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Hydrodynamic interaction between two cylinders in planar shear flow of viscoelastic fluid

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Abstract

Particle-particle interaction is of great importance in the study of suspension rheology. In this research we have investigated the hydrodynamic interaction between two identical cylinders in viscoelastic fluids numerically as a model problem for the study of viscoelastic suspension. We confine two neutrally buoyant cylinders between two parallel plates and impose a shear flow. We determine the migration velocity of two cylinders. The result shows that cylinders move toward or away from each other depending upon the initial distance between them and that there is an equilibrium distance between two cylinders in viscoelastic fluids regardless of the initial distance. In the case of Newtonian fluid, there is no relative movement as expected. The results partly explain the chaining phenomena of spherical particles in shear flows of viscoelastic fluids.

Keywords **:** microstructure formation, chaining, particle migration, shear thinning, normal stress difference

1. Introduction

The flow of fluid with suspended particles and the motion of these particles in the suspension have been of great interest in many industrial processes such as materials development. This is because the properties of materials produced by the processing of suspension are determined by the microstructure developed during the flow of suspension. Until now, most studies have been focused on suspensions dispersed in Newtonian fluids and only a few studies have been reported on the flow of suspension in viscoelastic medium despite of its practical importance. But there is a growing interest on the flow of viscoelastic suspension and the microstructure formation in the suspension.

It has been well documented that non-colloidal, spherical particles suspended in polymeric fluids form shear-induced structures such as chains or aggregates when the suspension is sheared either between two parallel plates (Michele *et al.*, 1977; Giesekus, 1981; Lyon *et al.*, 2001) or subjected to planar or circular Poiseuille flows (Jefri and Zahed, 1989; Tehrani, 1994). However, one of the present authors (Kim, C.) and his coworkers have observed different microstructures developed in the shear flow of viscoelastic suspension (Won and Kim, 2002; Jung *et al.*, 2002): In a shear thinning xanthan gum solution in aqueous sugar solution, the particles form string-like structures when they are sheared between two parallel plates as observed in the literature (Michele *et al.*, 1977; Giesekus, 1981; Lyon *et al.*, 2001). However, in non-shear thinning Boger fluids (*e.g.*, 500 ppm polyacrylamide in the mixture of ethylene glycol and glycerin), the particles were distributed evenly without forming such string-like structures; Kim *et al.* (2000) observed that, in their studies on the diffusivity of spherical particles in viscoelastic solution, particles dispersed in Boger fluid did not get aggregated or chained when sheared between two concentric cylinders. They also reported reduced hydrodynamic diffusivity in non-shear thinning viscoelastic fluids; Kim (2001) also reported that the same suspension did not show any aggregation when subjected to torsional flows between two parallel disks or Poiseuille flows in a tube. Therefore, from the series of experimental observations, we can conclude that aggregation and/or chaining of particles in viscoelastic fluid do not appear to be always the case.

The migration of particles in suspension results from the approach or separation of particles in suspension. Therefore, to understand the particle migration and microstructure formation in suspension, it is necessary to understand the hydrodynamic interaction among particles. Since it would be a formidable task to solve a full 3-D problem on particle-particle interaction in viscoelastic fluid, in this paper, we have considered the relative motion of two cylinders placed side by side between two parallel plates and formulated the problem in two-dimensional space. The result shows that, in non-shear thinning Maxwell fluid, there is an equilibrium distance between two cylinders.

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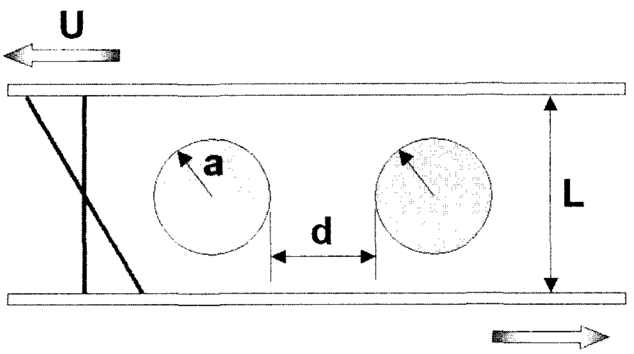


Fig. 1. Schematic diagram of the flow geometry with two neutrally buoyant cylinders.

This implies that particles do not get chained in such fluids. We believe that the result obtained here can shed light on the understanding of the hydrodynamic interaction among particles and microstructure formation in viscoelastic fluid.

2. Formulation of the problem

Let us consider a layer of fluid of depth L confined between two parallel plates shown in Fig. 1. The lower and upper plates are moving in the opposite direction with the same speed U. Between the two plates, two neutrally buoyant cylinders of radius a are placed so that their axes are parallel to the vorticity direction of the base flow (simple shear flow), the plane joining the axes of two cylinders coincides with the mid-plane between two fluid boundaries and the shortest distance between two cylinders is d. Depending upon the fluid rheology and velocity of the fluid boundary, two cylinders will migrate toward or away from each other while rotating. When two cylinders are separated far enough, the rate of rotation of one cylinder is not affected by the presence of the other cylinder. Also the flow is symmetric with respect to the axis of cylinder, and therefore the axis of the cylinder does not move from its initial position. As the two cylinders become closer, the presence of one cylinder affects the rotation of the other cylinder and the flow strength between two cylinders will be changed accordingly. Then due to the fore-aft asymmetry, the cylinders will have a migration velocity toward or away from each other in addition to upward or downward velocity.

Since we consider a mobility problem in which the motion of the particles is not designated *a priori*, we formulated the problem as the motion of liquid cylinder that is immiscible with the immersing fluid and sufficiently viscous to act as a solid cylinder.

To investigate the effect of shear thinning on the chaining of particles in suspension, we will consider one Newtonian and two different nonlinear viscoelastic constitutive equations: Upper convected Maxwell (UCM) model and White-Metzner model (UCM model with Bird-Carreau viscosity law). The former has non-shear thinning viscosity while the latter has shear thinning viscosity. Both viscoelastic models have first normal stress difference. By comparing the results from both models, we will be able to understand the physics of chaining phenomena. For completeness, we write two nonlinear viscoelastic models as follows:

UCM model:

$$\underline{\underline{T}}+\lambda\left(\frac{D\underline{\underline{T}}}{Dt}-\underline{\underline{T}}\cdot\nabla\underline{v}-\nabla\underline{v}^T\cdot\underline{\underline{T}}\right)=2\eta\underline{\underline{D}} \tag{1}$$

Bird-Carreau viscosity law:

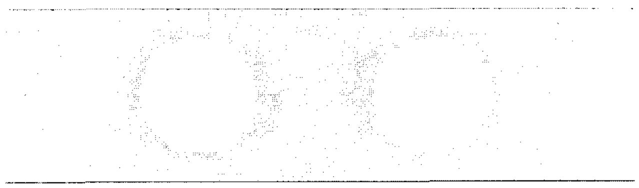
$$\eta=(\eta_0-\eta_\infty)(1+\Lambda^2\dot{\gamma}^2)^{\frac{n-1}{2}}+\eta_\infty \tag{2}$$

where $\underline{\underline{T}}$ is extra stress tensor, $\underline{v}$ velocity vector, $\underline{\underline{D}}$ rate of deformation tensor, λ material relaxation time, η viscosity, η_0 zero-shear-rate viscosity, η_∞ infinite-shear-rate viscosity (this value was set to zero for simple calculation.), K consistency factor, n power-law index, and Λ natural time (inverse of the shear rate at which the fluid changes from Newtonian to power-law behavior; this value was 500 to clearly expose the shear thinning effect.).

3. Numerical method

To solve the above problem we need a numerical technique. In this research we used POLYFLOW™ software based on FEM. To circumvent the troublesome computational difficulty caused by the lack of appropriate boundary conditions for mobility problems in POLYFLOW™, two neutrally buoyant cylinders were assumed as fluid with very high viscosity ($>10^6$), which is immiscible with the immersing fluid. It has been found that this assumption is appropriate for describing the interface conditions between the cylinders and fluids, guaranteeing the continuity of the velocity field and solid body rotation of the cylinder. Utilizing above conditions, the relative migration velocity of two cylinders with different spatial distances between them was investigated for each model.

The detailed description of this flow regime is as follows: The width of plates is 60, depth between plates 3, radius of cylinders 1, velocities of upper and lower plates moving in opposite direction -10 and 10, respectively, fluid viscosity is set to 1 in the zero-shear-rate state (The present calculation was not so much affected by other viscosity values because the inertia force term in equation of motion was neglected.), and material relaxation time of viscoelastic Maxwell fluids 0.05 (This value was chosen to successfully obtain the all simulated data under the various geometric conditions considered in this study, albeit this is so small.). The number of nodal points at the surface of the cylinders is 200, guaranteeing the acceptable accuracy for the flow field near the cylinders. Total elements required in the fluid



'ig. 2. Distribution of elements and nodes around the cylinders when d=1.

egion and inside the cylinders are appropriately chosen lepending on the distance, between two cylinders, d. For xample, the number of total elements was 12,216 (5,146 n flow regime and 3,535 in each cylinder) when d is equal o the diameter of the cylinder as shown in Fig. 2.

ł. Results and discussion

To validate the method adopted in this research, the flow ›ehavior around a single cylinder submerged in Newtonian ›r viscoelastic fluids was checked first. It was found that he flow is symmetric with respect to the axis of cylinder ınd the axis of the cylinder does not move from its initial ›osition. Also, for the case of Newtonian fluid, the rate of otation of cylinder is half of the shear rate when the disance between two confining plates (solid boundaries) is ufficiently large. It has been also found that the force xerted on the cylinder vanishes. From these numerical esults, the appropriateness of our method for solving the nobility problems for force free particles is validated.

When two cylinders are placed in viscoelastic fluid under hear flow, we notice qualitative differences in the motion ›f cylinders due to their hydrodynamic interaction that ould not be seen in the case of single cylinder: They will nove toward or apart each other and they will tend to move ıpward or downward. Fig. 3 shows rotational speed of the ylinders in Maxwell and White-Meztner fluids. First we ıote that the rotational speed has a maximum value when ve plot the speed against the distance. When two cylinders ıre placed very close, we expect that the rotation will be everely hindered by the presence of the other cylinder. As he distance becomes larger, the rotational speed increases. 3ut if the distance becomes larger than a certain critical 'alue, it decreases due to the reduced flow strength ›etween two cylinders. When two cylinders are separated ar enough, the rotational rate will be that of a single cylnder. We note that the rotational speed of the cylinder in Maxwell fluid is higher than that in White-Metzner fluid egardless of the distance between cylinders. It appears that he reduced shear stress due to the reduced viscosity of luid is responsible.

Fig. 4 shows the dependence of upward velocity of left ylinder on the distance between two cylinders. It can be een from this figure that the left (right) cylinder will

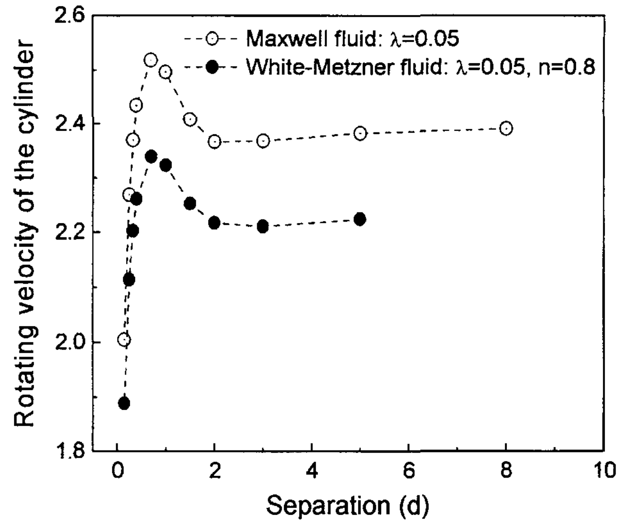


Fig. 3. Rotating velocity of the cylinder in viscoelastic fluids.

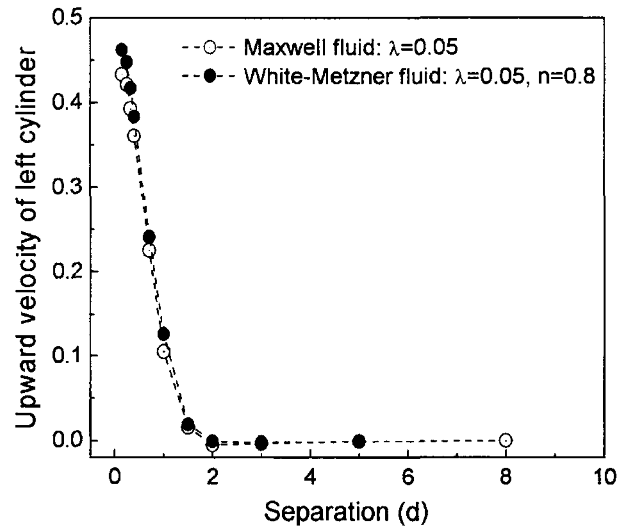


Fig. 4. Upward velocity of the left cylinder in viscoelastic fluids. The upward velocity is not dependent on fluid model.

strongly tend to move upward (downward) when two cylinders are separated closely (d < 1.5). In the case of a single cylinder problem, there is no such velocity component due to fore-aft symmetry of the flow field. But in the case of two-cylinders problems, the movement of the lower plate induces to rush fluid toward the interstice between the lower plate and the left cylinder, while in the upper right region of the left cylinder, there is no such rush of fluid between the upper plate and the left cylinder due to the presence of the right cylinder. In the region between two cylinders, there is only circulating flow as will be discussed below. This will cause the left cylinder to tend to move upward. For the right cylinder, the opposite motion is expected. These upward or downward velocities are also observed in the case of Newtonian fluid since this is caused

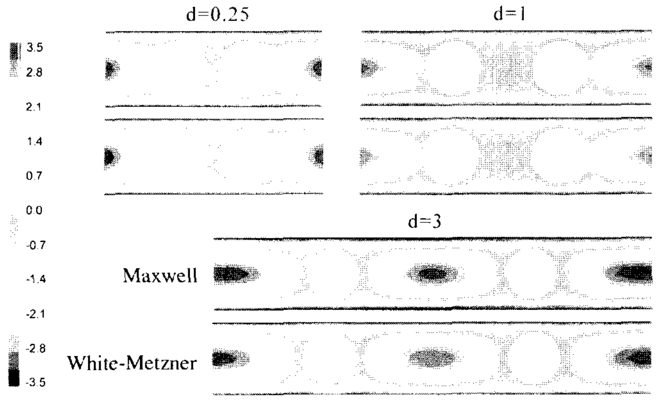


Fig. 5. Streamlines around two cylinders in viscoelastic shear flows for differing separation. When the separation is small, there is no vortex between two cylinders and only an extensional flow develops. When the distance is large, there develops a vortex. An extensional flow develops between the vortex and the cylinder. The strength of the vortex between two cylinders is stronger in non-shear thinning Maxwell fluid.

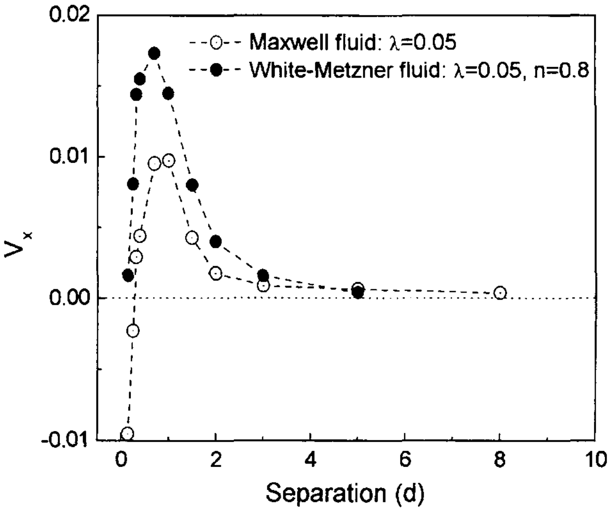


Fig. 6. Migration velocity of the left cylinder along the flow direction. The migration velocity is a function of distance between cylinders. The vanishing of migration velocity means that there is an equilibrium distance. The migration velocity is larger in shear thinning velocity.

not by the fluid rheology but by the asymmetry of the geometry.

The streamlines of Maxwell and White-Metzner fluids around the two cylinders are shown in Fig. 5. Since the cylinders almost block the simple shear flow, fluid turns around near the noses of the cylinders in outer regions of both cylinders. Between two cylinders, we observe a vortical motion of fluid when two cylinders are far apart. We note that the strength of the vortex is weaker in shear thinning White-Metzner fluid and therefore the fore-aft asymmetry is stronger. The difference will result in different migration behavior. When two cylinders are placed closely, there develops an extensional flow between two cylinders. The vortex and the flow adjacent to the cylinder are always linked by an extensional flow. Therefore there is a flow type change from a vortical motion to an extensional flow as the distance between cylinders increases. Fig. 6 shows the flow-direction velocity component of the left cylinder, representing the relative movement of the cylinder (The right cylinder also has same but opposite signed velocity component.). We note that the cylinders have migration velocities toward or away from each other depending upon the initial distance between them in addition to upward and downward velocities. Also the migration velocity changes the sign when the distance between cylinders decreases from the large separation case. From the vanishing of the migration velocity, we expect that there exist an equilibrium distance between two cylinders in viscoelastic fluids regardless of the initial distance. In the case of Newtonian or inelastic power-law fluids, it has been found that there is no such relative movement. This means that the driving force for the migration is the first normal stress difference as expected. In the Maxwell fluid, there is an equilibrium distance at near $d = d_e \approx 0.28$. That is, two cylinders will move away from each other by repulsion when distance, d is shorter than d_e, whereas they will move toward each other when d is larger than d_e. Even though we have not been able to determine the equilibrium distance for the shear thinning fluid due to numerical difficulties, it appears that the distance is shorter in shear thinning fluid than in non-shear thinning fluid. This means that the chaining is stronger in shear thinning fluid. Finally, there will be no interaction between two cylinders when they are separated far enough. We also observe that the migration velocity is larger in shear thinning fluids. This is also observed in the migration of spherical particles.

This result has some implications for the chaining mechanism of particles in shear thinning fluid and distribution of particles in non-shear thinning Boger fluids. If two cylinders are separated from each other by a short distance, the shear rate between two spheres will be large because two neighboring cylinders are rotating in the same direction. This will result in a normal stress difference between two cylinders that will push them apart. Depending on the degree of imbalance in normal forces between the outer and interstice regions, particles will move toward or away from each other. In a shear thinning fluid, the vortical motion between two cylinders is severely diminished as seen in Fig. 5. Therefore the normal force from the interstice region cannot support the normal force from the outer region, hence the two cylinders get closer. In a non-shear thinning fluid, the normal force from the interstice region remains large and comparable to the one from the outer region, hence two cylinders cannot get closer as in the case of shear thinning fluid. When the two cylinders get sufficiently close, even in a shear thinning fluid, two cylinders cannot move toward each other due to strong extensional flow. Hence there is an equilibrium distance even in shear thinning fluid even though it is smaller than that of a non-shear thinning fluid.

In the case of spherical particles, once two particles approach to each other closely, flow between the two particles becomes very weak and thus the normal stress difference becomes negligibly small there. This will accelerate the chaining phenomenon because the resisting force does not exist any longer.

5. Conclusion

In this study we have considered a hydrodynamic interaction problem between two cylinders placed in a viscoelastic fluid under shear flow between two solid boundaries. We have found that there is an equilibrium distance between two cylinders and the equilibrium distance is smaller in shear thinning fluid, implying the chaining is stronger in shear thinning fluid. Also the migration velocity is larger in shear thinning fluid, implying faster chaining. Even though the implications here are only qualitative, the result obtained here can shed light on the understanding of the hydrodynamic interaction among particles and microstructure formation in viscoelastic fluid.

Acknowledgments

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