On Nonovershooting or Monotone Nondecreasing Step Response of Second-Order Systems

Byung-Moon Kwon, Myung-Eui Lee, and Oh-Kyu Kwon

Abstract: This paper has shown that the impulse and the unit step responses of 2nd-order systems can be computed by an analytic method. Three different 2nd-order systems are investigated: the prototype system, the system with one LHP(left half plane) real zero and the system with one RHP(right half plane) real zero. It has also shown that the effects of the LHP or the RHP zero are very serious when the zero is getting closer to the origin on the complex plane. Based on these analytic results, this paper has presented two sufficient and necessary conditions for 2nd-order linear SISO(single-input/single-output) stable systems to have the nonovershooting and the monotone nondecreasing step response, respectively. The latter condition can be extended to the sufficient conditions for the monotone nondecreasing step response of high-order systems.

Keywords: second-order system, step response, impulse response, monotonicity, nonovershoot, transient response

I. Introduction

In many control problems, it is often required that the step response of the system displays no extrema such as undershoots or overshoots. If this requirement is not satisfied, it sometimes causes serious damages to the environments or to the system itself [1]. Moreover, the local extrema are also unacceptable for certain control plants [2]. Hence, it could be practical and theoretical interest in control problems that the step response of the closed-loop system does not have any local extrema. It is noted that the step response has no local extrema in the whole control time if and only if it is monotonically nondecreasing in the transient response. In the recent years, many works were devoted to find explicit conditions for the nonovershooting or the monotone nondecreasing step response [2], [3], [4], [5], [6]. These works however have been deal with the sufficient conditions to the specified system with the only real zeros and real poles.

The prototype 2nd-order system has been well-studied in the classical textbooks since the impulse and the unit step responses can be computed by analytic methods [7], [8]. For the 2nd-order system with one LHP(left half plane) or RHP(right half plane) real zero, however, the effects of the zero have been illustrated only through some numerical examples [7], [8]. Even though they have qualitatively shown that the peak undershoot and the maximum overshoot increase significantly in the step response when the RHP and the LHP real zero approach the origin, respectively [7], [8], [9], analytical results are not given yet to compute the exact values.

In this paper, the impulse and the unit step responses of all 2nd-order systems are computed by the analytic method. Using the Laplace transform technique, it is shown that the peak undershoot and maximum overshoot of those systems can be analyzed on the time-domain and explicitly presented by some analytic equations. These analytic results would be useful understanding about the effects of the LHP or the RHP real zero in 2nd-order systems. Based on these analyses, we present the sufficient and necessary conditions for the nonovershooting or the monotone nondecreasing step response of 2nd-order systems. The proposed results can be extended to the sufficient conditions for the monotone nondecreasing step response of the high-order systems. They will be used very easily for the controller design since these conditions are formulated by the only pole-zero configurations of the systems.

The layout of this paper is organized as follows: In Section 2, we summarize the impulse and the unit step responses of 2nd-order systems, and analyze transient response specifications such as the peak undershoot or the maximum overshoot. In Section 3, we present the sufficient and necessary conditions for the nonovershooting or the monotone nondecreasing step response of the systems. The concluding remarks are given in Section 4.

II. Transient response analysis of second-order systems

In this section, the impulse and the unit step responses of 2nd-order systems are summarized. These time-domain responses give some useful understanding for the transient characteristics, e.g., overshoot, undershoot, settling time, etc., which have been computed through three steps [11] as follows:

1) Compute the impulse response $g(t)$ and the unit step response $y(t)$ of 2nd-order systems using Laplace transform method provided that the systems are relaxed at time $t = 0$ and stable.
2) Compute the peak undershoot time t_p and the maximum overshoot time t_m, which are the first and the second time points such that $g(t) = 0$, respectively.
3) Compute the peak undershoot $Y_p = y(t_p)$ and the maximum overshoot $Y_m = y(t_m)$.

Manuscript received: Nov., 29, 2001., Accepted: Apr., 30, 2002.

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※ This work was supported by INHA UNIVERSITY Research Grant.(INHA-20672)

Note that the system has the monotone nondecreasing step response if and only if $g(t) \geq 0$, and the minimum phase system does not have undershoot phenomena [3], [4], [6], [9]. Here, three types of the systems are investigated: the prototype system, the system with one LHP real zero, and the system with one RHP real zero. It is noted that [11] has dealt with only the last case.

1. Prototype second-order system

Let us consider the prototype 2nd-order system as follows:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{1}$$

where ω_n is the undamped natural frequency and ζ is the damping ratio of the system. It can be described by four cases with respect to the value of damping ratio ζ. Since the system has been already well-studied in the control textbooks [7], [8], it is briefly summarized in this section.

1.1 Undamped case ($\zeta = 0$)

When $\zeta = 0$, system (1) has two complex conjugate imaginary poles at $s = \pm j\omega_n$. For $teq0$, the impulse response $g(t)$ and the unit step response $y(t)$ can be calculated by

$$g(t) = \omega_n \sin \omega_n t, \tag{2}$$

$$y(t) = 1 - \cos \omega_n t, \tag{3}$$

respectively.

1.2 Underdamped case ($0 < \zeta < 1$)

In this case, system (1) has two complex conjugate poles as follows:

$$p_1 = -\zeta\omega_n + j\omega_d,\ \bar{p}_1 = -\zeta\omega_n - j\omega_d, \tag{4}$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency. For $t \geq 0$, the impulse and the unit step responses can be written by

$$g(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_d t, \tag{5}$$

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi_1), \tag{6}$$

where

$$\phi_1 = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}. \tag{7}$$

Step response (6) has the maximum overshoot

$$Y_m = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \tag{8}$$

at the maximum overshoot time

$$t_m = \frac{\pi}{\omega_d}. \tag{9}$$

Equation (8) shows that the maximum overshoot of the unit step response is the function of the only damping ratio ζ.

1.3 Critically damped case ($\zeta = 1$)

If $\zeta = 1$, the two poles of system (1) are nearly equal at $s = -\omega_n$. Hence, the impulse and the unit step responses can be computed by

$$g(t) = \omega_n^2 t e^{-\omega_n t}, \tag{10}$$

$$y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \tag{11}$$

for $t \geq 0$, respectively. In this case, system (1) has the monotone nondecreasing step response since $g(t) \geq 0$ for $t \geq 0$.

1.4 Overdamped case ($\zeta > 1$)

In the overdamped case, system (1) has two distinct real poles as follows:

$$p_2 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2-1},\ p_3 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}. \tag{12}$$

For $t \geq 0$, the impulse and the unit step responses can be calculated as follows:

$$g(t) = A_1\left[e^{p_2 t} - e^{p_3 t}\right], \tag{13}$$

$$y(t) = 1 - A_1\left[\frac{e^{p_3 t}}{p_3} - \frac{e^{p_2 t}}{p_2}\right], \tag{14}$$

where

$$A_1 = \frac{\omega_n}{2\sqrt{\zeta^2-1}}. \tag{15}$$

Since $p_3 < p_2 < 0$, or equivalently, $g(t) \geq 0$ for $t \geq 0$, the overdamped prototype 2nd-order system has the monotone nondecreasing step response like the critically damped case.

2. Second-order system with one LHP real zero

Consider the 2nd-order system with LHP real zero at $s = -a$ as follows:

$$G(s) = \frac{\frac{\omega_n^2}{a}(s+a)}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{16}$$

where $a > 0$. Note that the DC gain of system (16) is normalized by 1. If the LHP zero infinitely approaches the left side on the complex plane, then system (16) is the same as the prototype 2nd-order system.

2.1 Undamped case ($\zeta = 0$)

In the undamped case, the impulse and the unit step responses can be calculated by

$$g(t) = A_2 \sin(\omega_n t + \phi_2), \tag{17}$$

$$y(t) = 1 - \frac{A_2}{\omega_n}\sin(-\omega_n t + \phi_3) \tag{18}$$

for $t \geq 0$, where

$$\begin{aligned} A_2 &= \frac{\omega_n}{a}\sqrt{a^2+\omega_n^2}. \\ \phi_2 &= \tan^{-1}\frac{\omega_n}{a},\quad \phi_3 = \tan^{-1}\frac{a}{\omega_n}. \end{aligned} \tag{19}$$

When the LHP real zero goes to $-\infty$ on the complex plane, $g(t)$ and $y(t)$ are the same as those of the undamped prototype 2nd-order system.

2.2 Underdamped case ($0 < \zeta < 1$)

For $t \geq 0$, the impulse and the unit step responses can be calculated as follows:

$$\begin{aligned} g(t) &= \frac{\omega_n^2}{a}e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{a-\zeta\omega_n}{\omega_d}\sin\omega_d t\right] \\ &= \begin{cases} A_3 e^{-\zeta\omega_n t}\sin(\omega_d t + \phi_4), & a > \zeta\omega_n, \\ \frac{\omega_n}{\zeta}e^{-\zeta\omega_n t}\cos\omega_d t, & a = \zeta\omega_n, \\ A_3 e^{-\zeta\omega_n t}\sin(-\omega_d t + \phi_5), & a < \zeta\omega_n, \end{cases} \end{aligned} \tag{20}$$

$$\begin{aligned} y(t) &= 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta a - \omega_n}{a\sqrt{1-\zeta^2}}\sin\omega_d t\right] \\ &= \begin{cases} 1 - \frac{A_3}{\omega_n}e^{-\zeta\omega_n t}\sin(\omega_d t + \phi_6), & a > \frac{\omega_n}{\zeta}, \\ 1 - e^{-\zeta\omega_n t}\cos\omega_d t, & a = \frac{\omega_n}{\zeta}, \\ 1 - \frac{A_3}{\omega_n}e^{-\zeta\omega_n t}\sin(-\omega_d t + \phi_7), & a < \frac{\omega_n}{\zeta}, \end{cases} \end{aligned} \tag{21}$$

where

$$A_3 = \frac{\omega_n}{a}\sqrt{\frac{a^2 - 2\zeta\omega_n a + \omega_n^2}{1-\zeta^2}},$$
$$\phi_4 = \tan^{-1}\frac{\omega_d}{a - \zeta\omega_n},\quad \phi_5 = \tan^{-1}\frac{\omega_d}{\zeta\omega_n - a}, \tag{22}$$
$$\phi_6 = \tan^{-1}\frac{a\sqrt{1-\zeta^2}}{\zeta a - \omega_n},\quad \phi_7 = \tan^{-1}\frac{a\sqrt{1-\zeta^2}}{\omega_n - \zeta a}.$$

Step response (21) has the maximum overshoot

$$Y_m = \begin{cases} \frac{1}{a}e^{-\frac{\zeta(\pi-\phi_4)}{\sqrt{1-\zeta^2}}}\sqrt{a^2 - 2\zeta\omega_n a + \omega_n^2}, & a > \zeta\omega_n, \\ \frac{\sqrt{1-\zeta^2}}{\zeta}e^{-\frac{\zeta\pi}{2\sqrt{1-\zeta^2}}}, & a = \zeta\omega_n, \\ \frac{1}{a}e^{-\frac{\zeta\phi_5}{\sqrt{1-\zeta^2}}}\sqrt{a^2 - 2\zeta\omega_n a + \omega_n^2}, & a < \zeta\omega_n, \end{cases} \tag{23}$$

which occurs at the maximum overshoot time

$$t_m = \begin{cases} \frac{\pi - \phi_4}{\omega_d}, & a > \zeta\omega_n, \\ \frac{\pi}{2\omega_d}, & a = \zeta\omega_n, \\ \frac{\phi_5}{\omega_d}, & a < \zeta\omega_n, \end{cases} \tag{24}$$

respectively. Note that if the LHP zero infinitely approaches the left side on the complex plane, these specifications are the same as the cases of the prototype 2nd-order system. However, the maximum overshoot becomes infinitely large if it is getting close to the origin on the complex plane.

2.3 Critically damped case ($\zeta = 1$)

When $\zeta = 1$, the impulse and the unit step responses of system (16) can be calculated by

$$g(t) = \frac{\omega_n^2}{a}\left[1 + (a - \omega_n)t\right]e^{-\omega_n t}, \tag{25}$$

$$y(t) = 1 - \left[1 + \frac{\omega_n}{a}(a - \omega_n)t\right]e^{-\omega_n t}, \tag{26}$$

respectively. If $-a > -\omega_n$, *i.e.* the LHP zero is larger than the poles, then system (16) has the maximum overshoot

$$Y_m = \left[1 + \frac{\omega_n}{a}\right]e^{-\frac{\omega_n}{\omega_n - a}} \tag{27}$$

at the maximum overshoot time

$$t_m = \frac{1}{\omega_n - a}. \tag{28}$$

When the LHP zero is getting closer to the origin on the complex plane, the maximum overshoot becomes infinitely large. However, system (16) has the monotone nondecreasing step response since $g(t) \geq 0$ for $t \geq 0$ if $-a < -\omega_n$, or equivalently, if the LHP zero is smaller than the poles.

2.4 Overdamped case ($\zeta > 1$)

In the overdamped case, the impulse and the unit step responses can be computed by

$$g(t) = \frac{A_1}{a}\left[(a + p_2)e^{p_2 t} - (a + p_3)e^{p_3 t}\right], \tag{29}$$

$$y(t) = 1 - \frac{A_1}{a}\left[\frac{a + p_3}{p_3}e^{p_3 t} - \frac{a + p_2}{p_2}e^{p_2 t}\right] \tag{30}$$

for $t \geq 0$, respectively. If $-a > p_2$, *i.e.* the LHP zero is larger than the dominant pole, then system (16) has the maximum overshoot

$$Y_m = \frac{A_1}{a}\left[\frac{a + p_2}{p_2}e^{p_2 t_m} - \frac{a + p_3}{p_3}e^{p_3 t_m}\right], \tag{31}$$

where t_m is the maximum overshoot time, which is given by

$$t_m = \frac{1}{p_2 - p_3}\ln\frac{a + p_3}{a + p_2}. \tag{32}$$

It is noted that $g(t) \geq 0$ in (29) if and only if $-a < p_2$. In other words, the overdamped 2nd-order system with an LHP real zero has the monotone nondecreasing step response if and only if the LHP zero is smaller than the dominant pole.

3. Second-order system with one RHP real zero

Let us consider the 2nd-order system with one RHP real zero at $s = b$ as follows:

$$G(s) = \frac{-\frac{\omega_n^2}{b}(s - b)}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{33}$$

where $b > 0$ [11]. Note that system (33) has the initial undershoot in the step response owing to one RHP real zero [9], [10], [11], [6]. Similar to the system with an LHP real zero, if the RHP zero infinitely approaches the right side on the complex plane, then system (33) is the same as the prototype 2nd-order system.

3.1 Undamped case ($\zeta = 0$)

In this case, the impulse and the unit step responses of system (33) can be calculated by

$$g(t) = A_4\sin(\omega_n t - \phi_8), \tag{34}$$

$$y(t) = 1 - \frac{A_4}{\omega_n}\sin(\omega_n t + \phi_9), \tag{35}$$

where

$$A_4 = \frac{\omega_n}{b}\sqrt{b^2 + \omega_n^2},$$
$$\phi_8 = \tan^{-1}\frac{\omega_n}{b},\quad \phi_9 = \tan^{-1}\frac{b}{\omega_n}, \tag{36}$$

for $t \geq 0$. Equations (34) and (35) imply that the system (33) has the peak to peak values of $\pm A_4$ and $1 \pm A_4/\omega_n$ in the impulse and the unit step responses, respectively. These become infinitely large when the real RHP zero is getting closer to the origin on the complex plane. Also, when the real RHP zero goes to ∞, those responses have the bounds as follows:

$$-\omega_n \leq \lim_{b\to\infty} g(t) \leq \omega_n, \tag{37}$$

$$0 \leq \lim_{b\to\infty} y(t) \leq 2, \tag{38}$$

which are the same results as the prototype 2nd-order system.

3.2 Underdamped case ($0 < \zeta < 1$)

In the underdamped case, the impulse and the unit step responses can be computed by

$$g(t) = A_5 e^{-\zeta\omega_n t}\sin(\omega_d t - \phi_{10}), \tag{39}$$

$$y(t) = 1 - \frac{A_5}{\omega_n}e^{-\zeta\omega_n t}\sin(\omega_d t + \phi_{11}), \tag{40}$$

where

$$A_5 = \frac{\omega_n}{b}\sqrt{\frac{b^2 + 2\zeta\omega_n b + \omega_n^2}{1-\zeta^2}},$$
$$\phi_{10} = \tan^{-1}\frac{\omega_d}{b+\zeta\omega_n},\ \phi_{11} = \tan^{-1}\frac{b\sqrt{1-\zeta^2}}{\zeta b + \omega_n}. \tag{41}$$

The peak undershoot Y_p and the maximum overshoot Y_m occur at the peak undershoot time t_p and the maximum overshoot time t_m, respectively, and these are explicitly formulated as

$$Y_p = 1 - \frac{1}{b}e^{-\zeta\omega_n t_p}\sqrt{b^2 + 2\zeta\omega_n b + \omega_n^2}, \tag{42}$$

$$t_p = \frac{\phi_{10}}{\omega_d}, \tag{43}$$

$$Y_m = \frac{1}{b}e^{-\zeta\omega_n t_m}\sqrt{b^2 + 2\zeta\omega_n b + \omega_n^2}, \tag{44}$$

$$t_m = \frac{\phi_{10} + \pi}{\omega_d}. \tag{45}$$

If the RHP real zero is getting closer to the origin on the complex plane, the peak undershoot and maximum overshoot extremely large, *i.e.*

$$\lim_{b\to 0^+} Y_p = -\infty, \tag{46}$$

$$\lim_{b\to 0^+} Y_m = \infty. \tag{47}$$

Note that when the RHP zero goes to ∞ on the complex plane, these specifications are the same as those of prototype 2nd-order system.

3.3 Critically damped case ($\zeta = 1$)

If $\zeta = 1$, the impulse and the unit step responses can be written by

$$g(t) = \frac{\omega_n^2}{b}\left[(b+\omega_n)t - 1\right]e^{-\omega_n t}, \tag{48}$$

$$y(t) = 1 - \left[1 + \frac{\omega_n}{b}(b+\omega_n)t\right]e^{-\omega_n t}, \tag{49}$$

for $t \geq 0$, respectively. Step response (49) has the peak undershoot

$$Y_p = 1 - \left[1 + \frac{\omega_n}{b}\right]e^{-\frac{\omega_n}{\omega_n + b}}, \tag{50}$$

which occurs at the peak undershoot time

$$t_p = \frac{1}{\omega_n + b}. \tag{51}$$

The overshoot however does not appear in step response (49). Moreover, if the real RHP zero is as far as infinitely to the right on the complex plane, the undershoot phenomenon does not appear anymore, and if it is getting closer to the origin, the peak undershoot becomes infinitely large.

3.4 Overdamped case ($\zeta > 1$)

In the overdamped case, the impulse and the unit step responses can be computed by

$$g(t) = \frac{A_1}{b}\left[(b-p_2)e^{p_2 t} - (b-p_3)e^{p_3 t}\right], \tag{52}$$

$$y(t) = 1 - \frac{A_1}{b}\left[\frac{p_2 - b}{p_2}e^{p_2 t} - \frac{p_3 - b}{p_3}e^{p_3 t}\right], \tag{53}$$

respectively. The peak undershoot is given by

$$Y_p = 1 - \frac{A_1}{b}\left[\frac{p_2 - b}{p_2}\left(\frac{b-p_3}{b-p_2}\right)^{\frac{p_2}{p_2-p_3}} - \frac{p_3 - b}{p_3}\left(\frac{b-p_3}{b-p_2}\right)^{\frac{p_3}{p_2-p_3}}\right], \tag{54}$$

which occurs at the peak undershoot time

$$t_p = \frac{1}{2\omega_n\sqrt{\zeta^2 - 1}}\ln\frac{b-p_3}{b-p_2}. \tag{55}$$

It is noted that the overdamped 2nd-order system with the RHP real zero does not have the overshoot in the step response like the critically damped case.

III. Nonovershooting or monotone nondecreasing step response

Let us investigate the nonovershooting or the monotone nondecreasing condition in the step response of 2nd-order systems based on the analyses of the previous section. Such condition is often required in the control system design even though it causes a slower response. First of all, the next result states the condition for the nonovershooting step response of the 2nd-order systems.

Theorem 1 : The 2nd-order systems have the nonovershooting step responses if and only if

$$\zeta \geq 1, \tag{56}$$

where ζ is the damping ratio of the systems.

Proof : From Section II, the overshoot only appears in the undamped and the underdamped cases of all 2nd-order systems. Hence, the systems have the nonovershooting step response if and only if they are the critically damped and the overdamped ones, which completes the proof. ■

Also, we can obtain the sufficient and necessary condition for the monotone nondecreasing step responses of 2nd-order systems.

Theorem 2 : The 2nd-order systems have the monotone nondecreasing step responses if and only if

(1) $\zeta \geq 1$

(2) $z < p_d$,

where ζ, z and p_d are the damping ratio, the zero and the dominant pole of the systems, respectively.

Proof : The prototype 2nd-order system has the nonnegative impulse response if and only if it is the critically damped and the overdamped cases. The 2nd-order system with an LHP real zero also has the nonnegative impulse response if and only if it is the critically damped and the overdamped ones with $z < p_d$. Finally, the system with an RHP real zero does not have the nonnegative impulse response. Hence, it comes from the combination of three cases, which completes the proof. ■

Based on the results of Section II, it can be seen that the 2nd-order system with RHP real zero always has the undershoot phenomenon in the step response without relation with the value of ζ. If $\zeta \geq 1$, however, the step response of the 2nd-order system does not have the overshoot even though it has an RHP real zero. Note that some papers provide some sufficient conditions to avoid extrema in the step response of SISO systems with real zeros and poles [3], [4], [6].

Theorem 2 can be extended to the sufficient conditions for high-order systems as follows:

Lemma 1 : Let us consider transfer functions $\{G_i(s)\}$ for $i = 1, 2, \cdots, n$, with the monotone nondecreasing step response, then the series system

$$G_s(s) = G_1(s)G_2(s)\cdots G_n(s) \tag{57}$$

also has the monotone nondecreasing step response.

Proof : In the time-domain, $G_s(s)$ can be written by

$$g_s(t) = g_1(t) * g_2(t) * \cdots * g_n(t), \; ^{\forall}t \geq 0, \tag{58}$$

where '$*$' denotes the convolution. Since $g_i(t) \geq 0$ for $i = 1, 2, \cdots, n$, $g_s(t) \geq 0$ for $t \geq 0$, which follows from the definition of convolution. Hence, $G_s(s)$ also has the monotone nondecreasing step response, which completes the proof. ■

Lemma 2 : If $\{G_i(s)\}$ for $i = 1, 2, \cdots, n$, have the monotone nondecreasing step response, then the parallel system

$$G_p(s) = G_1(s) + G_2(s) + \cdots + G_n(s) \tag{59}$$

also has the monotone nondecreasing step response.

Proof : Since $g_i(t) \geq 0$, for $i = 1, 2, \cdots, n$, the impulse response $g_p(t)$ is nonnegative, *i.e.*

$$g_p(t) = g_1(t) + g_2(t) + \cdots + g_n(t) \geq 0, \; ^{\forall}t \geq 0, \tag{60}$$

or equivalently, $G_p(s)$ also has the monotone nondecreasing step response, which completes the proof. ■

Hence, the total system formed by series or parallel connections of the subsystems with the monotone nondecreasing step response also has the monotone nondecreasing step response. It is noted that the series system has the slower step response than those of all subsystems. In the case of the system without finite zeros, it always has the monotone nondecreasing step response if the system has the only real poles.

Example 1 : Let us consider a series system $G_s(s) = G_1(s)G_2(s)G_3(s)$, where the subsystems are

$$\begin{aligned} G_1(s) &= \frac{6}{s+6}, \\ G_2(s) &= \frac{20}{(s+4)(s+5)}, \\ G_3(s) &= \frac{1.5(s+2)}{(s+1)(s+3)}. \end{aligned} \tag{61}$$

All these subsystems have the monotone nondecreasing step response. From Lemma 1, the total system $G_s(s)$ also has the monotone nondecreasing step response, which can be verified in Fig. 1.

Example 2 : Let us consider a parallel system $G_p(s) = G_4(s) + G_5(s) + G_6(s)$, where the subsystems are

$$\begin{aligned} G_4(s) &= \frac{1}{s+6}, \\ G_5(s) &= \frac{10}{(s+4)(s+5)}, \\ G_6(s) &= \frac{0.5(s+2)}{(s+1)(s+3)}. \end{aligned} \tag{62}$$

From Lemma 2, the total system $G_p(s)$ also has the monotone nondecreasing step response since all the subsystems have the monotone nondecreasing step response. It can be shown in Fig. 2.

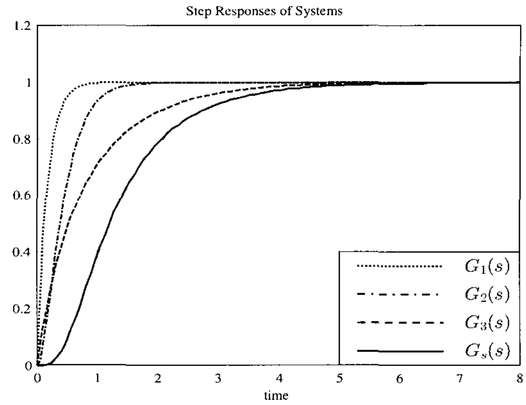

Fig. 1. Step responses of the systems in Ex. 1.

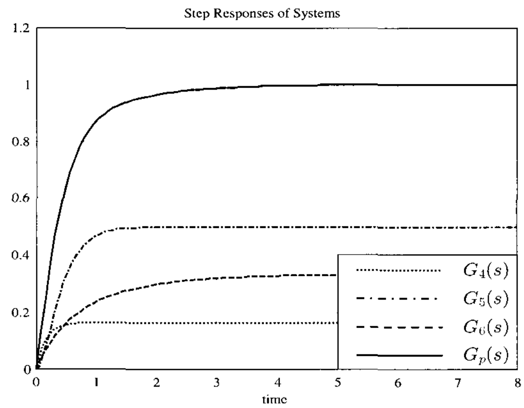

Fig. 2. Step responses of the systems in Ex. 2.

IV. Conclusions

In this paper, we have analyzed the characteristics of three different 2nd-order systems: the prototype system, the system with LHP real zero, and the system with RHP real zero. We also have computed the peak undershoot Y_p, the peak undershoot time t_p, the maximum overshoot Y_m, and the maximum overshoot time t_m in the unit step response of the systems. Analytic results in this paper will provide some useful understanding about the effects of the LHP or the RHP real zero in 2nd-order systems. Moreover, it has presented the sufficient and necessary conditions for the nonovershooting or the monotone nondecreasing step response of 2nd-order systems. It also has formulated the sufficient condition for the monotone nondecreasing step response of high-order systems.

Although the sufficient and necessary conditions for the nonovershooting or the monotone nondecreasing step response of 2nd-order systems are perfectly characterized by the pole-zero configurations, the conditions for high-order systems are the only sufficient ones, so it will require further research to find necessary conditions for those systems.

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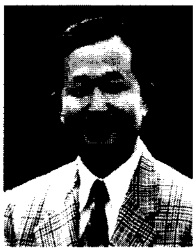

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