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BER and Throughput Analyses of the Analytical Optimum Chip Waveform

해석적 최적 칩파형의 BER과 전송성능(Throughput) 분석

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Abstract

The study on the chip waveform design to minimize multiple-access interference (MAI) and its performance evaluation are very important since chip waveform decides the signal quality and system capacity of the direct-sequence CDMA wireless communication system. This paper suggests the analytical chip waveform to minimize the MAI. The BER and throughput performances achieved by the proposed analytical optimum chip waveform are compared with those of the conventional chip waveforms in the Nakagami-m distribution frequency selective channel when the differential phase shift keying (DPSK) is employed in DS-CDMA system. From the numerical results, capacity and throughput are improved about 2 times and 1.4 times respectively when it is compared with the Kaiser chip waveform that is considered as one of the best in the conventional ones.

Key words : Chip Waveform, Multiple-Access Interference(MAI), Frequency Selective Channel, DS-CDMA

Ⅰ. Introduction

Code division multiple-access (CDMA) system is known to support more users than conventional time division multiple-access (TDMA) and frequency division multiple-access (FDMA). The CDMA is an interference-limited system because multiple-access interference (MAI) determines the signal quality and capacity of the direct-sequence CDMA (DS-CDMA) wireless communication system. Since MAI depends on the shape of the chip waveform, the design of chip waveform for the MAI minimization have been studied very much. In 1999, especially, M.A Landolsi and W.E Stark worked the design of chip waveform with minimal MAI under the constraints of bandwidth, phase and envelope. Even if they proposed chip waveforms with minimal MAI, they are too complex and difficult to implement in communication system because of computer generation. In this reason, we search for expressing the optimal chip waveforms in the analytical form. Also, the system capacity and throughput of optimal chip waveforms were not analyzed in the Rayleigh fading channel. The effects of the conventional chip waveforms on system performance were investigated by P.I. Dallas and F.N. Pavlidou in the Rayleigh, Nakagami-m distribution frequency selective fading channel[2]. B.K. Kok and M.A. Do considered the

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performance dependency on the chip waveforms in the Rician distributed frequency selective fading channel[3].

This paper proposes the analytical chip waveforms that are mathematically approximated forms of optimal chip waveform with minimal MAI in the ref.[1]. The system performances achieved by the analytical chip waveforms analyzed and compared with those of the conventional chip waveforms in the Nakagami-m distribution frequency selective channel when the differential phase shift keying (DPSK) is employed in DS-CDMA system. The conventional chip waveforms include half-sine, raised cosine, Kaiser and Blackman waveform.

Ⅱ. System and Channel Description

In a CDMA system with DPSK modulation, each of K users has a unique code sequence and the transmitter consists of k active simultaneous users. The transmitted signal of the k th user can be written by

$$s^k(t) = \sqrt{2P}c^k(t)b^k(t)\cos[w_c t + \phi^k] \tag{1}$$

where $c^k(t)$ is a code sequence of the k th user and may be expressed as

$$c^k(t) = \sum_{j=-\infty}^{\infty} c_j^k \psi_c(t - jT_c),\ c_j^k \in \{-1,1\} \tag{2}$$

and $b^k(t)$ is data waveform which can be given by

$$b^k(t) = \sum_{j=-\infty}^{\infty} b_j^k \psi_b(t - jT_b),\ b_j^k \in \{-1,1\} \tag{3}$$

In eq.(1), P is average transmitted power which is common to all users, w_c is carrier frequency and ϕ^k is phase angle of the k th user. The phase angle is assumed to be uniformly distributed in $[0,2\pi]$. In eqs.(2) and (3), T_c is the chip duration and T_b is the data bit duration, and c_j^k, b_j^k is j th direct sequence code and data bit of the k th user, respectively. The processing gaine is assumed to be $N = T_b/T_c$. $\Psi_b(t)$ is unit rectangular pulses of unit height and duration T_b, and $\Psi_c(t)$ is the chip waveform of duration T_c, and will be explained in detail in section IV.

The incoming signal to the receiver is given by

$$r(t) = \sqrt{2P}\sum_{k=1}^{K}\sum_{l=1}^{L} \beta_l^k c^k(t - \tau_l^k) b^k(t - \tau_l^k)\cos(w_c t + \theta_l^k) + n(t) \tag{4}$$

where L is the number of resolvable path, θ_l^k is the phase of the l th path of the k th user and $n(t)$ is AWGN with double-sided power spectral density $\eta_0/2$. The path phase θ_l^k is independent and assumed to be uniformly distributed in $[0,2\pi]$. The phase delay τ_l^k is also independent random variables with uniform distribution in $[0,2\pi]$. The receiver consists of the matched filter, the DPSK demodulator and the selection diversity component of order M $(1 \le M \le L)$. The selection diversity with order M chooses the path based on the largest peak of the M received correlated peaks.

The equivalent low pass impulse response of the channel can be written as

$$h^k(\tau) = \sum_{l=1}^{L} \beta_l^k e^{j\theta_l^k} \delta[\tau - \tau_l^k] \tag{5}$$

where the number of resolvable path L is given by

$$L = \left\lfloor \frac{T_m}{T_c} \right\rfloor + 1 \tag{6}$$

and the T_m is maximum delay spread of the channel.

Assuming Nakagami-m fading channel, the path gain β_l^k has the probability density function (pdf) as follows

$$p_\beta(r) = 2\left(\frac{m}{\Omega}\right)^m \frac{r^{2m-1}}{\Gamma(m)} \exp\left(\frac{-mr^2}{\Omega}\right),\ r \ge 0 \tag{7}$$

where m is

$$m = \frac{\Omega^2}{E[(r^2 - \Omega)^2]} \geq 0.5,\ \Omega = E[r^2]. \tag{8}$$

and $\Gamma(m)$ denotes gamma function. In Nakagami fading channel, m=1 corresponds to the Rayleigh fading channel, $m = \infty$ corresponds to the AWGN and $m = 0.5$ corresponds to the worst case fading condition. $m > 1$ approximates the Rician fading channel. In general, Rician factor R(average direct power/average scattered power) is related to m as follow

$$m = \frac{1}{1 - \left(\frac{R}{1+R}\right)^2}. \tag{9}$$

Ⅲ. Optimum Chip Waveforms

3-1 Conventional Chip Waveforms

The chip waveform $\Psi_c(t)$ is time-limited to the interval $[0, T_c]$ and normalized to have energy T_c, so that

$$\int_0^{T_c} \psi_c(t) dt = T_c. \tag{10}$$

The representative conventional chip waveforms are considered as follow [2], [3]

a) Half-Sine :

$$\psi_c(t) = \sqrt{2} \sin(\frac{\pi t}{T_c}) u(t). \tag{11}$$

b) Raised Cosine :

$$\psi_c(t) = \sqrt{\frac{2}{3}} \left[1 - \cos(\frac{2\pi t}{T_c})\right] u(t). \tag{12}$$

c) Blackman :

$$\psi_c(t) = c_1 \left[0.42 - 0.5\cos(\frac{2\pi t}{T_c}) + 0.08\cos(\frac{4\pi t}{T_c})\right] u(t). \tag{13}$$

d) Kaiser :

$$\psi_c(t) = c_2 \frac{I_0\left\{\beta\pi \sqrt{1 - \left[\frac{t - T_c/2}{T_c}\right]^2}\right\}}{I_0(\beta\pi)} u(t). \tag{14}$$

where $c_i (i=1,2)$ is chosen to satisfy the normalized energy condition, β is any real, positive number, $I_n(x)$ is the modified Bessel function of the first kind and n th order with argument x. $u(t)$ is a rectangular pulse defined by

$$u(t) = \begin{cases} 1 & 0 \leq t \leq T_c. \\ 0 & otherwise. \end{cases} \tag{15}$$

3-2 Proposed Analytical Chip Waveforms

In 1999, M.A. Landolsi and W.E. Stark proposed optimal chip waveforms generated by computer for various cases that are not in analytic forms [1]. We approximate and propose the analytical optimal chip waveforms for 4 kinds of the optimal chip waveforms in [1]. From now, we denote that $WT_c = 1.2$ for 99 %BW, $WT_c = 1.6$ for 99.9 %BW, $WT_c = 2.0$ for 99 %BW and $WT_c = 3.0$ for 99 %BW in [1] are the first, second, third and fourth chip waveform, where BW means bandwidth. The proposed analytical chip waveforms are expressed in Fig. 1 corresponding to 4 waveforms of the results in [1] and can be written

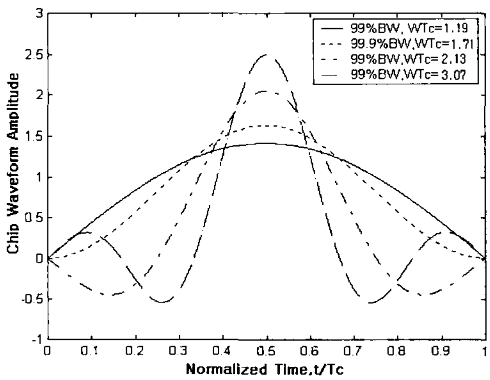

Fig. 1. Proposed analytical optimum chip waveforms.

as eqs. (16)~(19),

a) First chip waveform:

$$\psi_c(t) = \sqrt{2}\sin(\frac{\pi t}{T_c})u(t) \tag{16}$$

b) Second chip waveform:

$$\psi_c(t) = \sqrt{\frac{2}{3}}\left[1 - \cos(\frac{2\pi t}{T_c})\right]u(t) \tag{17}$$

c) Third chip waveform:

$$\psi_c(t) = k_1 \sin c(4(\frac{t - T_c/2}{T_c}))u(t) \tag{18}$$

d) Fourth chip waveform:

$$\psi_c(t) = k_2 \sin c(6(\frac{t - T_c/2}{T_c}))u(t) \tag{19}$$

where $k_i (i = 1,2)$ are constants satisfying the normalization condition. Among the proposed chip waveforms, the first and second chip waveforms are of the half-sine and raised cosine form. Third and fourth chip are similar to the sin c function.

To verify the similarity of each chip waveforms, we compare the power bandwidth and normalized mean-squared partial chip correlation of analytical chip waveforms with those of computer-generated chip waveforms. From Table 1 the mean-squared partial chip correlation of analytical chip waveforms is almost the same as computer-generated chip waveforms. And Table 2 shows that bandwidth of analytical waveforms is almost equal to that of computer-generated waveforms. Thus, we confirm that analytical chip waveforms are well approximated to the computer-generated chip waveforms.

Table 1. Mean-squared partial chip correlation of analytical chip waveforms.

Waveform \ Correlation	Analytical waveform	Computer-generated Waveform
First waveform	0.2914	0.2821
Second waveform	0.2414	0.2527
Third waveform	0.1317	0.1382
Fourth waveform	0.0865	0.0894

Ⅳ. Performance Analysis

4-1 BER Analysis

The BER of Nakagami-m distribution channel is given by [2]

$$P_{er}(k) = \int_0^\infty \frac{1}{2}\exp(-\gamma_b)\left\{M\sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^i \times \exp(\frac{-mi\gamma_b}{\bar{\gamma_c}})\left[\sum_{p=0}^{m-1}\frac{\left(m\gamma_b/\bar{\gamma_c}\right)^p}{p!}\right]^i\right\}$$
$$\left\{\times\exp(\frac{-m\gamma_b}{\bar{\gamma_b}})\left[\sum_{j=0}^{m-1}\left(\frac{m}{\bar{\gamma_c}}\right)^{j+1}\frac{\gamma_b^j}{j!} - \sum_{j=0}^{m-1}\left(\frac{m}{\bar{\gamma_c}}\right)^j\frac{j\gamma_b^{j-1}}{j!}\right]\right\}d\gamma_b \tag{20}$$

The average signal to noise ratio $\bar{\gamma_c}$ is

$$\bar{\gamma_c} = [(kL-1)\varepsilon_k^2 + N_0/(E_b\rho)]^{-1} \tag{21}$$

where E_b is bit energy, $\rho = (1/2)E[\beta^2]$ is the half-average power per path.

The interference parameter ε_k^2 is defined as [4], [5], [6]

$$\varepsilon_k^2 = 2\left\{\frac{1}{(NT_c)^3}[\mu_{k1}(0)m_\varphi + \mu_{k1}(1)m_\varphi']\right\} \tag{22}$$

where $\mu_{k1}(0)$, $\mu_{k1}(1)$ are obtained from aperodic

Table 2. Power bandwidth of analytical chip waveforms.

Waveform \ Bandwidth(WTc)	90 %	95 %	95 %	99.9 %	Notes
First waveform	0.77	0.91	1.19(1.2)	2.38	( ): Computer-generated
Second waveform	0.94	1.11	1.40	1.71(1.6)	
Third waveform	1.65	1.83	2.13(2.0)	3.00	
Fourth waveform	2.52	2.73	3.07(3.0)	3.89	

autocorrelation function of PN sequence as follow

$$\mu_{k1}(n) = \sum_{l=1-N}^{N-1} C_{k1}(l)C_{k1}(l+n) \tag{23}$$

and PN code aperiodic corrosscorrelation between user 1 and user k, $C_{k1}(n)$, can be written as

$$C_{k1}(n) = \begin{cases} \sum_{j=0}^{N-1-n} a_j^k a_{j+n}^1 & 0 \le n \le N-1 \\ \sum_{j=0}^{N-1+n} a_j^k a_{j+n}^1 & -(N-1) \le n \le 0\,. \\ 0 & otherwise \end{cases} \tag{24}$$

The normalized partial chip correlation, m_φ, m'_φ given in Table 1, is

$$m_\varphi = \int_0^{T_c} R_\varphi^2(\tau)d\tau \tag{25}$$

$$m'_\varphi = \int_0^{T_c} R_\varphi(\tau)\overline{R_\varphi}(\tau)d\tau \tag{26}$$

where autocorrelation function of chip waveform is given by

$$\overline{R_\varphi}(\tau) = \int_\tau^{T_c} \psi_a(t)\psi_a(t-\tau)dt\,. \tag{27}$$

Table 3. The normalized partial chip correlation for several chip waveforms.

WaveformShape	m_φ	m'_φ
Blackman	0.2081	0.0025
Kaiser	0.1570	9.7016×10^{-5}
First chip waveform	0.2941	0.0430
Second chip waveform	0.2414	0.0095
Third chip waveform	0.1317	0.00577
Fourth chip waveform	0.0865	0.00223

$$R_\varphi(\tau) = \overline{R_\varphi}(T_c - \tau), \quad \text{for } 0 \le \tau \le T_c\,. \tag{28}$$

In Table 1, we omit the normalized partial chip correlation for half-sine and raised cosine chip waveforms because they have the same normalized partial chip correlation with the first and second chip waveforms, respectively.

4-2 Throughput Analysis

The normalized throughput is defined as the average number of successfully received packets per time slot, normalized by the system capacity, [7] and it is given by

$$S = \frac{1}{W}\sum_{k=1}^{W} kP_k P_{cor}(k) \tag{29}$$

where packet success probability $P_{cor}(k)$ is

$$P_{cor}(k) = [1 - P_{er}(k)]^{N_d} \tag{30}$$

and the probability of having k simultaneous users p_k is

$$P_k = \binom{C}{k}\left[\frac{G}{C}\right]^k\left[1-\frac{G}{C}\right]^{C-k} \tag{31}$$

where G is an offered traffic, C is system capacity and N_d is the number of data bits per packet.

Ⅴ. Numerical Results and Discussion

The number of users supported by the system, for a specified BER, is an important design criterion and performance measure. It is assumed that Gold sequence is used, code length is N=127, T_c = 1 and selection diversity is applied into the receiver.

Fig. 2 and 3 show the differences in BER performance when the multipath fading parameter m=1(Rayleigh). Kaiser chip waveform is the best among the conventional waveforms and fourth chip waveform is the best among the proposed 4 kind of analytic chip waveforms. It can be observed that at the required BER=10^{-3} in Fig. 2, the capacity of Kaiser and fourth chip waveform are 2 and 4 users, respectively. In addition, when all the paths are resolved and selected at the receiver ($L = M$) and required BER =10^{-3} in Fig. 3, the capacity of Kaiser and fourth chip waveforms are 11 and 20 users,

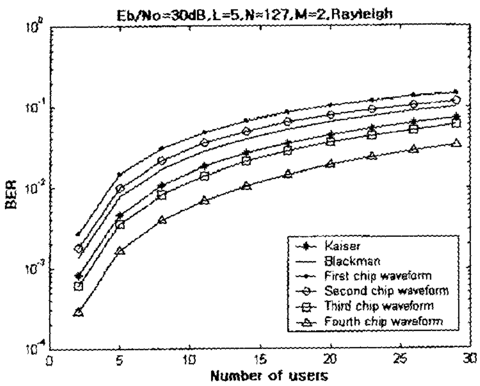

Fig. 2. BER in Rayleigh channel ($M=2$).

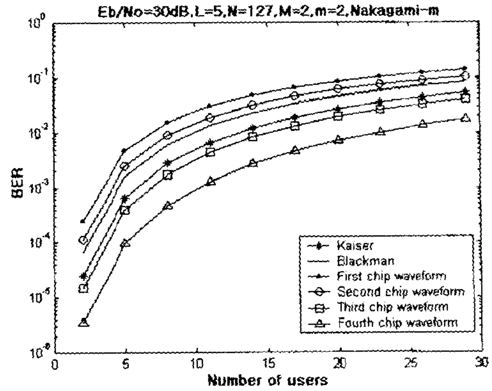

Fig. 4. BER in Nakagami-m channel ($M=2$).

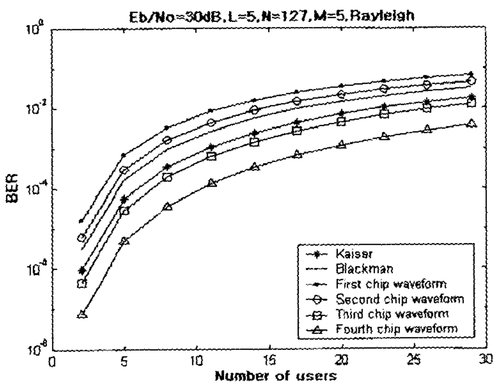

Fig. 3. BER in Rayleigh channel ($M=5$).

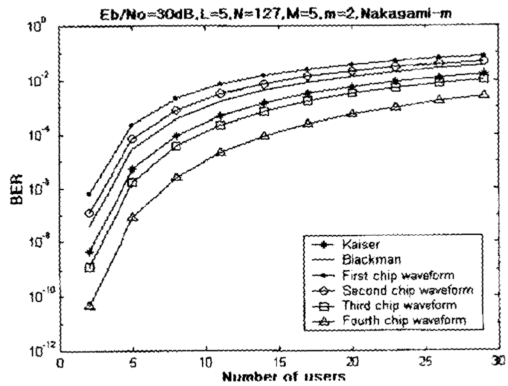

Fig. 5. BER in Nakagami-m channel ($M=5$).

respectively. We can find that the analytical optimum chip waveforms significantly improve the number of users supported by the system.

Fig. 4 and 5, which are the situations in Nakagami-m channel(at $m=2$), show better BER performance than Fig. 2 and 3. It can be observed that at the required BER=10^{-3} in Fig. 5, the capacity of Kaiser and fourth chip waveform are 14 and 25 ısers, respectively.

The throughput is also an important design ·iterion and performance measure. Fig. 6 and 7 ow the throughput performance in the multipath ling parameter $m=1$ (Rayleigh) according to the ered traffic G. When offered traffic $G=30$ in Fig the throughputs of Kaiser and fourth chip eforms are 0.58 and 0.8 respectively. In Fig. 7, hroughputs of all chip waveforms are decreased

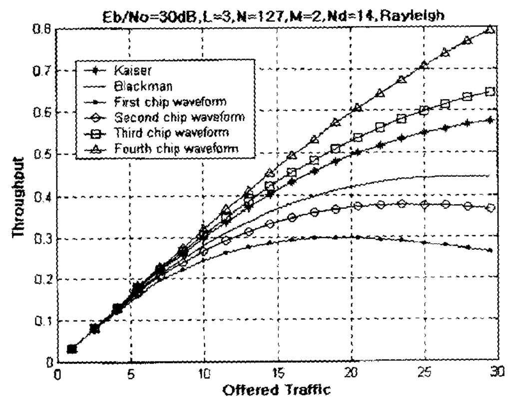

Fig. 6. Throughput in Rayleigh channel ($M=14$).

as data bits of a packet N_d are increased. From these figures, the proposed analytical optimum chip wave forms considerably improve the throughput performance.

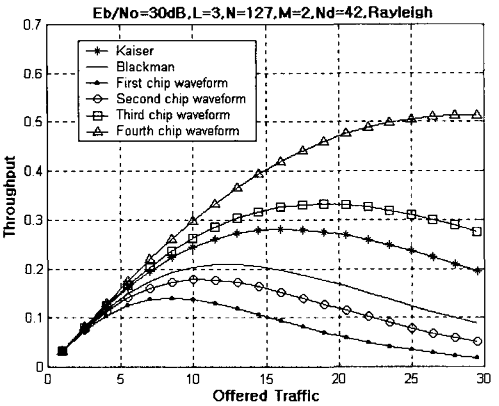

Fig. 7. Throughput in Rayleigh channel ($N_d = 42$).

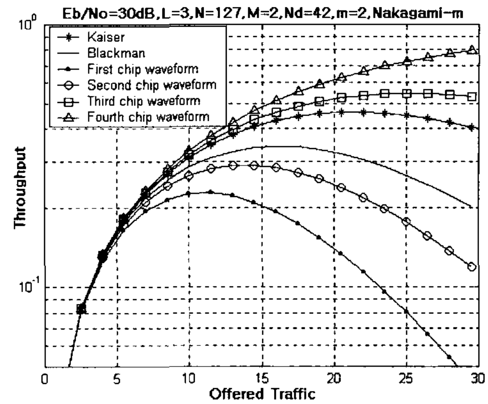

Fig. 9. Throughput in Nakagami-m channel (N_d=42).

Fig. 8 and 9 show the throughput performance with multipath fading parameter $m = 2$. When offered traffic $G = 30$ in Fig 8, the throughput of Kaiser and fourth chip waveforms is 0.75 and 0.9 respectively. As shown Fig. 8 and 9, the throughput of all chip waveforms is significantly decreased when data bits of a packet N_d are increased as like Fig. 6 and 7. From these figures, it is observed that the careful choice of data bits of a packet N_d in a given condition of the offered traffic and fading parameter m enhance the throughput performance.

From these computational results, the capacity of the proposed analytical chip waveforms is increased about nearly 2 times than that of conventional chip waveforms in the Rayleigh fading channel. Also, the throughput is enhanced about 1.4 times compared with conventional cases.

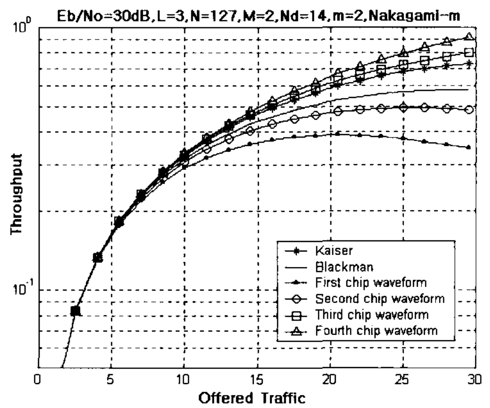

Fig. 8. Throughput in Nakagami-m channel (N_d=14).

Ⅵ. Conclusion

In this paper, analytical optimum chip waveform based on the result of ref. [1] is proposed and considered in the Nakagami-m distribution frequency selective channel. The system capacity and throughput performance of analytical optimum chip waveform is analyzed when DPSK modulation is used in DS-CDMA system. Numerical results show that the capacity and throughput of the proposed chip waveform are increased about 2 times and 1.4 times than those of the conventional Kaiser chip waveform. It can be concluded that the proposed analytical optimum chip waveform considerably improves the system performances.

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