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NORAD TLE CONVERSION FROM OSCULATING ORBITAL ELEMENT

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ABSTRACT

The NORAD type Two Line Element (TLE) was obtained from the osculating orbital elements by an iterative approximation procedure. The mathematical model was presented and computer program was developed for the conversion. The osculating orbital elements of the KOMPSAT-1 were converted into the NORAD TLE. Then the effect of the SGP4 atmospheric drag coefficient (B*) was analyzed by comparison of the orbit propagation results with different B* values.

Keywords: NORAD, TLE, SGP4, Orbital Element, KOMPSAT

1. INTRODUCTION

Two Line Element (TLE) is the mean orbital elements of the space object generated by the North American Aerospace Defense Command (NORAD). The NORAD TLE is estimated by their own orbit determination system using the observation data from world wide Space Surveillance Network (SSN). The SSN tracks about 8,500 space objects and consists of three primary types of sensors such as conventional radars, phased-array radars, and an optical system known as the ground-based electro-optical deep space surveillance (Kelso 1997). NORAD TLE has been widely used by the satellite communities who don't have their own orbit determination system. So, NORAD TLE is used as a basic input for the commercialized ground antenna control system and orbit analysis tools (Boelitz 1995, Beck & Boelitz 1997). The osculating orbital position and velocity of the Low Earth Orbit (LEO) satellite can be propagated from the NORAD TLE using the general perturbation formula such as Simplified General Perturbation 4 (SGP4) model (Hoots & Roehrich 1980).

However, general TLE users have to wait for the updated NORAD elements because the update of the TLE is totally dependent upon the NORAD activities. In order to overcome such situations, Jochim et al. (1996) in German Satellite Operations Center (GSOC) have developed their own orbit determination system for generating TLE type elements of their satellites. Recently Cho et al. (2002) developed the NORAD TLE type orbit determination system for the LEO satellite using GPS navigation solutions as the measurement data.

In the mean time, many amateur satellite observers have difficulties to use NORAD SGP4 orbit propagator for the space shuttle missions because NASA releases the orbital elements in osculating position and velocity vector instead of NORAD TLE format. So, the transformation program of the osculating position and velocity vector to NORAD TLE has been developed by Ernandes (1994) in MS-DOS environment. The name of the program is VEC2TLE and the program can be downloaded via web site only as executable file without the description of the mathematical models (URL 2002). So, the program in the UNIX environment is not applicable at the present time.

For the KOMPSAT-1 satellite mission operations, the antenna pointing data such as the azimuth, elevation, range, and range rate are used for the tracking of the satellite. The antenna pointing data are predicted from the osculating orbital element that is the result of the orbit determination. However, the antenna control system has an option to use NORAD TLE and the option provide a convenient method to control the early or late rising of the satellite, especially in the launch and early orbit phase operations and safe-hold mode operations. The UNIX operating system is used for the KOMPSAT-1 computer operation, so the development of our own code for converting the osculating orbital element into the NORAD TLE is requested.

In this paper, the mathematical model and computer program for deriving the NORAD TLE from osculating orbital elements are presented. Then, the orbit propagation using TLE with SGP4 model is performed and compared with the numerical orbit propagation for the KOMPSAT-1 spacecraft. The SGP4 type drag coefficient (B*) in NORAD TLE could not be derived from single set of the osculating orbital elements. So, the effect of the B* is analyzed using a series of SGP4 orbit propagations with the different B* values.

2. FORMULATION OF THE PROBLEM

The functional relationship between the NORAD mean orbital elements and the osculating orbital elements is expressed by

$$y_i = f_i(x_1, \cdots, x_6), \qquad i = 1, \cdots, 6 \tag{1}$$

where, y is the osculating elements, x is the NORAD mean elements, and f is the SGP4 model function in this case. Only six orbital elements are considered here because the other parameters in NORAD TLE such as the SGP4 atmospheric drag parameter B* cannot estimated from one set of the osculating elements.

In this problem, the NORAD elements x_i should be derived when the osculating elements y_i is given. Taylor series expansion of f around $x_i^{(a)}$ in Eq. (1) is given by (Walter 1967)

$$\begin{aligned} f_i(x_1, \cdots, x_6) &= f_i\left(x_1^{(a)}, \cdots, x_6^{(a)}\right) + \frac{\partial f_i}{\partial x_1^{(a)}}\left(x_1 - x_1^{(a)}\right) + \frac{\partial f_i}{\partial x_2^{(a)}}\left(x_2 - x_2^{(a)}\right) \\ &\quad + \cdots + \frac{\partial f_i}{\partial x_6^{(a)}}\left(x_6 - x_6^{(a)}\right) + \text{higher order terms}, \qquad i = 1, \cdots, 6. \end{aligned} \tag{2}$$

The higher order terms in Eq. (2) can be neglected under the assumption that a reasonably good approximation for the NORAD elements $x_i^{(a)}$ is available.

Eq. (2) is simplified as a system of six linear equations.

$$\Delta y_i = M \cdot \Delta x_i \tag{3}$$

where, Δy_i and Δx_i are the difference between the real one and approximate value, and M is the 6×6 matrix of the partial derivatives of f_i with respect to the six NORAD elements $\Delta x_i^{(a)}$.

$$M = \begin{bmatrix} \frac{\partial f_1}{\partial x_1^{(a)}} & \frac{\partial f_1}{\partial x_2^{(a)}} & \cdots & \frac{\partial f_1}{\partial x_6^{(a)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_6}{\partial x_1^{(a)}} & \frac{\partial f_6}{\partial x_2^{(a)}} & \cdots & \frac{\partial f_6}{\partial x_6^{(a)}} \end{bmatrix} \tag{4}$$

$$\Delta y_i = \left(y_i - y_i^{(a)}\right), \Delta x_i = \left(x_i - x_i^{(a)}\right)$$

Table 1. KOMPSAT-1 ECI orbital elements.

2001/02/13 00:00:29.0 UTC	Position (km)	Velocity (km/s)
X	-1799.56322	4.03338703
Y	3883.60987	-4.52428114
Z	-5632.97758	-4.41288927

Eq. (3) can be transcribed with $M^{-1} = c_{ij}$ as

$$x_i = x_i^{(a)} + \sum_{j=1}^{6} c_{ij} \left(y_j - y_j^{(a)} \right), \qquad i = 1, \cdots, 6. \tag{5}$$

One of the partial derivatives in Eq. (4) is simply expressed using a forward difference quotient as (Escobal 1965)

$$\frac{\partial f_1}{\partial x_1^{(a)}} = \frac{f\left(x_1^{(a)} + \Delta x_1^{(a)}, x_2^{(a)}, \cdots, x_6^{(a)}\right) - f\left(x_1^{(a)}, x_2^{(a)}, \cdots, x_6^{(a)}\right)}{\Delta x_1^{(a)}} \tag{6}$$

or using the symmetric difference quotient as (Serafin 1982)

$$\frac{\partial f_1}{\partial x_1^{(a)}} = \frac{f\left(x_1^{(a)} + \Delta x_1^{(a)}, \cdots, x_6^{(a)}\right) - f\left(x_1^{(a)} - \Delta x_1^{(a)}, \cdots, x_6^{(a)}\right)}{2 \cdot \Delta x_1^{(a)}} \tag{7}$$

A forward difference quotient was computed in this paper using an increment of 0.1 % of $x_1^{(a)}$ for $\Delta x_1^{(a)}$. Then, the NORAD elements can be derived by the iterative procedures as

$$x_i^{(a+1)} = x_i^{(a)} + \sum_{j=1}^{6} c_{ij} \left(y_j - y_j^{(a)} \right), \qquad i = 1, \cdots, 6. \tag{8}$$

The iteration can be finished when the difference between the given osculating elements and derived ones, i.e., $\left(y_j - y_j^{(a)}\right)$, are small enough for convergence. A convergence number of 1.0×10^{-8} was used in this paper.

3. CONVERSION INTO THE KOMPSAT-1 TLE

The osculating position and velocity vector of the KOMPSAT-1 in Table 1 is used as an input data for deriving TLE. The converted TLE using the program developed in this paper is shown in Table 2. VEC2TLE program version 9648 (URL 2002) was also executed to compare the results. The two results were identical except for the last digit of the mean motion (n) and argument of perigee (ω).

The SGP4 orbit propagation using the derived TLE was performed for 7 days from the epoch. Due to the fact that the atmospheric drag effect cannot be considered for deriving TLE from one set

Table 2. Converted NORAD mean elements.

n (rev/day)	e	i(deg)
14.62292261	.0006329	98.1516
Ω (deg)	ω (deg)	M (deg)
305.7348	85.2914	148.1408

Table 3. Atmospheric drag parameters.

Case	Case 1	Case 2	Case 3
Date	N/A	2001/02/13	2002/01/03
B*	0	3.4851×10^{-4}	1.1531×10^{-3}

of the position and velocity vector, the B* value for SGP4 propagation is obtained from NORAD. Table 3 shows three cases of the SGP4 atmospheric drag input parameters and adopted dates. The date in Case 2 is the same day of the epoch in Table 1. The drag parameters in Case 3 are obtained in almost 11 months later. So, the B* value in Case 3 is bigger than that of the Case 2 because there are no orbit raising maneuvers of the KOMPSAT-1 spacecraft in the time period. Table 4 shows three TLE sets for 7-day SGP4 orbit propagation.

4. COMPARISON OF THE SGP4 ORBIT PROPAGATION

Numerical orbit propagation using KOMPSAT-1 Mission Analysis and Planning System (MAPS) (Won et al. 1999) was performed to evaluate the SGP 4 propagation. Table 5 shows the spacecraft parameters, atmospheric drag coefficient (C_d), and solar radiation pressure coefficient (C_r) used in the numerical orbit propagation in MAPS. The position and velocity vectors in 30-second interval for 7 days are obtained from the epoch and state vector in Table 1.

Figure 1 shows the position differences between the numerical orbit propagation and three cases of the SGP4 propagation. The numerical orbit propagation is used as reference orbit for comparison. The position difference of 60 km is shown for a week in Case 1 where no atmospheric drag is applied to the SGP4. About 15 km difference is shown in 3 days. The same epoch day values of B* from NORAD is used in Case 2 and the position differences of 20 km for a week and below 10 km for three days are shown. In Case 3, the biggest position difference of 70 km among the three cases is presented for a week. However, it shows the smallest differences within 3 days periods.

Figure 2 shows the Radial, Along-track, and Cross-track (RAC) differences between the numerical orbit propagator and SGP4 in Case 2. Mostly, along-track difference is revealed and it is caused by the atmospheric drag effect.

Figure 3 enlarges the radial and cross-track differences in Figure 2. The difference of the cross-track direction is vibrated within $\pm 500m$ amplitude. For the radial direction, the amplitude of vibration is gradually increasing.

Figure 4 shows the along-track differences for the three cases. In Case 1, i.e., no atmospheric

Table 4. TLE format of KOMPSAT-1 ECI orbital elements.

Case 1	1 26032U 99070A 1044.00033565 .00000000 00000-0 00000-0 0 13 2 26032 98.1516 305.7348 0006329 85.2914 148.1408 14.62292261 59976
Case 2	1 26032U 99070A 1044.00033565 .00001712 00000-0 34851-3 0 18 2 26032 98.1516 305.7348 0006329 85.2914 148.1408 14.62292261 59976
Case 3	1 26032U 99070A 1044.00033565 .00006038 00000-0 11531-2 0 13 2 26032 98.1516 305.7348 0006329 85.2914 148.1408 14.62292261 59976

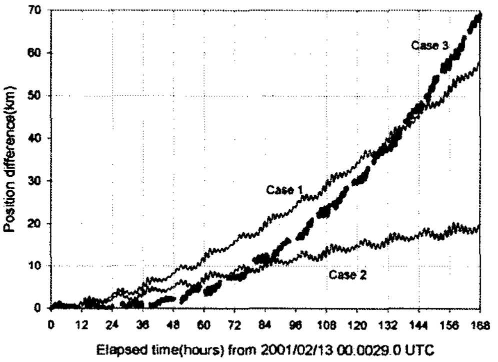

Figure 1. Position differences in three cases.

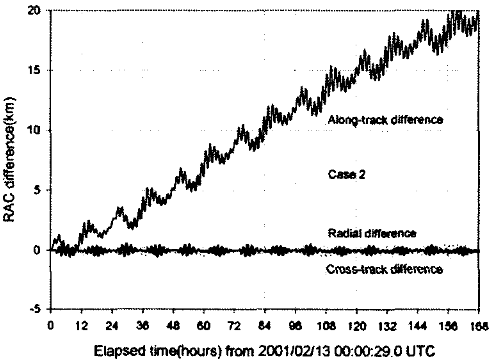

Figure 2. RAC differences in Case 2.

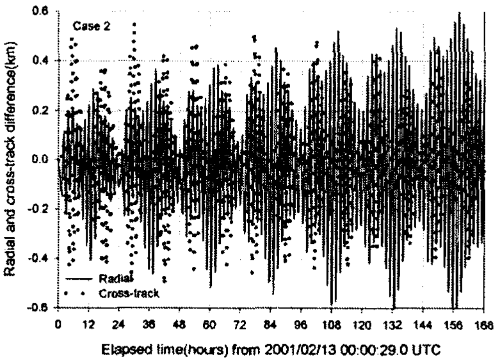

Figure 3. Radial and cross-track differences in Case 2.

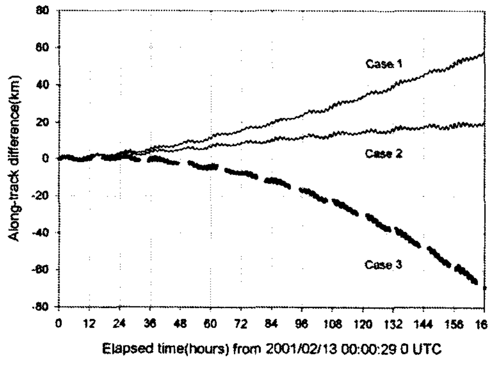

Figure 4. Along-track differences in three cases.

drag is applied in SGP4 propagator, the reference orbit by the MAPS goes in advance to the SGP4 orbit because orbital period of MAPS orbit is shorter than that of the SGP4 orbit. The atmospheric effect in Case 2 is still smaller than that of the MAPS. So, the reference orbit precedes the SGP4 orbit. On the other hand, the atmospheric effect in Case 3 is bigger than the reference orbit. The

Table 5. Parameters for numerical orbit propagation.

Parameter	Value	Parameter	Value
Area(m^2)	8.5	C_d	2.2
Mass(kg)	448	C_r	1.5

Table 6. Different B* values for SGP4 orbit propagation.

Case Number	B* Value	Case Number	B* Value
Case 4	1.0×10^{-4}	Case 6	1.0×10^{-3}
Case 5	5.0×10^{-4}	Case 7	5.0×10^{-3}

reference orbit falls behind the SGP4 orbit. This is because the B* value of Case 3 is obtained from the NORAD TLE in January 2002.

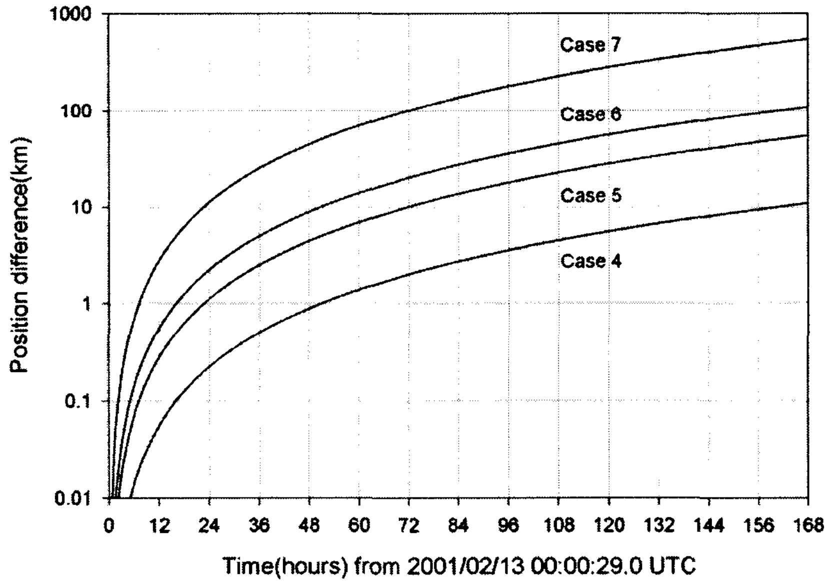

Figure 5. Position difference from different drag term.

A series of SGP4 orbit propagation was performed to examine the effect of the drag term B*. The converted NORAD type mean elements in Table 2 were used for the SGP4 orbit propagation. The SGP4 orbit propagations were performed for the four cases of the B* values as shown in Table 6.

Fig. 5 shows the position difference of the SGP4 orbit propagations with different B* values.

Table 7. Position differences for the Cases.

Case	Position Differences(km)			
	1 day	3 day	5 day	7 day
Case 4	0.22	2.02	5.59	10.97
Case 5	1.12	10.08	27.93	54.87
Case 6	2.24	20.16	55.88	109.77
Case 7	11.21	100.91	279.77	549.85

The orbit propagation with B*=0 is used for the reference orbit. The position differences show some linearity with the differences of the B* values. The B* value of Case 6 is ten times bigger than that of the Case 4 and then the position difference of the Case 6 is also ten times bigger than that of the Case 4. The same situation can be applicable to the Case 5 and Case 7. This shows the characteristics of the drag modeling in the SGP4 and the value plays key role for the accurate orbit prediction. Table 7 summarizes the position differences with the elapsed time.

5. CONCLUSIONS

Mathematical algorithm and computer program were developed for the conversion of osculating orbital elements into NORAD Two-Line-Element. The converted TLE was same as the result of the VEC2TLE. The orbit propagations using NORAD SGP4 model with three different B* values were evaluated with the numerical orbit propagation. And the effect of the B* values was analyzed by the SGP4 orbit propagation with the different B* values. The B* value for SGP4 model was verified as an important factor for orbit propagation accuracy.

It was not possible to derive the B* value from one set of the osculating orbital elements. However, the B* value can be estimated when the two sets of the osculating elements with enough time span are available. It will be reserved as a further study.

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