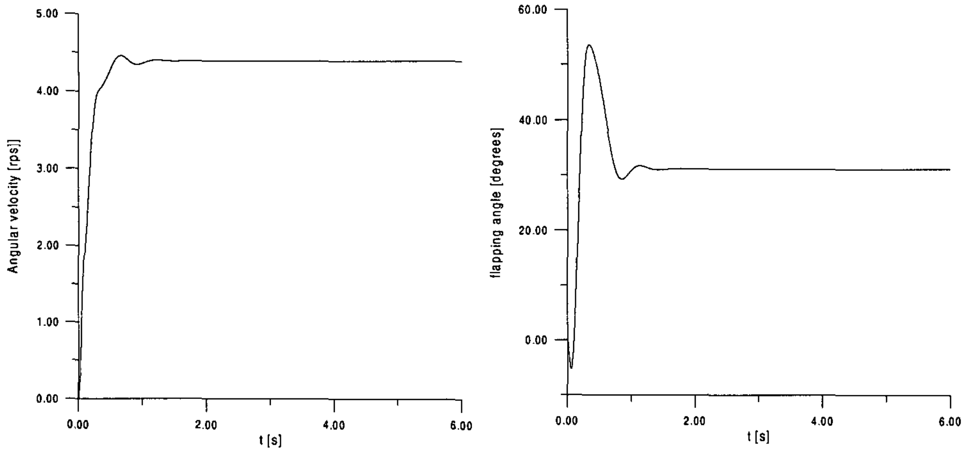


**Figure 6.** a) Angular velocity function $\Omega(t)$. b) Flapping angle function $\psi(t)$.

about 1 second) and the stable condition. In particular the steadying of the angular velocity $\Omega$ to the stationary value of 4.5 rps (rounds per second), and of the flapping angle $\psi$ to the value of about 30 degrees are visible.

## 4. Second experimental phase: computer experiments

Once the deterministic model is calibrated, it can be applied for previsional purposes. The computational code is exploited to simulate experimental phases oriented to the exploration of performance improvements, giving rise to a computer experiments phase.

For the computer simulation, we chose as control factors the design parameters which were most influent in the screening phase (Table 3), and for each of them we fixed three levels (Table 4). We decided to perform a full factorial design $3^2$ corresponding to nine parameter combinations. In each simulation the predicted value of the impact to the ground has been evaluated; the results are presented in Table 5. The predicted response surface of data obtained in this simulation phase is presented in Figure 7.

The improvement trend predicted in the screening phase, due to the increase of the level of the parameter B and the decrease of the parameter E, is confirmed. Nonetheless there's still a margin of gain as it is possible to note from the response surface in which no optimal zone is visible.

The application of specific methods for response surfaces, as, for instance, the *path of steepest ascent*, allows predicting even better performance values. Nonetheless we preferred to set up a further experimental phase aimed at validating the deterministic model and at checking the real improvement of the project.

**Table 4.** Parameters B and E levels for the computer experiments phase.

| | Level -1 | Level 0 | Level +1 |
|---|---|---|---|
| B (Blade length) | 130 mm | 140 mm | 150 mm |
| E (Width ) | 45 mm | 55 mm | 65 mm |

**Table 5.** Computer experiment results.

| Run | B | E | Simulated flight times (sec) |
|---|---|---|---|
| 1 | -1 | -1 | 5.621 |
| 2 | 0 | -1 | 5.789 |
| 3 | +1 | -1 | 5.929 |
| 4 | -1 | 0 | 5.523 |
| 5 | 0 | 0 | 5.768 |
| 6 | +1 | 0 | 5.971 |
| 7 | -1 | +1 | 5.201 |
| 8 | 0 | +1 | 5.600 |
| 9 | +1 | +1 | 5.894 |

## 5. Third experimental phase: deterministic model validation

The third phase is aimed at validating the deterministic model previously formulated by means of real experimental runs, and at searching for parameter combinations better than those previously experimented.

The design chosen is the same given in Table 5. Five replications were experimented for each run in the same experimental conditions of the screening phase and simulated in the computer experiment phase.

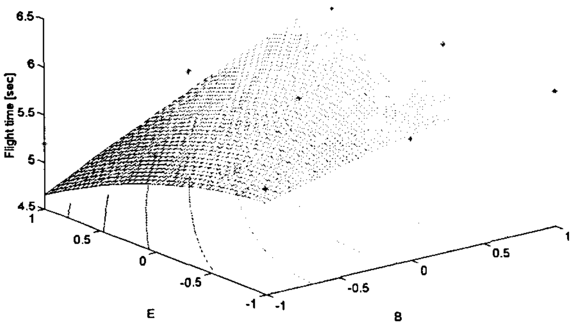


**Figure 7.** Predicted response surface for the results of computer experiments.

The execution order is completely randomised. In Table 6 the experimental design and the observations are presented.

The estimated response surface is presented in Figure 8. The shape of the surface is slightly different from that of Figure 7, hence we deduce that in the deterministic model the second order effects of the parameters B and E on the performance are not as strong as they really are. The comparison of the results of the simulated experimental phase with the results of the third phase obtained with the same parameter combinations, presented in Figure 9, enables one to validate the deterministic model, being its predictions generally satisfactory. The only weak point lies in the fact that the deterministic model would have suggested the parameter combination 6 (Table 5), while the real results indicate that the best design solution would be the combination 3 (Table 6). On the contrary,

**Table 6.** Plan and results of the third experimental phase.

| Run | B | E | Observations $t_{ij}$ (sec) | | | | | $\bar{t}$ | $s^2$ | $\bar{t}/s$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | -1 | 5.82 | 5.78 | 5.68 | 6.14 | 5.96 | 5.87 | 0.033 | 32.52 |
| 2 | 0 | -1 | 6.20 | 5.68 | 5.91 | 5.88 | 5.75 | 5.88 | 0.040 | 29.56 |
| 3 | +1 | -1 | 6.53 | 6.29 | 6.67 | 6.82 | 6.64 | 6.59 | 0.039 | 33.35 |
| 4 | -1 | 0 | 5.92 | 5.83 | 5.80 | 5.79 | 6.13 | 5.89 | 0.021 | 40.95 |
| 5 | 0 | 0 | 5.48 | 5.86 | 5.80 | 5.67 | 5.82 | 5.72 | 0.024 | 36.63 |
| 6 | +1 | 0 | 6.56 | 6.24 | 6.28 | 6.80 | 6.58 | 6.49 | 0.054 | 27.84 |
| 7 | -1 | +1 | 5.35 | 5.28 | 5.48 | 5.48 | 5.31 | 5.38 | 0.009 | 57.62 |
| 8 | 0 | +1 | 5.82 | 5.74 | 5.82 | 5.84 | 5.80 | 5.80 | 0.002 | 144.24 |
| 9 | +1 | +1 | 5.97 | 6.17 | 5.97 | 6.24 | 5.97 | 6.06 | 0.017 | 46.55 |

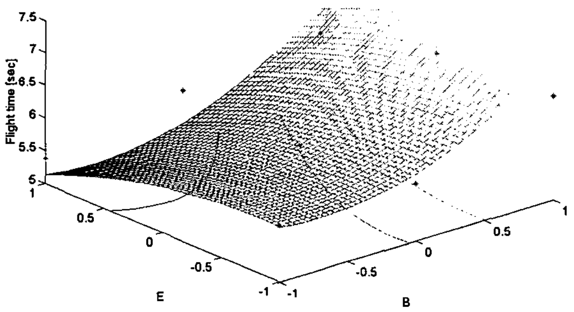


**Figure 8.** Predicted response surface for the results of the third experimental phase.

taking into account the variability too, the combination 8 (Table 6) represents the best compromise, that is the most robust to environmental uncontrolled sources of noise.

An important parameter in aerodynamics, often affecting the performance flight time, is the blade-width ratio. In the case of the paper helicopter, it is possible to compute this parameter as the ratio B/E. This parameter has been evaluated for each parameter combination previously experimented and the corresponding performance values obtained in the second and third experimental phases are presented in Figure 10. The analysis of Figure 10 shows in both cases that the body-width ratio has not a predominant effect on the performance. The trend is clear from computer experiments data which are arranged along three series of increasing values. The level of parameter B raises within each series, while the level of parameter E raises between the three series. In other words, a visible gain in performance is obtained only increasing the level of B. This feature would have been difficult to put in evidence without the aid of the results of computer experiment phase.

Another important parameter in aerodynamic analysis is the blade surface. In the case of paper helicopter this parameter is easily valued as the product B(E. From Figure 11 we can deduce a complete absence of any trend of performance due to the blade area. Instead it is possible to observe again the beneficial influence of the parameter B. From computer experiments data we can even observe, for the same level of B, a reduction of performance raising the level of parameter E. It means that a beneficial effect of the supporting surface cannot balance the resultant weight increase.

## 6. Concluding remarks

Thanks to the deterministic model so formulated we improved the knowledge of the phenomenon, which couldn't have been obtained otherwise. At the same time we improved the effectiveness of the subsequent experimental phase. Nonetheless, the model can be still further refined. In fact there are still some differences between the response surface regarding the real experimentation (Figure 8) and response surface regarding the virtual prototype (Figure 7) which tend to become significant especially near a zone of optimality.

The explanation of this behaviour can be assigned to the limitation of the hypotheses of the deterministic model formulated in section 3. Actually the hypothesis of indeformability is rather strict especially regarding the blade. A careful observation of the phenomenon reveals that, by increasing the level of the blade length (parameter B) and by decreasing the level of the blade width (E), the blade distorts under the effect

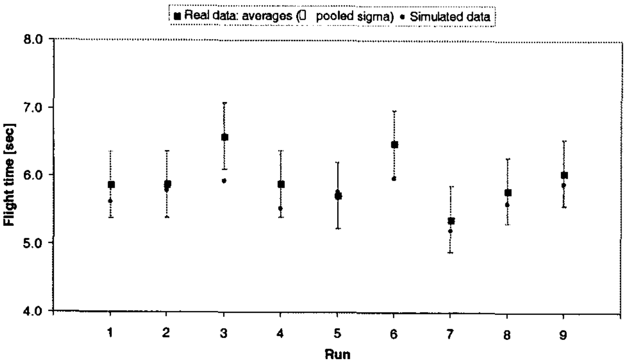


**Figure 9.** Comparison between computer experiment results and real data of the third experimental phase.

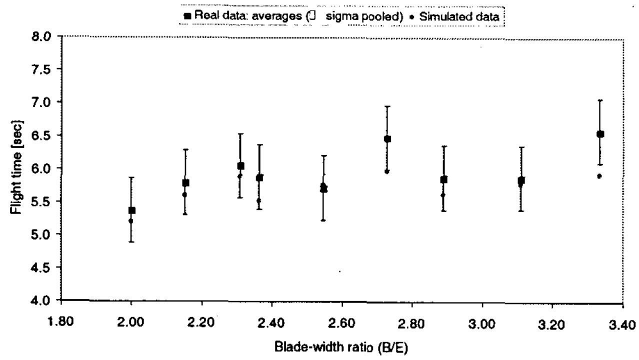


**Figure 10.** Effect of the blade-width ratio (B/E) on the flight time performance.

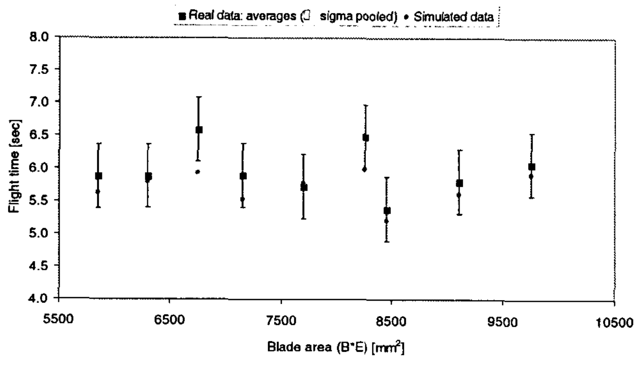


**Figure 11.** Effect of blade area (product B(E) on the flight time performance.

of its own weight and of the other forces acting on it during the motion.

Nonetheless the first conclusions can be so summarised:

1) numerically comparing the results of the third experimental phase with the results of the screening phase (Figure 12) we find an improvement of 28% of the best performance (in terms of $\overline{t}/s$);
2) the improvement between the third experimental phase and the initial performances taken as benchmark (Neu's prototype) is of 45%;
3) the deterministic model gives a good picture of the real phenomenon and it has a good approximation within the experimented variability field;
4) firstly from deterministic model predictions and secondly from real data follows that the body-width ratio hasn't a prevailing effect on the performance;
5) the performance is not affected by the blade surface, meaning that a beneficial effect in terms of supporting surface doesn't counterbalance the resultant weight increase;
6) the improved engineering knowledge of the physical phenomenon, provided by the deterministic model, allows expertise transfer and the computer simulation of the phenomenon; in this sense, according to Box (Nair, 1992), we effectively tried to *"discover the causal relationships and to understand the mechanics of how things happen"*;
7) the computer simulation by means of virtual prototype can drive to the reduction of time and cost of research & development, reaching the aim of producing *"better helicopters quicker"* (Box & Liu, 1999).

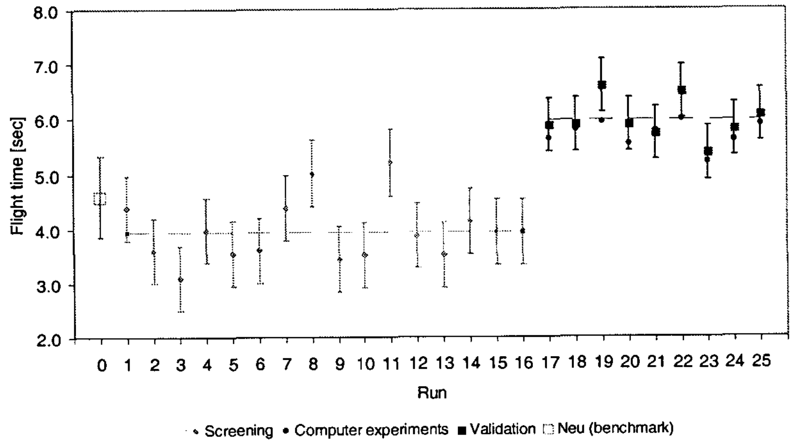


**Figure 12.** Design improvement flow through the real and simulated experimental phases.

## 7. Acknowledgements

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