

# Adaptive Feedback Linearization Control Based on Stator Fluxes Model for Induction Motors

Seok Ho Jeon and Jin Young Choi

**Abstract:** This paper presents an adaptive feedback linearization control scheme for induction motors using stator fluxes. By using stator fluxes as states, overparameterization is prevented and control inputs can be determined straightforwardly unlike in existing schemes. This approach leads to the decrease of the relative degree for the flux modulus and thus yields a simpler control algorithm than the prior results. In this paper, adaptation schemes are suggested to compensate for the variations of stator resistance, rotor resistance and load torque. In particular, the adaptation to the variation of stator resistance with a feedback linearization control is a new trial. In addition, to improve the convergence of rotor resistance estimation, the differences between stator currents and its estimates are used for the parameter adaptation. The simulations show that torque and flux are controlled independently and that the estimates of stator resistance, rotor resistance, and load torque converge to their true values. Actual experiments on a 3.7kw induction motor verify the effectiveness of the proposed method.

**Keywords:** feedback linearization, adaptive control, parameter estimation, induction motor

## I. Introduction

To achieve high performance of an induction motor, which is widely used due to its reliability, low cost, and easy maintenance, the field-oriented control scheme has been developed [1]. The field-oriented control scheme cannot achieve decoupling control between two output variables, i.e. torque and flux, while flux changes in a field weakening region. Various nonlinear control schemes have also been applied to induction motors. In particular, the feedback linearization methods that guarantee the decoupling characteristics in all operation regions have been proposed [7]-[15]. Passivity based methods have been applied to induction motors [16][17][18] and the backstepping technique has also been proposed [19]. Since some motor parameters such as rotor resistance and stator resistance are varied during the operation due to temperature, skin effect and so on, it is necessary to compensate for the parameter variations. An adaptive feedback linearization control scheme with respect to load torque and rotor resistance has already been proposed [10].

This paper attempts to design a simple control scheme by using the stator fluxes unlike the existing ones mentioned in the above. The model of a three phase balanced induction motor is described in a two-axis coordinate frame under the assumption of linear magnetic circuits. Provided that two variables are determined as states among stator currents ( $i_{ds}$ ,  $i_{qs}$ ), rotor currents ( $i_{dr}$ ,  $i_{qr}$ ), stator fluxes ( $\lambda_{ds}$ ,  $\lambda_{qs}$ ), and rotor fluxes ( $\lambda_{dr}$ ,  $\lambda_{qr}$ ), an induction motor can be described in a various fourth-order model [2]. Among the variables, stator currents are usually used since they can be directly measured by hall sensors, but rotor currents are not, since they cannot be measured due to the mechanical structure of induction motors. Although either stator fluxes or rotor fluxes can be chosen as states, the control scheme of induction motors based on rotor flux model is preferred in implementation of torque or speed controller than that based on stator flux model because the estimation of stator fluxes includes the integral, which makes

the estimated stator fluxes to be easily saturated due to offset or drift errors. So, in most cases, except for some special cases such as sensorless control, rotor flux model is adopted for the estimation and flux control. The field-oriented control based on rotor flux model can control torque and flux independently by each current controller. If the flux reference is constant, d-axis stator current controls flux magnitude and q-axis stator current controls motor torque. In addition, most feedback linearization control schemes and nonlinear control methods have been derived using rotor fluxes as states. A few field-oriented control schemes have used stator flux as states, which show robustness against motor inductance variation [3]-[6].

This paper aims to develop an adaptive feedback linearization control scheme using stator fluxes to show the advantages of the stator flux model over the rotor flux model. In deriving adaptive nonlinear control with respect to motor resistances, the stator flux model is simpler than the rotor flux model. The relative degree for flux modulus is decreased, control inputs can be determined straightforwardly, and overparameterization disappears. Among the parameters, in addition to rotor resistance and load torque whose adaptation schemes have already been developed, the stator resistance adaptation law is proposed. The adaptation of stator resistance is a new trial in nonlinear control for induction motors.

This paper uses two control loop; in the inner loop, motor torque and the flux modulus are controlled while estimating stator and rotor resistances, and in the outer loop, speed is controlled while estimating load torque. In the speed controller load torque is always estimated by a simple adaptive feedback linearization method. However, in the torque and flux controller, stator and rotor resistances cannot be guaranteed to be estimated if there does not exist motor torque. To improve the convergence of rotor resistance estimation, we estimate the stator currents with estimated parameters, and the difference from the actual currents is used as additional terms for the resistance adaptation. Using this method, rotor resistance can be estimated even when torque is zero, unless the flux modulus is constant.

The rest of this paper is organized as follows. Section II describes the key ideas and design approach. Section III designs

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a torque and flux controller in the inner loop, in which the convergence of tracking errors and parameter errors are analyzed. Section IV designs a speed controller in the outer loop. Section V and section VI show the simulation results and experiment results respectively for the proposed method. Conclusions are given in section VII.

## II. Statements of key idea and design approach

Using dq-transformation, the electrical dynamics of a three-phase induction motor can be described in a two-axis coordinates frame [2]. If we choose a stationary reference frame, it is described as

$$\begin{aligned} V_{ds} &= R_s i_{ds} + \frac{d}{dt} \lambda_{ds}, \\ V_{qs} &= R_s i_{qs} + \frac{d}{dt} \lambda_{qs}, \\ 0 &= R_r i_{dr} + \frac{d}{dt} \lambda_{dr} + \omega_r \lambda_{qr}, \\ 0 &= R_r i_{qr} + \frac{d}{dt} \lambda_{qr} - \omega_r \lambda_{dr}. \end{aligned} \quad (1)$$

The first two in (1) are stator circuit equations and the rest are rotor circuit equations. The relation between fluxes and currents are given by (2) under the assumption of linear magnetic circuits.

$$\begin{aligned} \lambda_{ds} &= L_s i_{ds} + L_m (i_{ds} + i_{dr}) = L_s i_{ds} + L_m i_{dr}, \\ \lambda_{qs} &= L_s i_{qs} + L_m (i_{qs} + i_{qr}) = L_s i_{qs} + L_m i_{qr}, \\ \lambda_{dr} &= L_r i_{dr} + L_m (i_{ds} + i_{dr}) = L_r i_{dr} + L_m i_{ds}, \\ \lambda_{qr} &= L_r i_{qr} + L_m (i_{qs} + i_{qr}) = L_r i_{qr} + L_m i_{qs}. \end{aligned} \quad (2)$$

Among the variables, if we choose rotor fluxes  $\lambda_{dr}$ ,  $\lambda_{qr}$  and stator currents  $i_{ds}$ ,  $i_{qs}$  as states, the rotor flux model is given by (3).

Rotor flux model:

$$\begin{aligned} \frac{d}{dt} \lambda_{dr} &= -\frac{R_r}{L_r} \lambda_{dr} - \omega_r \lambda_{qr} + \frac{R_r L_m}{L_r} i_{ds}, \\ \frac{d}{dt} \lambda_{qr} &= \omega_r \lambda_{dr} - \frac{R_r}{L_r} \lambda_{qr} + \frac{R_r L_m}{L_r} i_{qs}, \\ \frac{d}{dt} i_{ds} &= \frac{L_m R_r}{L_\sigma L_r^2} \lambda_{dr} + \frac{L_m}{L_\sigma L_r} \omega_r \lambda_{qr} \\ &\quad - \left( \frac{L_m^2 R_r + L_r^2 R_s}{L_\sigma L_r^2} \right) i_{ds} + \frac{1}{L_\sigma} V_{ds}, \\ \frac{d}{dt} i_{qs} &= -\frac{L_m}{L_\sigma L_r} \omega_r \lambda_{dr} + \frac{L_m R_r}{L_\sigma L_r^2} \lambda_{qr} \\ &\quad - \left( \frac{L_m^2 R_r + L_r^2 R_s}{L_\sigma L_r^2} \right) i_{qs} + \frac{1}{L_\sigma} V_{qs}. \end{aligned} \quad (3)$$

Electrical motor torque is described by using rotor fluxes as in (4)

$$T_e = \frac{3P}{2} \frac{L_m}{L_r} (\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}). \quad (4)$$

Also, if we use stator fluxes instead of rotor fluxes, the stator flux model is obtained.

Stator flux model:

$$\begin{aligned} \frac{d}{dt} \lambda_{ds} &= V_{ds} - R_s i_{ds}, \\ \frac{d}{dt} \lambda_{qs} &= V_{qs} - R_s i_{qs}, \\ \frac{d}{dt} i_{ds} &= -\frac{L_r R_s + L_s R_r}{L_r L_\sigma} i_{ds} + \frac{R_r}{L_r L_\sigma} \lambda_{ds} + \frac{\omega_r \lambda_{qs}}{L_\sigma} \\ &\quad - \omega_r i_{qs} + \frac{1}{L_\sigma} V_{ds}, \\ \frac{d}{dt} i_{qs} &= -\frac{L_r R_s + L_s R_r}{L_r L_\sigma} i_{qs} + \frac{R_r}{L_r L_\sigma} \lambda_{qs} - \frac{\omega_r \lambda_{ds}}{L_\sigma} \\ &\quad + \omega_r i_{ds} + \frac{1}{L_\sigma} V_{qs}. \end{aligned} \quad (5)$$

By using stator fluxes, electrical motor torque can be rewritten in (6).

$$T_e = \frac{3P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}). \quad (6)$$

We assume that all states are available and all parameters are constant. Among the parameters, stator resistance in addition to rotor resistance is assumed to be an unknown parameter. When controlling induction motors, the control outputs are flux modulus and motor speed, and the control inputs are stator voltages. The square of flux modulus is described as  $|\lambda_r|^2 = \lambda_{dr}^2 + \lambda_{qr}^2$  in the rotor flux model or  $|\lambda_s|^2 = \lambda_{ds}^2 + \lambda_{qs}^2$  in the stator flux model, and the dynamic of motor speed is given by (7)

$$\frac{d}{dt} \omega = \frac{T_e}{J} - \frac{T_l}{J}. \quad (7)$$

The following addresses some difficulties in deriving a nonlinear adaptive controller using rotor fluxes. To obtain control inputs in the feedback linearization control scheme, the derivatives of control output are needed to the amount of the relative degree. The derivative of stator flux has stator voltages, but that of rotor flux does not, which can be shown in the first two equations of (3) and (5) respectively. Therefore, in the rotor flux model, two times derivative of the flux modulus is needed, which means that the relative degree is two. However, in the stator flux model, the relative degree for the flux modulus is only one, which makes the design of the feedback linearization controller simple. The above two models are transformed into a normal form for the designing of the feedback linearization controller. The following shows the nonlinear portions including the control input terms in the normal form. This equation is used to obtain the feedback linearization control inputs.

Nonlinear portion in normal form of the rotor flux model

$$Y_r = F_r(x_r) + B_r(x_r)\theta_r + G_r(x_r, \theta_r)u, \quad (8)$$

where  $x_r = (i_{ds}, i_{qs}, \lambda_{dr}, \lambda_{qr}, \omega)^T$ ,  $Y_r = (\dot{\omega}, |\dot{\lambda}_r|)^T$ ,

$$\theta_r = (R_r, R_s, R_r^2, R_r R_s)^T, \quad u = (V_{ds}, V_{qs})^T.$$

Nonlinear portion in normal form of the stator flux model

$$Y_s = F_s(x_s) + B_s(x_s)\theta_s + G_s(x_s)u, \quad (9)$$

where  $x_s = (i_{ds}, i_{qs}, \lambda_{ds}, \lambda_{qs}, \omega)^T$ ,  $Y_s = (\dot{\omega}, |\dot{\lambda}_s|)^T$ ,

$$\theta_s = (R_r, R_s)^T.$$

$Y_r, Y_s$  are the derivatives of control outputs by the relative degrees,  $u$  is the control input and  $\theta_r, \theta_s$  are unknown parameter factors which consist of stator and rotor resistances. The matrices  $F_r, B_r, G_r, F_s, B_s$ , and  $G_s$  are given in the Appendix. In both cases, load torque is assumed to be zero, since it makes no difference between them. In addition to the relative degree issue, the rotor flux model has two problems which are absent in the stator flux model.

- The one is overparameterization, such that unknown parameter matrix  $\theta_r$  has  $R_r^2$  and  $R_r R_s$  as well as  $R_r$  and  $R_s$ .

- The other is that  $G_r(x_r, \theta_r)$  contains an unknown parameter  $R_r$  and cannot be linearly parameterized, which makes it extremely difficult to design the adaptive control scheme.

To solve these problems, the time varying state coordinate transformation depending on the parameter estimate has been used in [10], which requires a large amount of computation effort. Another solution is to use the reduced model of an induction motor, where the control inputs are not stator voltages but stator currents [12][13][18]. However, such a method has the assumption of ideal current controllers where stator currents are perfectly controlled by stator voltages. In the case of the stator flux model, such problems disappear and control inputs and parameter adaptations laws can be derived straightforwardly from (9).

To control motor speed, two control loops are used in this paper. If the loop gain of the torque controller is relatively so high that torque can be supposed to be its reference signal in the speed controller, torque and speed controllers can be designed independently. Torque and flux modulus tracking is achieved with the adaptation of rotor resistance and stator resistance in the inner loop and speed tracking is achieved with the adaptation of load torque in the outer loop. In other words, the inner loop controller is related with the electrical modeling of the induction motor as shown in (5), and the outer loop controller is related with the mechanical dynamics as shown in (7). Fig. 1 shows the control block diagram.

As a speed controller, a simple adaptive feedback linearization control scheme is used to estimate load torque. However, in this case, the other mechanical parameter, inertia  $J$  must be known. In some applications such as a rolling mill drive system, motor inertia may be varied during the process or there may be time varying load torque which cannot be assumed to

be constant, so it is necessary to design another speed controller. To design the speed controller and the torque controller independently makes it easy and efficient to modify the overall controller. Hence, the independent design approach is adopted in field-oriented control and also in feedback linearization control [11].

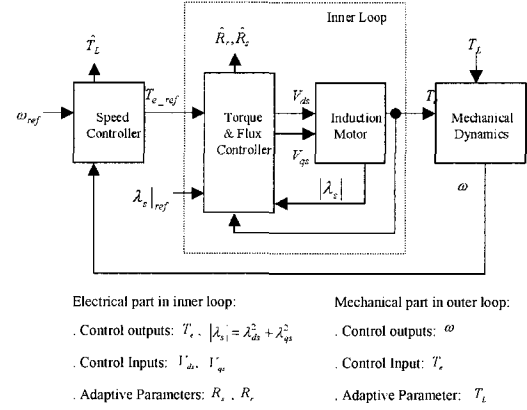


Fig. 1. Control block diagram.

### III. Torque and flux controller design.

Motor torque and square flux modulus are defined as outputs to be controlled, stator voltages are defined as control inputs, and stator resistance and rotor resistance are adaptive parameters. We assume that both resistances are constant, the reference signals for torque and square flux modulus are bounded along with their time derivatives, and all states are available bounded signals. The stator flux model shown in (5) is rewritten in matrix form.

$$\dot{x} = f_1(x) + f_2(x)R_r + f_3(x)R_s + g_1(x)V_{ds} + g_2(x)V_{qs},$$

where  $x = [\lambda_{ds}, \lambda_{qs}, i_{ds}, i_{qs}]^T$ ,

$$f_1(x) = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_\sigma} \omega_r \lambda_{qs} - \omega_r i_{qs} \\ -\frac{1}{L_\sigma} \omega_r \lambda_{ds} + \omega_r i_{ds} \end{bmatrix}, \quad f_2(x) = \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda_{ds} - L_s i_{ds}}{L_\sigma} \\ \frac{\lambda_{qs} - L_s i_{qs}}{L_\sigma} \end{bmatrix},$$

$$f_3(x) = \begin{bmatrix} -i_{ds} \\ -i_{qs} \\ -\frac{1}{L_\sigma} i_{ds} \\ -\frac{1}{L_\sigma} i_{qs} \end{bmatrix}, \quad g_1(x) = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{L_\sigma} \\ 0 \end{bmatrix}, \quad g_2(x) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{1}{L_\sigma} \end{bmatrix}. \quad (10)$$

Define control outputs  $y_1$  and  $y_2$  as motor torque and square flux modulus respectively.

$$y_1 = T_e = \frac{3P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}),$$

$$y_2 = |\lambda_s|^2 = \lambda_{ds}^2 + \lambda_{qs}^2. \quad (11)$$

Define error variables  $z_1, z_2$  between  $y_1, y_2$  and their references as

$$\begin{aligned} z_1 &= T_e - T_{e\_ref} = y_1 - y_{1\_ref}, \\ z_2 &= |\lambda_s| - |\lambda_s|_{ref} = y_2 - y_{2\_ref}, \end{aligned} \quad (12)$$

where  $y_{1\_ref} = T_{e\_ref}$  and  $y_{2\_ref} = |\lambda_s|_{ref}$ , and  $ref$  denotes the reference values of each variable. The derivatives of  $z_1, z_2$  are given by (13) using Lie derivatives,

$$\begin{aligned} \frac{d}{dt} z_1 &= L_{f_1} y_1 + L_{f_2} y_1 \cdot R_r + L_{f_3} y_1 \cdot R_s + L_{g_1} y_1 \cdot V_{ds} \\ &\quad + L_{g_2} y_1 \cdot V_{qs} - \dot{y}_{1\_ref}, \\ \frac{d}{dt} z_2 &= L_{f_3} y_2 \cdot R_s + L_{g_1} y_2 \cdot V_{ds} + L_{g_2} y_2 \cdot V_{qs} - \dot{y}_{2\_ref}, \end{aligned}$$

where  $L_{f_1} y_1 = \frac{3P}{2} \left[ -\frac{1}{L_\sigma} (\lambda_{ds}^2 + \lambda_{qs}^2) + i_{ds} \lambda_{ds} + i_{qs} \lambda_{qs} \right] \omega_r$ ,

$$\begin{aligned} L_{f_2} y_1 &= -\frac{3P}{2} \cdot \frac{L_s}{L_r L_\sigma} (i_{qs} \lambda_{ds} - i_{ds} \lambda_{qs}) = -\frac{L_s}{L_r L_\sigma} T_e, \\ L_{f_3} y_1 &= -\frac{3P}{2} \cdot \frac{1}{L_\sigma} (i_{qs} \lambda_{ds} - i_{ds} \lambda_{qs}) = -\frac{1}{L_\sigma} T_e, \\ L_{g_1} y_1 &= \frac{3P}{2} \cdot \frac{1}{L_\sigma} (L_\sigma i_{qs} - \lambda_{qs}), \\ L_{g_2} y_1 &= -\frac{3P}{2} \cdot \frac{1}{L_\sigma} (L_\sigma i_{ds} - \lambda_{ds}), \\ L_{f_3} y_2 &= -2(\lambda_{ds} i_{ds} + \lambda_{qs} i_{qs}), \\ L_{g_1} y_2 &= 2\lambda_{ds}, \\ L_{g_2} y_2 &= 2\lambda_{qs}. \end{aligned} \quad (13)$$

$L_f h$  is the Lie derivative of  $h$  with respect to  $f$ . Determine state feedback control inputs as follows

$$\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = D^{-1} \begin{bmatrix} -L_{f_1} y_1 - L_{f_2} y_1 \cdot \hat{R}_r - L_{f_3} y_1 \cdot \hat{R}_s + \dot{y}_{1\_ref} - c_1 z_1 \\ -L_{f_3} y_2 \cdot \hat{R}_s + \dot{y}_{2\_ref} - c_2 z_2 \end{bmatrix} \quad (14)$$

where  $D = \begin{bmatrix} L_{g_1} y_1 & L_{g_2} y_1 \\ L_{g_1} y_2 & L_{g_2} y_2 \end{bmatrix}$ ,  $\hat{R}_r$  and  $\hat{R}_s$  are the estimates of  $R_r$  and  $R_s$  respectively and  $c_1, c_2$  are positive constant gains. Now, we check whether  $D$  is nonsingular or not,

$$\det D = \frac{3P}{L_\sigma} [\lambda_{qs} (L_\sigma i_{qs} - \lambda_{qs}) + \lambda_{ds} (L_\sigma i_{ds} - \lambda_{ds})]. \quad (15)$$

(15) can be rewritten as (16) using the relations between stator fluxes and rotor fluxes, i.e.  $L_\sigma i_{ds} - \lambda_{ds} = -\frac{L_m}{L_r} \lambda_{dr}$

and  $L_\sigma i_{qs} - \lambda_{qs} = -\frac{L_m}{L_r} \lambda_{qr}$ , which can be easily derived from (2).

$$\begin{aligned} \det D &= -\frac{3P}{L_\sigma} \frac{L_m}{L_r} (\lambda_{ds} \lambda_{dr} + \lambda_{qs} \lambda_{qr}) = -\frac{3P}{L_\sigma} \frac{L_m}{L_r} \bar{\lambda}_{dqs} \cdot \bar{\lambda}_{dqr} \\ &= -\frac{3P}{L_\sigma} \frac{L_m}{L_r} \left| \bar{\lambda}_{dqs} \right| \left| \bar{\lambda}_{dqr} \right| \cos \alpha. \end{aligned} \quad (16)$$

where  $\bar{\lambda}_{dqs}$  and  $\bar{\lambda}_{dqr}$  are the stator flux vector and the rotor flux vector respectively, that is,  $\bar{\lambda}_{dqs} = (\lambda_{ds}, \lambda_{qs})$  and  $\bar{\lambda}_{dqr} = (\lambda_{dr}, \lambda_{qr})$ .  $\alpha$  is the angle between the stator flux vector and the rotor flux vector. From (16),  $D$  is singular when the stator flux vector is orthogonal to the rotor flux vector. However, such a case is physically impossible since leakage inductances  $L_{ls}, L_{lr}$  which make the angle  $\alpha$  are such small values [2]. Therefore,  $D$  is singular only at the start-up of the motor.

Define parameter errors as follows.

$$\tilde{R}_r = R_r - \hat{R}_r, \quad \tilde{R}_s = R_s - \hat{R}_s. \quad (17)$$

By applying control input (14) to (13), the error dynamics of  $z_1, z_2$  are derived as

$$\frac{d}{dt} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = C \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + W^T(t) \begin{bmatrix} \tilde{R}_r \\ \tilde{R}_s \end{bmatrix}, \quad (18)$$

where  $C = \begin{bmatrix} -c_1 & 0 \\ 0 & -c_2 \end{bmatrix}$ ,  $W^T(t) = \begin{bmatrix} L_{f_2} y_1 & L_{f_3} y_1 \\ 0 & L_{f_3} y_2 \end{bmatrix}$ .

From (12), the errors  $z_1, z_2$  inevitably occur whenever each reference changes even when the estimates of both resistances are true values. So, if we use  $z_1, z_2$  directly for the parameter adaptation, instantaneous fluctuation occurs in the estimates of both resistances. To overcome it, a reference model is introduced. Define the reference model where unknown parameters disappear using new states  $z_{1M}, z_{2M}$ ,

$$\frac{d}{dt} \begin{bmatrix} z_{1M} \\ z_{2M} \end{bmatrix} = C \begin{bmatrix} z_{1M} \\ z_{2M} \end{bmatrix}. \quad (19)$$

Define error variables  $e_1, e_2$  between  $z_1, z_2$  and  $z_{1M}, z_{2M}$  respectively.

$$e_1 = z_1 - z_{1M}, \quad e_2 = z_2 - z_{2M}. \quad (20)$$

From (18) and (19), their dynamics is given by (21).

$$\frac{d}{dt} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = C \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + W^T(t) \begin{bmatrix} \tilde{R}_r \\ \tilde{R}_s \end{bmatrix}. \quad (21)$$

Adaptive laws of  $\hat{R}_r$  and  $\hat{R}_s$  are determined as in (22) to make the error dynamics (21) asymptotically stable.

$$\begin{aligned} \frac{d}{dt} \hat{R}_r &= \gamma_a L_{f_2} y_1 e_1 = -\gamma_a \frac{L_s}{L_r L_\sigma} T_e \cdot e_1, \\ \frac{d}{dt} \hat{R}_s &= \gamma_b \{ L_{f_3} y_1 \cdot e_1 + L_{f_3} y_2 \cdot e_2 \} \\ &= -\gamma_b \frac{1}{L_\sigma} T_e \cdot e_1 - 2\gamma_b (\lambda_{ds} i_{ds} + \lambda_{qs} i_{qs}) e_2. \end{aligned} \quad (22)$$

**Theorem 1:** If state feedback control is given by (14) and the parameter adaptation laws are given by (22), then,

i)  $e_1, e_2 \in L_\infty \cap L_2$ , and zero tracking errors are achieved,

$$\text{i.e. } \lim_{t \rightarrow \infty} |T_e - T_{e\_ref}| = 0, \quad \lim_{t \rightarrow \infty} |\lambda_s| - |\lambda_s|_{ref} = 0.$$

ii) If the persistent excitation condition is satisfied, i.e.  $\int_t^{t+T} W(\tau)W^T(\tau)d\tau$  is positive definite for some  $T > 0$  and every  $t \geq 0$ , then, the parameter errors  $(\tilde{R}_r, \tilde{R}_s) = 0$  is exponentially stable.

**Proof of theorem 1 :** i) Define a Lyapunov function

$$V_0 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2\gamma_a}\tilde{R}_r^2 + \frac{1}{2\gamma_b}\tilde{R}_s^2, \quad (23)$$

where  $\gamma_a, \gamma_b$  are positive real numbers and  $e_1, e_2$  are defined in (20) and the dynamics are given in (21). The derivative of  $V_0$  is

$$\begin{aligned} \frac{d}{dt}V_0 = & -c_1e_1^2 - c_2e_2^2 + \tilde{R}_r \{L_{f_2}y_1 \cdot e_1 - \frac{1}{\gamma_a} \frac{d}{dt}\hat{R}_r\} \\ & + \tilde{R}_s \{L_{f_3}y_1 \cdot e_1 + L_{f_3}y_2 \cdot e_2 - \frac{1}{\gamma_b} \frac{d}{dt}\hat{R}_s\}. \end{aligned} \quad (24)$$

The parameter adaptation laws for  $\hat{R}_r$  and  $\hat{R}_s$  as shown in (22) yield

$$\frac{d}{dt}V_0 = -c_1e_1^2 - c_2e_2^2. \quad (25)$$

This guarantees that  $e_1, e_2, \tilde{R}_r, \tilde{R}_s$  are bounded and  $e_1, e_2 \in L_2$ . Since  $R_r, R_s$  are assumed to be constant parameters,  $\hat{R}_r$  and  $\hat{R}_s$  are also bounded from (17). Matrix  $W^T(t)$  contains only bounded signals and it follows that  $\dot{e}_1, \dot{e}_2$  are also bounded from (21). From Barbalat's lemma,

$$\lim_{t \rightarrow \infty} |e_1| = 0, \quad \lim_{t \rightarrow \infty} |e_2| = 0, \quad (26)$$

which implies that

$$\lim_{t \rightarrow \infty} z_1 = \lim_{t \rightarrow \infty} z_{1M} = 0, \quad \lim_{t \rightarrow \infty} z_2 = \lim_{t \rightarrow \infty} z_{2M} = 0, \quad (27)$$

because  $z_{1M}, z_{2M}$  exponentially converge to zero from (19), so zero tracking errors of torque and flux modulus are achieved. ■

ii) The proof of the convergence of parameter errors is shown in [20, pp.367-370]. ■

**Remark 1 :** Now, we check the persistent excitation condition,

$$W \cdot W^T = \begin{bmatrix} \frac{L_s^2}{L_r^2 L_\sigma^2} T_e^2 & \frac{L_s}{L_r L_\sigma^2} T_e^2 \\ \frac{L_s}{L_r L_\sigma^2} T_e^2 & 4(\lambda_{ds} i_{ds} + \lambda_{qs} i_{qs})^2 \end{bmatrix}. \quad (28)$$

If motor torque  $T_e$  is zero such as in the steady state with no load torque, the persistent excitation condition cannot be satisfied, since  $\int_t^{t+T} W(\tau)W^T(\tau)d\tau$  is not positive definite. Also, (22) shows that if motor torque  $T_e$  is zero, the update of rotor resistance does not occur. But if flux is changing in zero torque region, rotor current exists in rotor circuit, which means

that there is a possibility to estimate rotor resistance without torque. To use this situation, a new method using a stator current estimator is proposed as follows.

We design the estimator of stator currents,  $\hat{i}_{ds}, \hat{i}_{qs}$  as (29)

$$\begin{aligned} \frac{d}{dt}\hat{i}_{ds} = & -\frac{\hat{R}_s}{L_\sigma}i_{ds} - \frac{\hat{R}_r}{L_r L_\sigma}(L_s i_{ds} - \lambda_{ds}) \\ & + \frac{1}{L_\sigma}\omega_r \lambda_{qs} - \omega_r i_{qs} + \frac{1}{L_\sigma}V_{ds} + c_3 \tilde{i}_{ds}, \\ \frac{d}{dt}\hat{i}_{qs} = & -\frac{\hat{R}_s}{L_\sigma}i_{qs} - \frac{\hat{R}_r}{L_r L_\sigma}(L_s i_{qs} - \lambda_{qs}) \\ & - \frac{1}{L_\sigma}\omega_r \lambda_{qs} + \omega_r i_{ds} + \frac{1}{L_\sigma}V_{qs} + c_4 \tilde{i}_{qs}, \end{aligned} \quad (29)$$

where  $c_3, c_4$  are positive constants and  $\tilde{i}_{ds}, \tilde{i}_{qs}$  are current errors defined by  $\tilde{i}_{ds} = i_{ds} - \hat{i}_{ds}$  and  $\tilde{i}_{qs} = i_{qs} - \hat{i}_{qs}$  respectively. This is the same as modeling (5) except that estimated parameters are used and current error terms  $c_3 \tilde{i}_{ds}, c_4 \tilde{i}_{qs}$  are added.

From (10) and (29), error dynamics (21) can be modified as

$$\frac{d}{dt} \begin{bmatrix} e_1 \\ e_2 \\ \tilde{i}_{ds} \\ \tilde{i}_{qs} \end{bmatrix} = C' \begin{bmatrix} e_1 \\ e_2 \\ \tilde{i}_{ds} \\ \tilde{i}_{qs} \end{bmatrix} + W'^T(t) \begin{bmatrix} \tilde{R}_r \\ \tilde{R}_s \end{bmatrix}, \quad (30)$$

$$\text{where, } C' = - \begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix},$$

$$W'^T(t) = \begin{bmatrix} L_{f_2}y_1 & L_{f_3}y_1 \\ 0 & L_{f_3}y_2 \\ \frac{\lambda_{ds} - L_s i_{ds}}{L_r L_\sigma} & -\frac{i_{ds}}{L_\sigma} \\ \frac{\lambda_{qs} - L_s i_{qs}}{L_r L_\sigma} & -\frac{i_{qs}}{L_\sigma} \end{bmatrix}. \quad (31)$$

Adaptive laws for  $\hat{R}_r$  and  $\hat{R}_s$  are designed to make the error dynamics (30) asymptotically stable.

$$\begin{aligned} \frac{d}{dt}\hat{R}_r = & -\frac{\gamma_1}{L_r L_\sigma} \{L_s T_e \cdot e_1 + (\lambda_{ds} - L_s i_{ds})\tilde{i}_{ds} \\ & + (\lambda_{qs} - L_s i_{qs})\tilde{i}_{qs}\}, \\ \frac{d}{dt}\hat{R}_s = & -\frac{\gamma_2}{L_\sigma} \{T_e \cdot e_1 - 2L_\sigma(\lambda_{ds} i_{ds} + \lambda_{qs} i_{qs}) \cdot e_2 \\ & - i_{ds} \tilde{i}_{ds} - i_{qs} \tilde{i}_{qs}\}. \end{aligned} \quad (32)$$

**Theorem 2 :** If state feedback control is given by (14), the estimates of stator currents are given by (29) and parameter adaptation laws are given by (32), then

i)  $e_1, e_2, \tilde{i}_{ds}, \tilde{i}_{qs} \in L_\infty \cap L_2$ , zero tracking errors are achieved, i.e.  $\lim_{t \rightarrow \infty} |T_e - T_{e\_ref}| = 0, \quad \lim_{t \rightarrow \infty} |\lambda_s| - |\lambda_s|_{ref} = 0,$

and the current errors converge to their actual values, i.e.

$$\lim_{t \rightarrow \infty} |\tilde{i}_{ds}| = 0, \quad \lim_{t \rightarrow \infty} |\tilde{i}_{qs}| = 0.$$

ii) If the persistent excitation condition is satisfied, i.e.  $\int_t^{t+T} W'(\tau)W'(\tau)d\tau$  is positive definite for some  $T>0$  and every  $t \geq 0$ , then, the parameter errors  $(\tilde{R}_r, \tilde{R}_s) = 0$  are exponentially stable.

**Proof of theorem 2 :** The procedure is the same as in *theorem 1*. First, define a Lyapunov function as

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}\tilde{i}_{ds}^2 + \frac{1}{2}\tilde{i}_{qs}^2 + \frac{1}{2}\frac{1}{\gamma_1}\tilde{R}_r^2 + \frac{1}{2}\frac{1}{\gamma_2}\tilde{R}_s^2, \quad (33)$$

where  $\gamma_1, \gamma_2$  are positive real numbers. Compared to (23), current error terms  $\tilde{i}_{ds}, \tilde{i}_{qs}$  are added.

Its derivative is given by

$$\begin{aligned} \frac{d}{dt}V = & -c_1e_1^2 - c_2e_2^2 - c_3\tilde{i}_{ds}^2 - c_4\tilde{i}_{qs}^2 \\ & + \tilde{R}_r \{L_{f2}y_1 \cdot e_1 + \frac{1}{L_r L_\sigma} \{(\lambda_{ds} - L_s i_{ds})\tilde{i}_{ds} \\ & + (\lambda_{qs} - L_s i_{qs})\tilde{i}_{qs}\} - \frac{1}{\gamma_1} \frac{d}{dt} \hat{R}_r \} \\ & + \tilde{R}_s \{L_{f3}y_1 \cdot e_1 + L_{f3}y_2 \cdot z_2 - \frac{1}{L_\sigma} i_{ds} \tilde{i}_{ds} \\ & - \frac{1}{L_\sigma} i_{qs} \tilde{i}_{qs} - \frac{1}{\gamma_2} \frac{d}{dt} \hat{R}_s \}. \end{aligned} \quad (34)$$

Applying adaptive laws (32) to (34) yield (35).

$$\frac{d}{dt}V = -c_1e_1^2 - c_2e_2^2 - c_3\tilde{i}_{ds}^2 - c_4\tilde{i}_{qs}^2. \quad (35)$$

Similar to the proof of theorem 1, this guarantees that  $e_1, e_2, \tilde{R}_r, \tilde{R}_s, \tilde{i}_{ds}, \tilde{i}_{qs}$  are bounded and  $e_1, e_2, \tilde{i}_{ds}, \tilde{i}_{qs} \in L_2$ . Since  $i_{ds}, i_{qs}$  are assumed to be bounded signals,  $W'^T$  is also bounded and it follows that  $\tilde{i}_{ds}, \tilde{i}_{qs}$  are bounded from (30). From Barbalat's lemma,

$$\lim_{t \rightarrow \infty} |\tilde{i}_{ds}| = 0, \quad \lim_{t \rightarrow \infty} |\tilde{i}_{qs}| = 0. \quad (36)$$

The remaining proof for the tracking and parameter estimation is the same as that of theorem 1.

**Remark 2 :** Compared to (22), the adaptation laws given by (32) for  $\hat{R}_r$  has additional terms, so the update of  $\hat{R}_r$  will be carried out using current errors even though there does not exist motor torque. We can see the difference in the persistent excitation condition,

$$W' \cdot W'^T = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}, \quad (37)$$

$$\text{where } w_{11} = \frac{L_s^2}{L_r L_\sigma^2} T_e^2 + \left( \frac{\lambda_{ds} - L_s i_{ds}}{L_r L_\sigma} \right)^2 + \left( \frac{\lambda_{qs} - L_s i_{qs}}{L_r L_\sigma} \right)^2,$$

$$\begin{aligned} w_{12} = w_{21} = & \frac{L_s}{L_r L_\sigma^2} T_e^2 - \frac{\lambda_{ds} - L_s i_{ds}}{L_r L_\sigma^2} i_{ds} - \frac{\lambda_{qs} - L_s i_{qs}}{L_r L_\sigma^2} i_{qs}, \\ w_{22} = & \frac{1}{L_\sigma^2} T_e^2 + 4(\lambda_{ds} i_{ds} + \lambda_{qs} L_s i_{qs}) + \frac{1}{L_\sigma^2} (i_{ds}^2 + i_{qs}^2). \end{aligned} \quad (38)$$

Even though motor torque  $T_e$  is zero, unless flux is constant, the persistent excitation condition can be satisfied due to the additional term in (38). However, if flux is constant with zero torque, which means that rotor currents are zero, i.e.  $\lambda_{ds} = L_s i_{ds}$  and  $\lambda_{qs} = L_s i_{qs}$ , the persistent excitation condition is not satisfied, since  $w_{11} = w_{12} = w_{21} = 0$ . If rotor currents are absent, any kind of model based adaptation algorithms should not estimate the rotor resistance, because rotor resistance disappears in the modeling of induction motors, which is shown by  $f_2(x) = 0$  in (10).

#### IV. Speed controller design

To estimate load torque, we design a simple adaptive feedback linearization controller with the assumption that the inertia  $J$  is a known constant, load torque  $T_L$  is an unknown constant, and the reference of speed and its derivative are all bounded signals. The mechanical dynamics shown in (7) is rewritten by

$$\frac{d}{dt}\omega = \frac{T_e}{J} - \frac{T_L}{J}, \quad (39)$$

where we define motor torque  $T_e$  as control input and motor speed  $\omega$  as control output respectively. The derivation of the speed controller is simple, because it is only a 1<sup>st</sup> order system. Define output error variable  $z_3$  as

$$z_3 = \omega - \omega_{ref}, \quad (40)$$

where  $\omega_{ref}$  is the speed reference. We determine control input  $T_e$  as

$$T_e = J\omega_{ref} + \hat{T}_L - J \cdot c_5 z_3, \quad (41)$$

where  $c_5$  is a positive constant. Then, by applying (41) to (39), the error dynamics of  $z_3$  is written by

$$\dot{z}_3 = -c_5 z_3 - \frac{\tilde{T}_L}{J}, \quad (42)$$

where parameter estimation error  $\tilde{T}_L$  is defined as  $\tilde{T}_L = T_L - \hat{T}_L$ . Define the reference model where an unknown variable  $\tilde{T}_L$  does not appear

$$\dot{z}_{3M} = -c_5 z_{3M}. \quad (43)$$

Define error variable  $e_3$  between  $z_3$  and  $z_{3M}$  as

$$e_3 = z_3 - z_{3M}. \quad (44)$$

From (42) and (43), its derivative is given by

$$\dot{e}_3 = -c_5 e_3 - \frac{\tilde{T}_L}{J}. \quad (45)$$

We design the load torque adaptation law as

$$\frac{d}{dt} \hat{T}_L = -\gamma_3 \frac{1}{J} e_3. \quad (46)$$

**Theorem 3 :** If state feedback control is given by (41), and the adaptation law of load torque is given in (46), then,  $e_3 \in L_\infty \cap L_2$ ,  $\tilde{T}_L \in L_\infty$ , and zero tracking error is achieved, i.e.  $\lim_{t \rightarrow \infty} |\omega - \omega_{ref}| = 0$ . In addition, the estimation error of the load torque also converges to zero, i.e.  $\lim_{t \rightarrow \infty} |\tilde{T}_L| = 0$ .

**Proof of theorem 3 :** Define a Lyapunov function as

$$V_s = \frac{1}{2} e_3^2 + \frac{1}{2\gamma_3} \tilde{T}_L^2. \quad (47)$$

From (45), its derivative is given by

$$\frac{d}{dt} V_s = -c_5 e_3^2 + \tilde{T}_L \left\{ -\frac{1}{J} e_3 - \frac{1}{\gamma_3} \frac{d}{dt} \hat{T}_L \right\}. \quad (48)$$

The load torque adaptation law (46) yields

$$\frac{d}{dt} V_s = -c_5 e_3^2. \quad (49)$$

This guarantees that  $e_3$ ,  $\tilde{T}_L$  are bounded and  $e_3 \in L_2$ . From (45),  $\dot{e}_3$  is also bounded. From barbalat's lemma,  $\lim_{t \rightarrow \infty} |e_3| = 0$ , which means that  $\lim_{t \rightarrow \infty} |z_3| = \lim_{t \rightarrow \infty} |z_{3M}| = 0$ , because  $z_{3M}$  exponentially converges to zero from (43). Since  $\lim_{t \rightarrow \infty} |e_3| = 0$  and  $J$  is a positive constant, the estimation error of the load torque  $\tilde{T}_L$  also converges to zero from (45). ■

In the case that the tracking of speed is only a matter of concern, another controller such as PI may be used as a speed controller.

## V. Simulations

The proposed control algorithm is investigated for the motor whose data are listed in Table I, the square modulus of flux  $|\lambda_s|$  is set to its rated value 0.21Wb at 0.01sec, and speed reference starts from 0 to 1800r/min at 1sec. To avoid the singularity in computing control inputs  $V_{ds}, V_{qs}$ , the initial values of stator voltages  $V_{ds}, V_{qs}$ , are all set to 0.1v in 10msec. The load torque is applied at 3sec to the amount of 10Nm. Fig. 2 is the case where all of the parameters are known, Fig. 3 is the case of -20% initial error in rotor resistance and +50% initial errors in stator resistance, and Fig. 4 shows the case of +50% initial error in rotor resistance and -50% error in stator resistance. The stator current estimator proposed in section III is not used yet. In each Fig, (a), (b), (c), (d), (e), (f), (g), and (h) show speed  $\omega$  and its reference  $\omega_{ref}$ , torque  $T_e$  and its reference  $T_{e\_ref}$ , square flux modulus  $|\lambda_s|$  and its reference  $|\lambda_{s\_ref}|$ , load torque  $T_L$  and its estimate  $\hat{T}_L$ , normalized  $R_r$ , normalized  $R_s$ , torque and that of the reference model, and square flux modulus and that of the reference model, respectively. Figs (a) and (c) show that decoupling control between speed and flux is achieved and Figs (b) and (c) show that motor torque and flux modulus are also controlled independently. Fig (d) shows that the estimate of load torque converges to the true value in all cases. Rotor resistance is up-

dated so that the difference between actual torque and that of reference model, which is shown in (g), converges to zero. However, Fig (e) shows that rotor resistance can not be updated when motor torque is zero. Fig (f) shows that even though there does not exist motor torque, the estimate of stator resistance converges to its true value using flux error  $e_2$  shown in (h).

Table 1. Motor Parameters.

Power	3.7 [Kw]	$L_r$	29.97 [mH]
$R_s$	0.31 [ $\Omega$ ]	$L_s$	29.97 [mH]
$R_r$	0.41 [ $\Omega$ ]	$L_m$	28.92 [mH]
Pole pairs	2	$J$	0.03 [kgm <sup>2</sup> ]

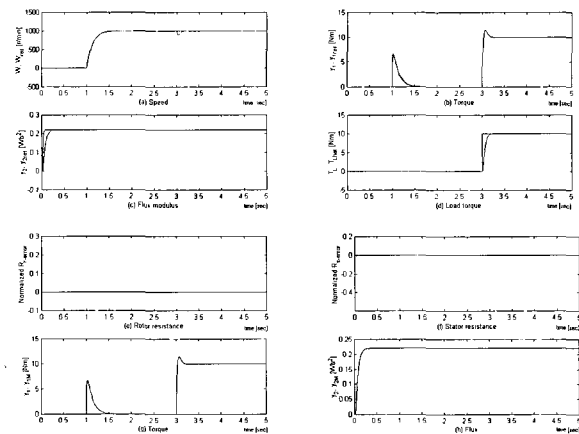


Fig. 2. Simulation with known parameters

$$(\hat{R}_r(0) = R_r \text{ and } \hat{R}_s(0) = R_s).$$

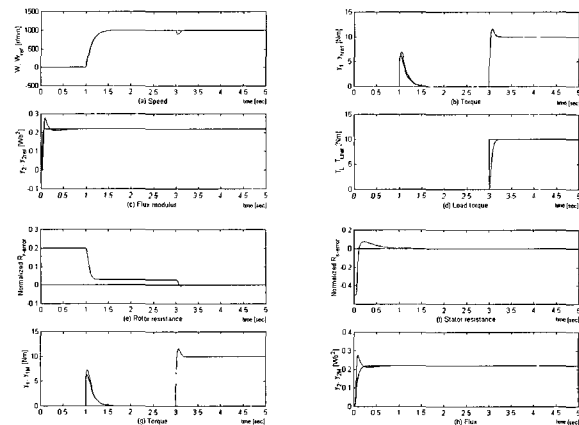
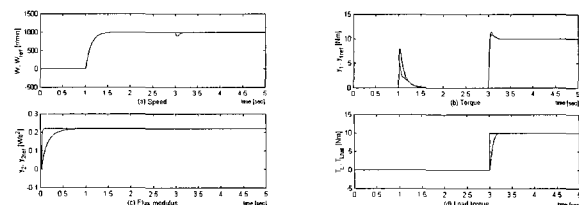


Fig. 3. Simulation with +50% initial parameters error

$$(\hat{R}_r(0) = R_r * 0.8, \hat{R}_s(0) = R_s * 1.5).$$



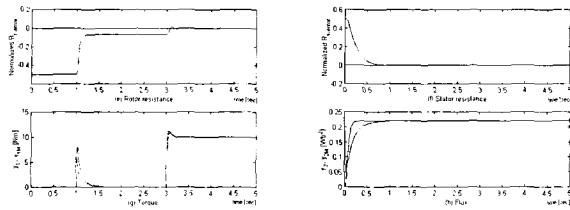


Fig. 4. Simulation with -50% initial parameters error ( $\hat{R}_r(0) = R_r * 1.5, \hat{R}_s(0) = R_s * 0.5$ ).

To overcome the fact that rotor resistance is not updated when motor torque does not exist, the proposed stator current estimator is added. Fig. 5 is the simulation of -20% initial error in rotor resistance and +50% initial errors in stator resistance and Fig. 6 is the simulations of +50% initial error in rotor resistance and -50% initial errors in stator resistance. Figs (i), (j), (k) and (l) show  $V_{ds}, \tilde{i}_{ds}, V_{qs}$ , and  $\tilde{i}_{qs}$  respectively. Fig. (e) shows that using the current errors shown in (j) and (l),  $\hat{R}_r$  can be estimated even though there is no electrical torque. Figs (i) and (k) show that the control inputs, that is, stator voltage  $V_{ds}, V_{qs}$ , are within their available range.

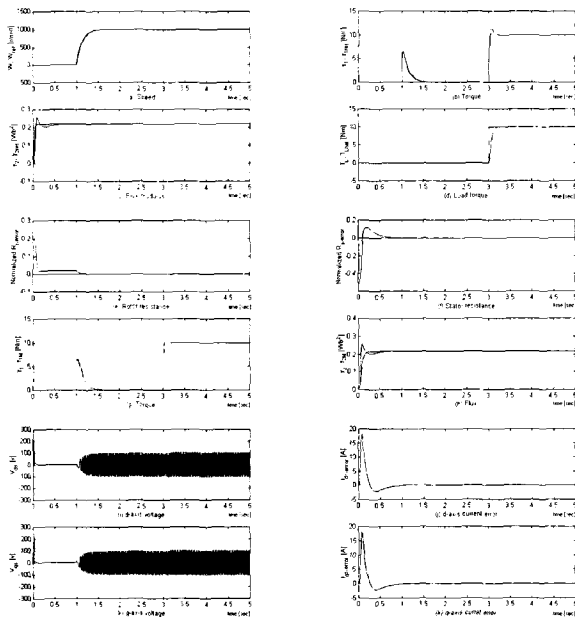


Fig. 5. Simulation using a current estimator with +50% initial errors ( $\hat{R}_r(0) = R_r * 0.8, \hat{R}_s(0) = R_s * 1.5$ ).

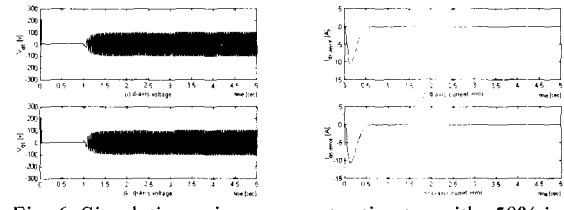
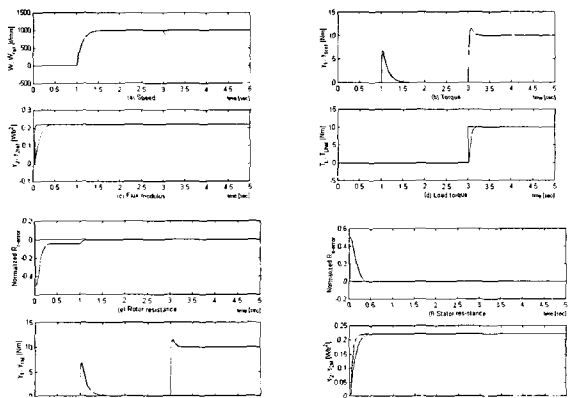


Fig. 6. Simulation using a current estimator with -50% initial errors ( $\hat{R}_r(0) = R_r * 1.5, \hat{R}_s(0) = R_s * 0.5$ ).

VI. Experiments

The proposed adaptive feedback linearization control scheme is implemented for a 3.7kw induction motor whose parameters are used in the simulations. The controller consists of TMS320C40 floating point digital signal processor for the computation of the nonlinear control, 14bit A/D converter for current sensing, 12 bit D/A converter for monitoring the internal variables through an oscilloscope, one RS-232 serial port for communicating with a PC, a pulse counter for measuring motor speed, and gating driver circuits for PWM inverter. PWM switching frequency is 2.5kHz, and the sampling time of the proposed torque and flux controller is 200  $\mu$ sec. The stator currents and motor speed are measured in every sampling time through hall-sensors and an encoder whose resolution is 1024 pulse  $rev^{-1}$ , respectively. Because stator flux measurement is not available, we use the flux observer proposed in [21]. Since we assume that all states are available, actual parameters are used in the flux observer. To combine the proposed algorithm with an adaptive flux observer is our next research goal. Load torque is zero and the speed controller is operated every 3msec. To control the flux modulus, PI is used in the outer loop as an additional flux controller, whose sampling time is the same as that of the speed controller. Fig. 7 shows the block diagram of the experiment set.

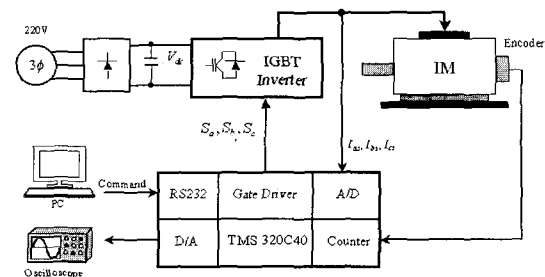
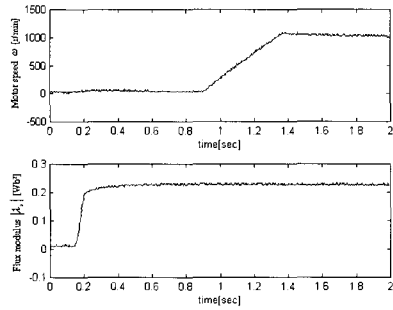


Fig. 7. Experiments set.

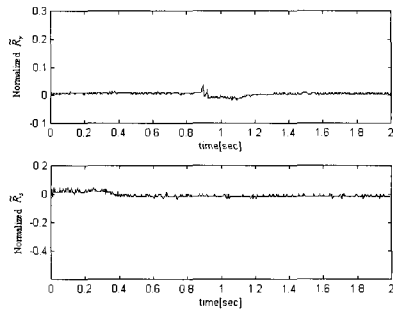
Fig. 8 is the case of no initial parameter errors in both resistances, and Fig. 9 is the case of +20% initial error of the rotor resistance and -50% initial error of the stator resistance respectively, in which the stator current estimator is not used. Flux reference is set to its rated value and speed reference is increased to 1000r/min after flux build-up. The speed reference ramped to 0.5 sec rising time to guarantee sufficient time interval when torque exists to satisfy persistent excitation condition during parameter adaptation. In each Fig, (a) shows the motor speed  $\omega$  and flux modulus  $|\lambda_s|$ , and (b) shows the normalized resistance errors of  $\hat{R}_r$  and  $\hat{R}_s$  respectively.



Fig (a) shows that the decoupling control between speed and flux is achieved. Fig. 8 shows that both resistances remain the initial values, which are actual ones. Fig. 9 shows that both resistance errors tend to their true values, however, rotor resistance is not updated until the motor speed starts to increase, which is a similar result to the simulations.

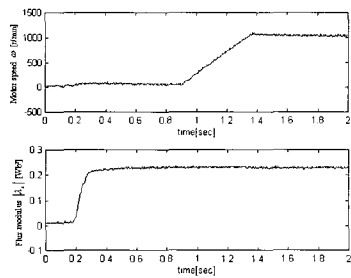


(a) Motor speed  $\omega$  and flux modulus  $|\lambda_s|$

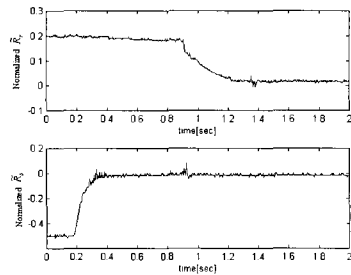


(b) Normalized  $\tilde{R}_r$  and  $\tilde{R}_s$

Fig. 8. Experiment results with  $\hat{R}_r(0) = R_r$  and  $\hat{R}_s(0) = R_s$ .



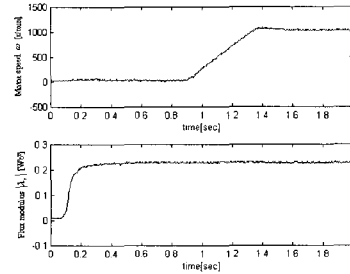
(a) Motor speed  $\omega$  and flux modulus  $|\lambda_s|$



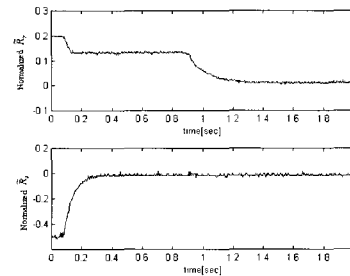
(b) Normalized  $\tilde{R}_r$  and  $\tilde{R}_s$

Fig. 9. Experiment results with  $\hat{R}_r(0) = R_r * 0.8$  and  $\hat{R}_s(0) = R_s * 1.5$ .

Fig. 10 is the case where the estimation of rotor resistance is performed with the stator current estimator. Although motor torque is zero, rotor resistance is updated during the change of flux, since rotor current exists. The parameter convergence rates are different between simulations and experiments because adaptive gains  $c_1, c_2, c_3, c_4, \gamma_1, \gamma_2$  determined by trial-error are some different.



(a) Motor speed  $\omega$  and flux modulus  $|\lambda_s|$



(b) Normalized  $\tilde{R}_r$  and  $\tilde{R}_s$

Fig. 10. Experiment results using a stator current estimator with  $\hat{R}_r(0) = R_r * 0.8$  and  $\hat{R}_s(0) = R_s * 1.5$

### VII. Conclusion

The presented adaptive control method based on the stator flux model has strong merits in that i) the relative degree for the flux modulus is reduced to one, which makes the control algorithm simple; ii) control inputs can be determined straightforwardly without state-space change of coordinates depending on parameter estimates; iii) overparameterization disappears, which yields an easy design of adaptive laws for the parameters. In addition, stator resistance as well as rotor resistance and load torque is also estimated, which has not been performed in prior works. To solve the problem that rotor resistance cannot be estimated without motor torque, we design a stator current estimator which improves the convergence of rotor resistance estimation. From another point of view, the proposed adaptive feedback linearization controller for torque and flux modulus can be considered as the current controller in the field-oriented control. Also, an additional flux controller such as PI may be used in the outer loop. The effectiveness and performance of the proposed method are verified by simulations and experiments.

### Appendix

The derivations of (8) and (9).

From the induction motor modeling as shown in (3)-(6)

and the mechanical dynamic equation (7), (8) and (9) can be computed directly with zero load torque.

Nonlinear portion in normal form of the rotor flux model;

$$Y_r = F_r(x_r) + B_r(x_r)\theta_r + G_r(x_r, \theta_r)u, \quad (50)$$

where

$$F_r = \begin{pmatrix} -\frac{3P}{2} \frac{L_m}{JL_r} \omega_r [\lambda_{dr} i_{ds} + \lambda_{qr} i_{qs} + \frac{L_m}{L_\sigma L_r} (\lambda_{dr}^2 + \lambda_{qr}^2)] \\ 0 \end{pmatrix}$$

$$G_r = \begin{pmatrix} -\frac{3P}{2} \frac{L_m}{JL_r L_\sigma} \lambda_{qr} & \frac{3P}{2} \frac{L_m}{JL_r L_\sigma} \lambda_{dr} \\ \frac{2L_m}{L_r L_\sigma} \lambda_{dr} \cdot R_r & \frac{2L_m}{L_r L_\sigma} \lambda_{qr} \cdot R_r \end{pmatrix},$$

$$B_r = \begin{pmatrix} b_{r_{-11}} & b_{r_{-12}} & 0 & 0 \\ b_{r_{-22}} & 0 & b_{r_{-23}} & b_{r_{-24}} \end{pmatrix},$$

$$b_{r_{-11}} = -\frac{1}{J} \left( \frac{1}{L_r} + \frac{L_m^2}{L_\sigma L_r^2} \right) \cdot \frac{3PL_m}{2L_r} (\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}),$$

$$b_{r_{-12}} = -\frac{1}{JL_\sigma} \cdot \frac{3PL_m}{2L_r} (\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}),$$

$$b_{r_{-22}} = \frac{2L_m}{L_r} \omega_r (\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}),$$

$$b_{r_{-23}} = \left( 1 + \frac{L_m^2}{L_\sigma L_r} \right) (\lambda_{dr}^2 + \lambda_{qr}^2) + L_m (i_{ds}^2 + i_{qs}^2) \\ - \left( 3L_m + \frac{L_m^2}{L_\sigma L_r} \right) (\lambda_{dr} i_{ds} + \lambda_{qr} i_{qs}),$$

$$b_{r_{-24}} = -\frac{2L_m}{L_r L_\sigma} (\lambda_{dr} i_{ds} + \lambda_{qr} i_{qs}).$$

Nonlinear portion in normal form of the stator flux model;

$$Y_s = F_s(x_s) + B_s(x_s)\theta_r + G_s(x_s)u \quad (51)$$

$$\text{where } F_s = \begin{pmatrix} \frac{3P}{2J} \omega_r [\lambda_{ds} i_{ds} + \lambda_{qs} i_{qs} - \frac{1}{L_\sigma} (\lambda_{ds}^2 + \lambda_{qs}^2)] \\ 0 \end{pmatrix},$$

$$B_s = \begin{pmatrix} -\frac{3PL_s}{2JL_\sigma L_r} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) & -\frac{3P}{2JL_\sigma} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \\ -2(\lambda_{ds} i_{ds} + \lambda_{qs} i_{qs}) & 0 \end{pmatrix}$$

$$G_s = \begin{pmatrix} \frac{3P}{2JL_\sigma} (L_\sigma i_{qs} - \lambda_{qs}) & -\frac{3P}{2JL_\sigma} (L_\sigma i_{ds} - \lambda_{ds}) \\ 2\lambda_{ds} & 2\lambda_{qs} \end{pmatrix}$$

### References

- [1] F. Blaschke, "The principle of field orientation applied to the new transvector closed-loop control system for rotating field machines," *Siemens-Rev.*, Vol. 39, pp. 217-220, 1972.
- [2] Peter Vas, *Electrical Machines and Drives*, Clarendon Press:Oxford, 1992.
- [3] Xingyi Xu, Rik De Doncker, and Donald W. Novotny, "A stator flux oriented induction machine drive," *PESC '88*, pp. 870-876, 1988.
- [4] W. L. Erdman and R. G. Hoft, "Induction machine field orientation along airgap and stator flux," *IEEE Trans. Energy Conversion*, vol. 5, no. 1, pp. 115-120, 1990.
- [5] Xingyi Xu, and Donald W. Novotny, "Implementation of direct stator flux orientation control on a versatile dsp based system," *IEEE Trans. Ind. Applicat.*, vol. 27, no. 4, pp. 694-700, 1991.
- [6] T. G. Habetler, F. Profumo, G. Griva, M. Pastorelli, and Alberto Bettini, "Stator resistance tuning in a stator-flux field-oriented drive using an instantaneous hybrid flux estimator," *IEEE Tran. Power Electronics*, vol. 13, no. 1, pp. 125-133, 1998.
- [7] Z. Krzeminski, "Nonlinear feedback and control strategy of the induction motor," in *Proc. IFAC Nonlin. Contr. Systems Design Sympos.*, pp. 162-167, Bordeaux, France, 1992.
- [8] A. De Luca, and G. Ulvi, "Design of exact nonlinear controller for induction motors," *IEEE Trans. Automat. Contr.* Vol. 34, No. 12, pp. 1304-1307, Dec. 1989.
- [9] Dong-II Kim, In-Joong Ha and Myoung-Sam Ko, "Control of induction motors via feedback linearization with input-output decoupling," *Int. J. Contr.*, vol. 51, pp. 863-883, 1990.
- [10] R. Marino, S. Peresada, and P. Valigi, "Adaptive input-output linearizing control of induction motors," *IEEE Trans. Automat. Contr.*, vol. 38, no. 2, pp. 208-221, 1993
- [11] Th. Von Raumer, J. M. Dion, L. Dugard, and J. L. Thomas, "Applied nonlinear control of an induction motor using digital signal processing," *IEEE Trans. Contr. Syst. Technol.*, Vol. 2, No. 4, pp. 327-335, 1994.
- [12] R. Marino, S. Peresada, and P. Tomei, "output feedback control of current-fed induction motors with unknown rotor resistance," *IEEE Trans. Contr. Syst. Technol.*, Vol. 4, No. 4, pp. 336-346, 1996.
- [13] R. Marino, S. Peresada, and P. Tomei, "Adaptive observer-based control of induction motors with unknown rotor resistance," *Int. J. Adaptive Contr. Signal Proc.*, Vol 10, pp. 345-363, 1996.
- [14] John Chiasson, "A new approach to dynamic feedback linearization control of an induction motor," *IEEE Trans. Automat. Contr.*, Vol. 43, No. 3, pp. 391-396, 1998.
- [15] R. Marino, S. Peresada, and P. Tomei, "Global adaptive output feedback control of induction motors with uncertain rotor resistance," *IEEE Trans. Automat. Contr.*, Vol. 44, No. 5, pp. 967-983, 1999.
- [16] R. Ortega, P. Nicklasson, and G. Espinosa, "On speed control of induction motors," *Automatica*, Vol. 32, No. 3, pp. 455-460, 1996.
- [17] G. Espinosa-Perez and R. Ortega, "An output feedback globally stable controller for induction motors," *IEEE Trans. Automat. Contr.*, Vol. 40, No. 1, pp. 138-143, 1995.
- [18] Ki-Chul Kim, Romeo Ortega, and Jean-Paul Vilaino, "Theoretical and experimental comparison of two

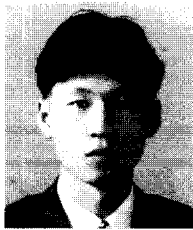
nonlinear controller for current-fed induction motors," *IEEE Trans. Contr. Syst. Technol.*, Vol. 5, No. 3, pp. 338-347, 1997.

- [19] I. Kanellakopoulos, P. T. Krein and F. Disilvestro, "Nonlinear flux-observer-based control of induction motor," in *American Control Conf.*, Chicago, IL., pp. 1700-1704, 1992.
- [20] R. Marino and P. Tomei, *Nonlinear Control Design – Geometric, Adaptive and Robust*. Englewood Cliffs, NJ:Prentice-Hall, 1995.
- [21] G. C. Verghese and S. R. Sanders, "Observers for flux estimation in induction machines," *IEEE Trans. Industrial Electronics*, Vol. 35, pp. 85-94, 1988.

#### Nomenclature

$V_{ds} (V_{qs})$	(q)-axis stator voltages
$i_{ds} (i_{qs})$	d(q)-axis stator currents
$i_{dr} (i_{qr})$	d(q)-axis rotor currents
$\lambda_{ds} (\lambda_{qs})$	d(q)-axis stator flux

$\lambda_{dr} (\lambda_{qr})$	d(q)-axis rotor flux
$P$	Number of pole pairs
$\omega$	Mechanical motor speed
$\omega_r$	Electrical motor speed [= $P\omega$ ]
$R_r$	Rotor resistance
$R_s$	Stator resistance
$L_{ls}$	Stator leakage inductance
$L_{lr}$	Rotor leakage inductance
$L_m$	Mutual inductance between stator and rotor
$L_s$	Stator self-inductance [= $L_{ls} + L_m$ ]
$L_r$	Rotor self-inductance [= $L_{lr} + L_m$ ]
$T_e$	Motor torque
$T_L$	Load torque
$J$	Moment of inertia of motor
$ \lambda_s ^2$	Modulus square of stator flux [= $\lambda_{ds}^2 + \lambda_{qs}^2$ ]
$ \lambda_r ^2$	Modulus square of rotor flux [= $\lambda_{dr}^2 + \lambda_{qr}^2$ ]
$L_\sigma$	Stator transient inductance [= $\frac{L_s L_r - L_m^2}{L_r}$ ]



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