The Effect of a Wing on the Heat Loss from a Modified Rectangular Fin

Hyung Suk Kang*

Key words: A modified rectangular fin, Wing, Heat loss, Biot number, Relative increasing rate

Abstract

A modified asymmetric rectangular fin is analysed using the two-dimensional separation of variables method. This modified rectangular fin is made by attaching the wing on the top side of a rectangular fin. Heat loss from each side of this modified rectangular fin is calculated. The relative increasing ratio of heat loss between a modified rectangular fin and a rectangular fin is presented as a function of dimensionless fin volume, wing height and the location of the wing. Especially, to show the remarkable effect of the wing on the heat loss, the relative increasing ratios of heat loss between two different volume increasing methods are listed.

Nomenclature

a : dimensionless beginning point of the wing
a' : beginning point of the wing [m]
b : dimensionless ending point of the wing
b' : ending point of the wing [m]
Bi_i : Biot number of the ith fin surface $(=h_i l/k)$
H : dimensionless height of the wing
H' : the height of the wing [m]
h_i : heat transfer coefficient of the ith fin surface [W/m^2℃]
k : thermal conductivity [W/m℃]
l : one half fin root height [m]
L : dimensionless fin length $(=L'/l)$
L' : fin length [m]
q : heat loss from the fin per unit width [W/m]
Q : dimensionless heat loss from the fin, $q/(k\theta_0)$
T : temperature [℃]
T_w : fin root temperature [℃]
T_∞ : ambient temperature [℃]
V : dimensionless fin volume of a fin $(=V'/l)$
V' : the volume of a fin per unit width [m^2]
x : dimensionless coordinate along the fin length $(=x'/l)$
x' : coordinate along the fin length [m]
y : dimensionless coordinate along the fin height $(=y'/l)$

* Division of Mechanical and Mechatronics Engineering, Kangwon National University, Hyoza-dong, Chunchon, Kangwon-do 200-701, Korea

y' : coordinate along the fin height [m]

Greek symbols

θ : dimensionless temperature $(T-T_\infty)/(T_w-T_\infty)$

θ_0 : adjusted fin root temperature $(=T_w-T_\infty)$ [℃]

λ_n : eigenvalues ($n=1, 2, 3, \cdots$)

Subscripts

0 : root

1 : top side of a modified rectangular fin

2 : bottom side of a modified rectangular fin

3 : tip side of a modified rectangular fin

∞ : surrounding

mr : modified rectangular fin

r : rectangular fin

w : wall

1. Introduction

Finned surfaces have been widely used to enhance the rate of heat transfer in many thermal engineering applications, for instance, for the cooling of combustion engines, the air-conditioning, and other heat transfer equipment. The most commonly used fins in industry are longitudinal rectangular, triangular, trapezoidal profile fins and annular fin. Lots of papers about these shapes of fins have been published. For example, Sen and Trinh[(1)] studied the rate of heat transfer from a rectangular fin governed by a power law-type temperature dependence while Kang[(2)] analyzed the performance of a thermally asymmetric rectangular fin using three-dimensional analytical method. Razelos and Satyaprakash[(3)] presented an analysis of trapezoidal profile longitudinal fins that delineate their thermal performance and Kraus et al.[(4)] were concerned with longitudinal fins of rectangular, trapezoidal and triangular profile. Also Abrate and Newnham[(5)] modeled heat conduction in an array of triangular fins with an attached wall and Kang and Look[(6)] investigated dimensionless heat loss from a geometrically symmetric, but thermally asymmetric, triangular fin. Increasing the heat dissipation of annular fins at a defined magnitude of mass is considered by Ullmann and Kalman[(7)] while Look[(8)] shows heat loss ratio between fin and bare pipe for a radial fin of uniform thickness on a pipe. Also studies on the pin fin have been published. Su and Hwang[(9)] developed the analytical transient solutions when the heat dissipation is convected from the lateral surface and fin tip to the surroundings in a two-dimensional pin fin while Gerencser and Razani[(10)] investigate the optimal pin fin array of variable cross section for a given fin material per unit base area. Recently studies on the more unique shape of fin have been reported. Kundu and Das[(11)] presents a numerical technique for the determination of the performance of eccentric annular fins with a variable base temperature. Bejan and Almogbel[(12)] reports the geometric optimization of T-shaped fin assemblies, where the objective is to maximize the global thermal conductance of the assembly, subject to total volume and fin-material constraints. Usually most of the studies on the fin assume that the heat transfer coefficients for all surfaces of the fin are the same. But no literature seems to be available which presents a modified rectangular fin with unequal heat transfer coefficients.

This paper analyzes a thermally asymmetric modified rectangular fin and shows the effect of the wing on the heat loss using the two-dimensional separation of variables method. A rectangular fin is modified by attaching the wing on the top side of the fin. In this study the top surface Biot number Bi_1, the bottom surface Biot number Bi_2 and the fin tip Biot number Bi_3 are independent each other. Heat losses from each side of this modified rectangular fin are compared with the variation of dimensionless fin length. The relative increasing ratio of heat loss between a modified rectangular fin and a rectangular fin is presented as a function of dimensionless fin volume, wing height and the location of the wing when surrounding Biot numbers are equal. Especially, to show the remarkable effect of the wing on the heat loss, the relative increasing ratios of heat loss between two different volume increasing methods are listed. For simplicity, the root temperature and the thermal conductivity of the fin's material are assumed constant as well as steady-state.

2. Two-dimensional analysis

For a modified asymmetric rectangular fin, illustrated in Fig. 1, the dimensionless governing differential equation is given by equation (1).

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=0 \tag{1}$$

Three standard boundary conditions and one energy balance equation serve as the required problem formulation. They are

$$\theta|_{x=0}=1 \tag{2}$$

$$\frac{\partial\theta}{\partial y}\bigg|_{y=-1}-\mathrm{Bi}_2\cdot\theta|_{y=-1}=0 \tag{3}$$

$$\frac{\partial\theta}{\partial x}\bigg|_{x=L}+\mathrm{Bi}_3\cdot\theta|_{x=L}=0 \tag{4}$$

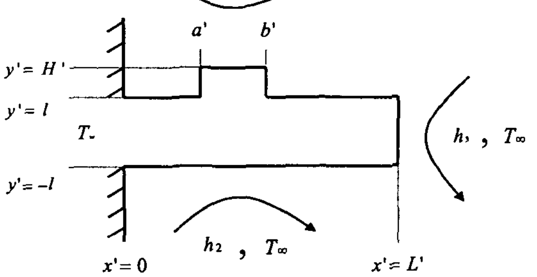


Fig. 1 Geometry of a modified asymmetric rectangular fin.

$$\begin{aligned}-\int_{-1}^{1}\frac{\partial\theta}{\partial x}\bigg|_{x=0}dy = \mathrm{Bi}_1\bigg\{&\int_0^a\theta|_{y=1}dx \\ &+\int_1^H\theta|_{x=a}dy+\int_a^b\theta|_{y=H}dx \\ &+\int_1^H\theta|_{x=b}dy+\int_b^L\theta|_{y=1}dx\bigg\} \\ &+\mathrm{Bi}_2\int_0^L\theta|_{y=-1}dx+\mathrm{Bi}_3\int_{-1}^{1}\theta|_{x=L}dy\end{aligned} \tag{5}$$

By solving equation (1) with three boundary conditions listed as equation (2) through equation (4), the dimensionless temperature can be obtained by the separation of variables procedure. The result is

$$\theta(x,y)=\sum_{n=1}^{\infty}N_n\cdot f(x)\cdot f(y) \tag{6}$$

where,

$$f(x)=\cosh(\lambda_n x)-f_n\cdot\sinh(\lambda_n x) \tag{7}$$

$$f(y)=\cos(\lambda_n y)+g_n\cdot\sin(\lambda_n y) \tag{8}$$

$$N_n=\frac{4\sin(\lambda_n)}{[\{2\lambda_n+\sin(2\lambda_n)\}+g_n^2\{2\lambda_n-\sin(2\lambda_n)\}]} \tag{9}$$

f_n and g_n are expressed by

$$f_n=\frac{\lambda_n\cdot\tanh(\lambda_n L)+\mathrm{Bi}_3}{\lambda_n+\mathrm{Bi}_3\cdot\tanh(\lambda_n L)} \tag{10}$$

$$g_n=\frac{\mathrm{Bi}_2-\lambda_n\cdot\tan(\lambda_n)}{\lambda_n+\mathrm{Bi}_2\cdot\tan(\lambda_n)} \tag{11}$$

Eigenvalues λ_n are obtained by using an energy balance equation (equation (5)) and is listed as equation (12) to equation (21).

$$\begin{aligned}&\{\mathrm{Bi}_2\cdot\sin(2\lambda_n)-\lambda_n\cdot AA_n\} \\ &\quad\cdot[\mathrm{Bi}_1\cdot\lambda_n\cdot\{\sinh(\lambda_n L)+BB_n\} \\ &\quad+\mathrm{Bi}_1\cdot\mathrm{Bi}_3\cdot\{\cosh(\lambda_n L)+CC_n-1\}] \\ &\quad+\{DD_n-\mathrm{Bi}_2\cdot AA_n-\lambda_n\cdot\sin(2\lambda_n)\} \\ &\quad\cdot(\mathrm{Bi}_1\cdot\lambda_n\cdot EE_n+\mathrm{Bi}_1\cdot\mathrm{Bi}_3\cdot FF_n) \\ &\quad-GG_n\cdot(\mathrm{Bi}_1\cdot\lambda_n\cdot BB_n+\mathrm{Bi}_1\cdot\mathrm{Bi}_3\cdot CC_n) \\ &\quad+\lambda_n\cdot HH_n\cdot II_n=0\end{aligned} \tag{12}$$

where,

$$AA_n = \sin^2(\lambda_n) - \cos^2(\lambda_n) \quad (13)$$

$$BB_n = \sinh\{\lambda_n(L-b)\} - \sinh\{\lambda_n(L-a)\} \quad (14)$$

$$CC_n = \cosh\{\lambda_n(L-b)\} - \cosh\{\lambda_n(L-a)\} \quad (15)$$

$$DD_n = \lambda_n \cdot \sin\{\lambda_n(1+H)\} - \mathrm{Bi}_2 \cdot \cos\{\lambda_n(1+H)\} \quad (16)$$

$$EE_n = \cosh\{\lambda_n(L-b)\} + \cosh\{\lambda_n(L-a)\} \quad (17)$$

$$FF_n = \sinh\{\lambda_n(L-b)\} + \sinh\{\lambda_n(L-a)\} \quad (18)$$

$$GG_n = \mathrm{Bi}_2 \cdot \sin\{\lambda_n(1+H)\} + \lambda_n \cdot \cos\{\lambda_n(1+H)\} \quad (19)$$

$$HH_n = \lambda_n \cdot \sin(2\lambda_n) + 2\mathrm{Bi}_2 \cdot \sin^2(\lambda_n) - \mathrm{Bi}_2 \quad (20)$$

$$II_n = \mathrm{Bi}_3 - \mathrm{Bi}_3 \cdot \cosh(\lambda_n L) - \lambda_n \cdot \sinh(\lambda_n L) \quad (21)$$

The value of the heat loss per unit width from the modified asymmetric rectangular fin can be calculated with equation (22).

$$q_{mr} = -k\int_{-l}^{l} \left.\frac{\partial T}{\partial x'}\right|_{x'=0} dy' \quad (22)$$

Then, the dimensionless heat loss from the fin can be expressed by

$$Q_{mr} = \frac{q_{mr}}{k\theta_0} = 2\sum_{n=1}^{\infty} N_n \cdot f_n \cdot \sin(\lambda_n) \quad (23)$$

The dimensionless heat loss from top side of a modified rectangular fin is written by equations (24)~(30).

$$Q_1 = \sum_{n=1}^{\infty} \mathrm{Bi}_1 \cdot \frac{N_n}{\lambda_n} \cdot (JJ_n \cdot KK_n + LL_n \cdot MM_n + NN_n \cdot OO_n) \quad (24)$$

where,

$$JJ_n = \sinh(\lambda_n a) - \sinh(\lambda_n b) + f_n \cdot \{\cosh(\lambda_n a) - \cosh(\lambda_n b)\} \quad (25)$$

$$KK_n = \cos(\lambda_n) - \cos(\lambda_n H) + g_n \cdot \{\sin(\lambda_n) - \sin(\lambda_n H)\} \quad (26)$$

$$LL_n = \sinh(\lambda_n L) + f_n \cdot \{\cosh(\lambda_n L) - 1\} \quad (27)$$

$$MM_n = \cos(\lambda_n) + g_n \cdot \sin(\lambda_n) \quad (28)$$

$$NN_n = \cosh(\lambda_n a) + \cosh(\lambda_n b) + f_n \cdot \{\sinh(\lambda_n a) + \sinh(\lambda_n b)\} \quad (29)$$

$$OO_n = \sin(\lambda_n H) - \sin(\lambda_n) - g_n \cdot \{\cos(\lambda_n H) - \cos(\lambda_n)\} \quad (30)$$

Also the equations for heat loss from bottom and tip sides are shown by equations (31)~(32).

$$Q_2 = \sum_{n=1}^{\infty} \mathrm{Bi}_2 \cdot \frac{N_n}{\lambda_n} \cdot \{\cos(\lambda_n) - g_n \cdot \sin(\lambda_n)\} \cdot \frac{\lambda_n \cdot \sinh(\lambda_n L) + \mathrm{Bi}_3\{\cosh(\lambda_n L) - 1\}}{\lambda_n \cdot \cosh(\lambda_n L) + \mathrm{Bi}_3 \cdot \sinh(\lambda_n L)} \quad (31)$$

$$Q_3 = \sum_{n=1}^{\infty} 2\mathrm{Bi}_3 \cdot \frac{N_n \cdot \sin(\lambda_n)}{\lambda_n \cdot \cosh(\lambda_n L) + \mathrm{Bi}_3 \cdot \sinh(\lambda_n L)} \quad (32)$$

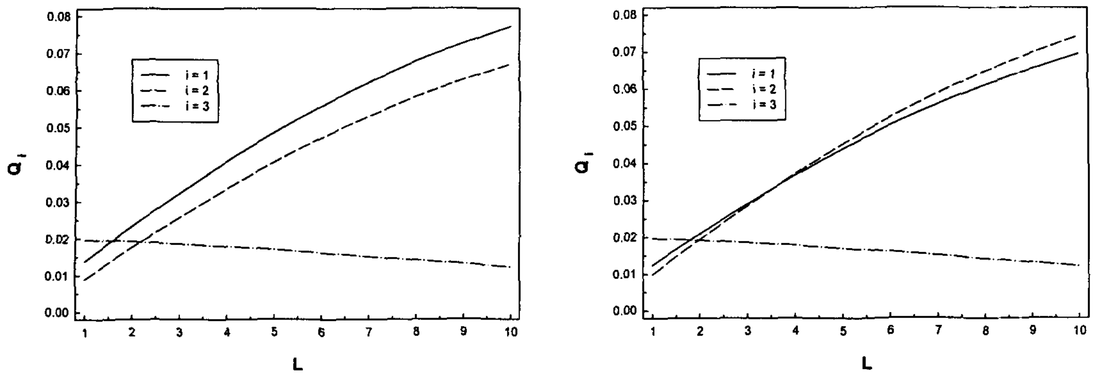
The volume of a modified rectangular fin per unit width is expressed as equation (33) and the dimensionless volume is written by equation (34).

$$V_{mr}' = \int_0^{a'} 2l\,dx' + \int_{a'}^{b'} \{2l + (H' - l)\}\,dx' + \int_{a'}^{L'} 2l\,dx' \quad (33)$$

$$V_{mr} = \frac{V'_{mr}}{l^2} = 2L + (H-1)\cdot(b-a) \quad (34)$$

3. Results

Figure 2(a) presents the variation of the dimensionless heat loss from each surface for a thermally asymmetric modified rectangular fin as the dimensionless fin length varies from 1 to 10 in the case of Bi_1=0.01, Bi_2=0.009, Bi_3=0.01, a=0.4 L, b=0.6 L and H=1.2. The heat loss from top and bottom sides increase while that from tip side decreases as the dimension-



(a) $Bi_1=0.01$, $Bi_2=0.009$ (b) $Bi_1=0.009$, $Bi_2=0.01$

Fig. 2 Heat loss from each surface vs. dimensionless fin length ($H=1.2$, $a=0.4\,L$, $b=0.6\,L$, $Bi_3=0.01$).

less fin length increases. Until about $L=1.5$ the heat loss from tip is the greatest and that from bottom is the smallest. The heat loss from top is the greatest and that from tip is the smallest for L greater than about 2.2. The heat loss from top is always greater than that from bottom since top Biot number is greater than bottom Biot number and the wing is attached on the top side.

Fig. 2(b) presents the variation of the dimensionless heat loss from each surface under the same condition as Fig. 2(a) but $Bi_1=0.009$ and $Bi_2=0.01$. The heat loss from tip is almost the same as that presented in Fig. 2(a). The heat loss from top is greater than that from bottom until about $L=3.5$ but the heat loss from bottom becomes greater than that from top for L greater than about 3.5. This phenomenon explains physically that the effect of wing on the heat loss is greater than the effect of given Biot number for short fin while the effect of Biot number on the heat loss becomes greater than the effect of wing as fin length increases.

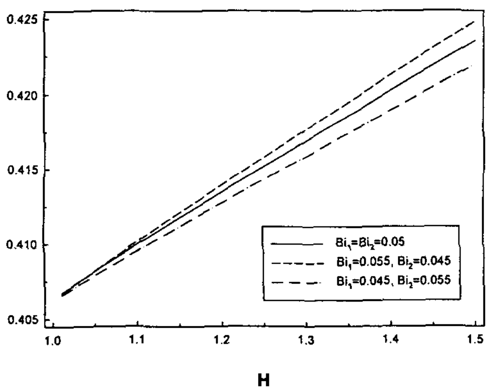
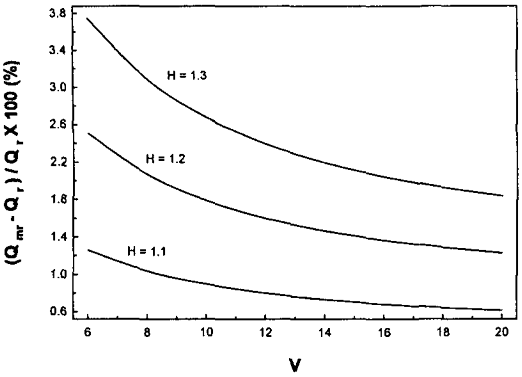


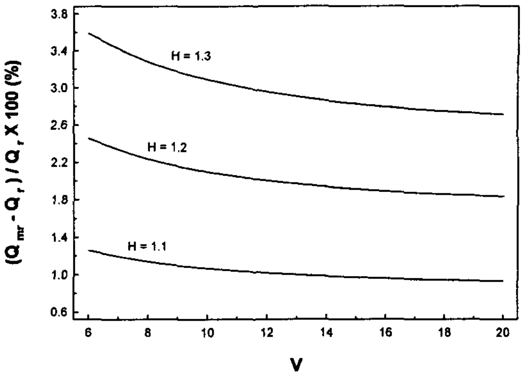
Fig. 3 Heat loss from an asymmetric modified rectangular fin as a function of dimensionless fin height ($L=6$, $a=0.4\,L$, $b=0.6\,L$, $Bi_3=0.05$).

The heat loss from an asymmetric modified rectangular fin versus dimensionless wing height in case of $Bi_3=0.05$, $a=2.4$, $b=3.6$ and $L=6$ is presented by Fig. 3. For given circumstances, the heat loss increases linearly as wing height increases for all three different cases of top and bottom Biot numbers. For very small height (i.e. $H=1.01$), the heat loss is the largest when top and bottom Biot numbers are same. As wing height increases, the heat loss becomes the largest when wing is attached on the side with higher value of Biot number.

Fig. 4(a) shows the relative increasing ratio of heat loss between a modified rectangular fin and a rectangular fin as a function of dimensionless fin volume for three values of wing height in case of $a=1$, $b=2$ and all Bis=0.01. The relative increasing ratio of heat loss decreases somewhat rapidly as V increases from 6 to 10 and this tendency is more noticeable



(a) All Bis=0.01



(b) All Bis=0.1

Fig. 4 Relative increasing rate of heat loss versus dimensionless fin volume (a=1, b=2).

as wing height increases. Since the heat loss from a modified fin is greater than that from a rectangular fin for the same volume, it is more effective to use a modified fin. This usefulness will be shown more clearly in Table 1.

Results for the same conditions as in Fig. 4 (a) except that all Biot numbers are 0.1 are depicted in Fig. 4(b). The relative increasing ratio of heat loss for all Bis=0.1 are equal to or a little less than those for all Bis=0.01 when dimensionless volume is 6. But these curves for all Bis=0.1 decrease slowly comparing to those for all Bis=0.01 as the volume increases. For one example, in case of H=1.2, the ratio for Bi=0.1 decreases from 2.45% to 1.83% while that for Bi=0.01 decreases from 2.51% to 1.22% as the dimensionless volume increases from 6 to 20. This phenomenon means physically that the modified fin is more useful for small Biot numbers when fin volume is small while it is more useful for large Biot numbers when fin volume is large.

The relative increasing ratios of heat loss with two different volume increasing methods are listed in Table 1. Fin volume can be increased by extending the fin length for a rectangular fin or by attaching the wing on the side for a modified rectangular fin. This table shows that heat loss is increased more remarkably if fin volume is increased by attaching the wing on the fin. For one example, when dimensionless fin length is 10 and all Biot numbers are 0.1, the heat loss is almost the same as that for L=10 even though dimensionless fin volume is increased 0.2 by extending the fin length while the heat loss increases 2.16% relative to that for L=10 if dimensionless fin volume is increased 0.2 by attaching the wing on the fin.

Fig. 5 presents the relative increasing ratio of heat loss as a function of dimensionless wing height for three different cases of surrounding

Table 1 The increasing rate of heat loss with volume increasing

	$\Delta Q/Q$ (at L) %			
	ΔV=0.2 by extending fin length		ΔV=0.2 by attaching the wing (a=0.5, b=1.5)	
L	All Bi=0.01	All Bi=0.1	All Bi=0.01	All Bi=0.1
2	3.12	1.83	6.46	5.09
5	1.32	0.28	3.18	2.73
10	0.45	0.01	1.74	2.16

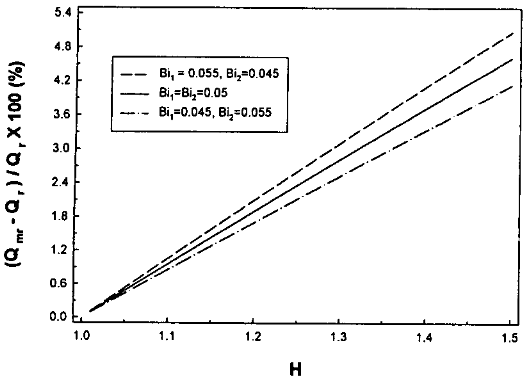


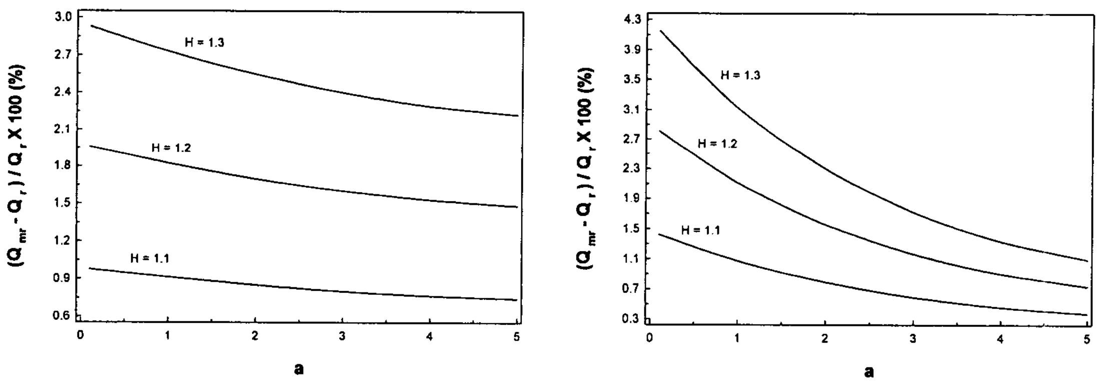
Fig. 5 Relative increasing rate of heat loss versus dimensionless wing height (a=1, b=2, V=12, Bi_3=0.05).

Biot numbers. For all three cases, this ratio increases linearly as wing height increases. It can be known that the modified fin is more useful if the wing is attached on the side with higher value of Biot numbers and this usefulness becomes remarkable as wing height increases.

The relative increasing ratio of heat loss versus the location of the wing in case of $b-a$= 0.8, V=12, and all Bis=0.01 is shown in Fig. 6(a). It can be known that this increasing ratio decreases slowly as the beginning point of wing moves from fin root direction to fin tip direction. It means physically that the modified rectangular fin becomes more useful when the wing is attached to the fin root as close as possible if the given variables are fixed.

Results for the same conditions as in Fig. 6 (a) except that all Biot numbers are 0.1 are presented in Fig. 6(b). The relative increasing ratio of heat loss for all Bis=0.1 are much larger than those for all Bis=0.01 when the wing is attached to the fin root direction very closely. But the values for all Bis=0.1 decreases very rapidly comparing to those for all Bis=0.01 as the wing location moves to fin tip direction. For one example, in case of H=1.2, the rate for Bi=0.1 decreases from 2.81% to 0.73% while that for all Bis=0.01 decreases from 1.96% to 1.48% as the beginning point of the wing moves from 0.1 to 5. This phenomenon means physically that the modified fin is more useful for small Biot numbers if the wing is located near the fin tip while that is more useful for large Biot numbers if the wing is located near the fin root.

Fig. 7 presents the ratios of heat loss from top surface to that from bottom surface as a function of top Biot number for three values of wing height. These ratios increase linearly as top Biot number increases. For given top Biot number, the ratio increases as wing height increases. It also can be noted that if top Biot number is n times of bottom Biot number, where n varies from 0.5 to 2.5, then the ratio



(a) All Bis=0.01 (b) All Bis=0.1

Fig. 6 Relative increasing rate of heat loss versus the location of wing ($b-a$=0.8, V=12).

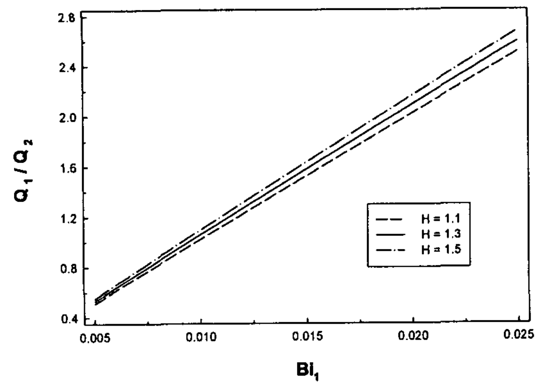


Fig. 7 Ratio of heat loss from top surface to that from bottom surface versus top Biot number ($L=10$, $a=4$, $b=6$, $Bi_2=Bi_3=0.01$).

is larger than n for all three wing heights. This fact shows apparently the effect of the wing of a modified rectangular fin on the heat loss.

4. Conclusion

From the two-dimensional analysis of an asymmetric modified rectangular fin presented here, the following conclusions can be drawn:

(1) A modified rectangular fin is more useful if the wing is attached on the side with higher value of Biot number when fin's surrounding condition is asymmetric and an average value of surrounding Biot numbers is equal.

(2) For the same fin volume, heat loss from a modified rectangular fin is greater than that from a rectangular fin. If an array of modified rectangular fins is made by one-low along the z-direction, the effect an array of these modified rectangular fins seems to be more remarkable.

(3) Relative increasing rate of heat loss between a modified rectangular fin and a rectangular fin decreases somewhat rapidly first and then decreases slowly as a wing moves from fin base direction to tip direction.

(4) The ratio of heat loss from top surface to that from bottom surface increases linearly as top Biot number increases with fixed bottom and tip Biot numbers.

Acknowledgement

This work was funded by (Kisunghoi professor sending abroad) of Kangwon National University in 2001.

References

1. Sen, A. K. and Trinh, S., 1986, An Exact Solution for the Rate of Heat Transfer from a Rectangular Fin Governed by a Power Law-Type Temperature Dependence, ASME J. Heat Trans., Vol. 108, pp. 457-459.
2. Kang, H. S., 2001, Three-Dimensional Performance Analysis of a Thermally Asymmetric Rectangular Fin, Int. Journal of Air-Conditioning and Refrigeration, Vol. 9, No. 2, pp. 94-101.
3. Razelos, P. and Satyaprakash, B. R., 1993, Analysis and Optimization of Convective Trapezoidal Profile Longitudinal Fins, ASME J. Heat Trans., Vol. 115, pp. 461-463.
4. Kraus, A. D., Snider, A. D. and Doty, L. F., 1978, An Efficient Algorithm for Evaluating Arrays of Extended Surface, ASME J. of Heat Trans., Vol. 100, pp. 288-293.
5. Abrate, S. and Newnham, P., 1995, Finite Element Analysis of Triangular Fin Attached to a Thick Wall, Computer & Structures, Vol. 57, No. 6, pp. 945-957.
6. Kang, H. S. and Look, D. C., 2001, Thermally Asymmetric Triangular Fin Analysis, AIAA Journal of Thermophysics and Heat Transfer, Vol. 15, No. 4, pp. 427-430.
7. Ullmann, A. and Kalman, H., 1989, Efficiency and Optimized Dimensions of Annular Fins of Different Cross-Section Shapes, Int. J. Heat Mass Transfer, Vol. 32, pp. 1105-1110.
8. Look, D. C., 1995, Fin on a Pipe (Insulated

Tip): Minimum Conditions for Fin to Be Beneficial, Heat Transfer Engineering, Vol. 16, No. 3, pp. 65-75.

9. Su, R. J. and Hwang, J. J., 1998, Analysis of Transient Heat Transfer in a Cylindrical Pin Fin, AIAA Journal of Thermophysics and Heat Transfer, Vol. 12, No. 2, pp. 281-283.

10. Gerencser, D. S. and Razani, A., 1995, Optimization of Radiative-Convective Arrays of Pin Fins Including Mutual Irradiation between Fins, Int. J. Heat Mass Transfer, Vol. 38, No. 5, pp. 899-907.

11. Kundu, B. and Das, P. K., 1999, Performance Analysis of Eccentric Annular Fins with a Variable Base Temperature, Numerical Heat Transfer, Part A, Vol. 36, pp. 751-766.

12. Bejan, A. and Almogbel, M., 2000, Constructal T-Shaped Fins, Int. J. Heat Mass Transfer, Vol. 43, pp. 2101-2115.