# Fuzzy Modeling of a surface Deformation for Virtual Environment

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#### Abstract

In this paper, a 3D model of the surface deformation is created in virtual environment. A proposed method is based on the fuzzy model and it is enough that only one rule set is added to the fuzzy model to model a surface deformation. Furthermore, the designer can easily determine which parameters should be used and how they should be changed in order to obtain the shapes as required. The proposed method is, thus, a simple, but effective technique that can also be used in practical applications. The results of the computer simulation are also given to demonstrate the validity of the proposed algorithm.

Key Words: Fuzzy model, Fuzzy display, Surface deformation, Virtual environment

#### I. Introduction

Recently, many studies have been conducted on virtual reality since there are a number of applications that use virtual reality. Virtual reality is a synthetic, three-dimensional, interactive environment typically generated by computer. In virtual reality, visual feedback is given primary consideration because it accomplishes much of the effect of immersion into virtual environment. Immersion into virtual reality depends heavily on good visual information. Providing good visual information requires a proper physical modeling of an environment. Thus, it is necessary to take a close look at physical modeling of the virtual environment to provide sufficient realism. Among the various components of physical modeling, this paper is focused on physical modeling of a surface deformation. Developing better algorithms for surface deformation model will improve the fidelity of visual realism of virtual environment.

In reality, surfaces are usefully deformable, so that a pressed virtual object will change shape in response to the user-applied forces. The visual realism can thus be added by the surface deformation models that are interactive and satisfy the real-time requirement of virtual environment. The methods for displaying a surface deformation can be largely classified as vertex-based and spline-based, depending on whether the object surface is represented by polygonal meshes or parametric equations[1]. In the vertex-based method, the deformation of one vertex will impact its neighbors, and therefore the object mesh look-up table needs supplemental information.

Another way of representing virtual objects is through parametric cubic surfaces. These have a local control behavior and the time needed to recompute the polynomial coefficients is greatly reduced. However, this local deformation technique does not suffice when several objects may need to be deformed simultaneously. Furthermore, in most of the existing free form deformation methods, users can not touch surfaces directly, but some special parameters called control points, weights, and so on[2]. In this method, there are many control points and it is hard to predict what deformation can be obtained after several parameters' change even if they are defined only by control points and knot vectors. Designer have to learn how many kinds of parameters they have and the effect of each parameter; therefore, each time they deform forms, users have difficulties in determining which parameters should used and how they should be changed in order to alter shapes as required.

A method for displaying a surface deformation has been proposed using the fuzzy model[3]. While the proposed method is simple since surface deformation is defined only by simple fuzzy rules, the shape deformed is coarse and may not seem to be accurate.

In this paper, to overcome the above problems, we propose a new method which is a simple, but effective technique, that can also be applied in practical applications. The approach selected in this paper is based on the fuzzy model for modeling a surface deformation.

This paper is organized as follows. In section II, a basic concept of the fuzzy model is explained and a new method for surface deformation modeling is suggested by utilizing the merits of the fuzzy model. In the next chapter, the results of computer simulation are given to demonstrate the validity of the proposed algorithm. Finally, the conclusion remarks are presented.

# II. Fuzzy model for elastic deformation

Fuzzy models are advantageous in their ability to accurately describe complex nonlinear systems. One of the outstanding models among them is the model suggested by Takagi and Sugeno in 1985[4]. Therefore, we use Takagi and Sugeno's fuzzy model to model a surface deformation. The method of

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identifying a fuzzy model is essential in understanding how the surface deformation is modeled by fuzzy model. But, since the identification of a fuzzy model is beyond the scope of this paper, for details, the author advises readers to refer to [5]-[9].

This fuzzy model is a nonlinear system model represented by fuzzy rules of the type:

$$R^{i}: If x_{1} \text{ is } A_{1}^{i} \text{ and } \cdots \text{ and } x_{m} \text{ is } A_{m}^{i}$$

$$then y^{i} = a_{0}^{i} + a_{1}^{i}x_{1} + \cdots + a_{m}^{i}x_{m}$$

$$(1)$$

where  $R^i$  ( $i=1,2,\cdots,n$ ) denotes the i th fuzzy rule,  $x_j$  ( $j=1,2,\cdots,m$ ) are input variables and  $y^i$  is an output from the i th implication. Furthermore,  $a^i_j$  is a consequent parameter and  $A^i_1, A^i_2, \cdots, A^i_m$  are membership functions representing a fuzzy subspace. The overall output of a fuzzy model is given as

$$\widehat{y} = \frac{\sum_{i=1}^{n} \omega^{i} y^{i}}{\sum_{i=1}^{n} \omega^{i}}, \quad \omega^{i} = \prod_{j=1}^{m} A_{j}^{i}(x_{j})$$
 (2)

where  $\hat{y}$  is an output inferred from the fuzzy model,  $\omega^i$  is a degree of match for the i th fuzzy rule and  $\prod$  denotes the minimum operation.

As shown in equations (1) and (2), this fuzzy model describes the nonlinear input-output relation with the piecewise linear equations for the fuzzy partition of the input space. The proposed algorithms for modeling a surface deformation are based on this fuzzy model. Deforming actions are used to create a new fuzzy rule set that defines the deformed shape of the object and this new fuzzy rule set is added to the original fuzzy model to model a surface deformation.

This new rule set is in the general form of

$$R^{i^{\text{new}}}$$
: If  $x$  is  $A_x^{i^{\text{new}}}$  and  $y$  is  $A_y^{i^{\text{new}}}$ ,  
then  $z^i = a^i x + b^i y + c^i$  (3)

where  $R^{i^{max}}$  ( i=1,2,3,4) denotes the fuzzy rule for the input space i.

The input space for a new rule set is divided into four regions as shown in Fig. 1, where the coordinate  $(x_0, y_0)$  corresponds to a point pressed on the surface of an object and  $d_0$  determines the area to be deformed. A larger value for  $d_0$  results in the deformation of a wider area.

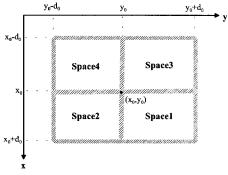


Fig. 1. Fuzzy partition of an input space for a new rule set

The graphical description of a consequent part for input space 1 is shown in Fig. 2. The value of  $z_0$  is the z coordinate on the surface without deformation. The value of  $z_1$  depends on both the force on and stiffness of the object and we define  $z_1$  as  $\alpha \frac{f}{k}$ , where f is the force which is applied to the surface, k is the stiffness of the object and  $\alpha$  is the calibration factor, where  $\alpha$  is defined as the ratio of the size of the virtual object to the size of the actual object. If a pressure is applied to the surface, the value of  $z_1$  is altered in proportion to pressure magnitude.

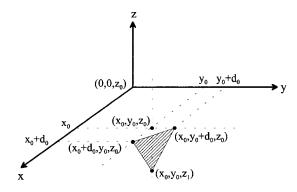


Fig. 2. The graphical description of a consequent part for input space 1

As shown in the Fig. 2, the hyperplane of a consequent part for input space 1 passes the three points  $(x_0+d_0,y_0,z_0)$ ,  $(x_0,y_0+d_0,z_0)$  and  $(x_0,y_0,z_1)$ . As an equivalent method, for each input space 2, 3 and 4, its hyperplane passes the points  $\{(x_0,y_0-d_0,z_0), (x_0+d_0,y_0,z_0), (x_0,y_0,z_1)\}$ ,  $\{(x_0-d_0,y_0,z_0), (x_0,y_0+d_0,z_0), (x_0,y_0,z_1)\}$  and  $\{(x_0,y_0-d_0,z_0), (x_0-d_0,y_0,z_0), (x_0,y_0,z_1)\}$  respectively.

The parameters of the new fuzzy rule are, thus, easily determined as follows:

For space 1, 
$$a^{1} = (z_{0} - z_{1})/d_{0}$$
,  $b^{1} = (z_{0} - z_{1})/d_{0}$ ,  $c^{1} = z_{1} - (z_{0} - z_{1})(x_{0} + y_{0})/d_{0}$ , for space 2,  $a^{2} = (z_{0} - z_{1})/d_{0}$ ,  $b^{2} = -(z_{0} - z_{1})/d_{0}$ ,  $c^{2} = z_{1-(z_{0}} - z_{1})(x_{0} - y_{0})/d_{0}$ , for space 3,  $a^{3} = -(z_{0} - z_{1})/d_{0}$ ,  $b^{3} = (z_{0} - z_{1})/d_{0}$ ,  $c^{3} = z_{1} + (z_{0} - z_{1})(x_{0} - y_{0})/d_{0}$ , and for space 4,  $a^{4} = -(z_{0} - z_{1})/d_{0}$ ,  $b^{4} = -(z_{0} - z_{1})/d_{0}$ ,  $c^{4} = z_{1} + (z_{0} - z_{1})(x_{0} + y_{0})/d_{0}$ . (4)

The  $A_x^{i^{\text{mex}}}$  and  $A_y^{i^{\text{mex}}}$  are membership functions and in this paper, all membership functions are Gaussian-like and are fully described by their modal values  $p_x^i$ ,  $p_y^i$  and spreads  $q_x^i$ ,  $q_y^i$  as follows:

$$A_x^{i^{\text{new}}} = \exp\left\{-\left(\frac{x-p_x^i}{q_x^i}\right)^2\right\}$$
 and

$$A_y^{i,\text{new}} = \exp\left\{-\left(\frac{x - p_y^i}{q_y^i}\right)^2\right\} \tag{5}$$

where modal values  $p_x^i$  and  $p_y^i$  of these membership functions correspond to the following values respectively.

For space 1, 
$$p_x^1 = x_0 + \frac{d_0}{2}$$
 and  $p_y^1 = y_0 + \frac{d_0}{2}$ ,

for space 2, 
$$p_x^2 = x_0 + \frac{d_0}{2}$$
 and  $p_y^2 = y_0 - \frac{d_0}{2}$ ,

for space 3,  $p_x^3 = x_0 - \frac{d_0}{2}$  and  $p_y^3 = y_0 + \frac{d_0}{2}$  and for space 4,

$$p_x^4 = x_0 - \frac{d_0}{2}$$
 and  $p_y^4 = y_0 - \frac{d_0}{2}$  (6)

The values for spreads  $q_x^i$ ,  $q_y^i$  are all defined as  $d_0$ .

The algorithm for modeling of a surface deformation is as follows:

**Step1**: Display the original surface using the fuzzy model of the type:

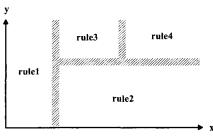
$$R^i$$
: If  $x$  is  $A_x^i$  and  $y$  is  $A_y^i$ ,  
then  $z^i = a_0^i x + b_0^i y + c_0^i$  (7)

**Step2**: If a force is applied to the surface, apply a new rule set  $R^{i^{\text{new}}}$  for surface deformation. This new rule set is in the general form of

$$R^{i^{new}}$$
: If  $x$  is  $A_x^{i^{new}}$  and  $y$  is  $A_y^{i^{new}}$ ,  
then  $z^i = a^i x + b^i y + c^i$  (8)

where  $R^{i^{***}}(i=1,2,3,4)$  denotes the fuzzy rule for the input space i.

**Step 3**: Add the newly generated rule set  $R^{i^{mw}}$  of a surface deformation to the original fuzzy model as shown in Fig. 3 and a new fuzzy model is reconstructed to model surface deformation.



(a) without deformation

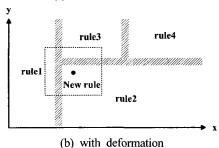


Fig. 3. Fuzzy partition of the input space

**Step4**: If another force is applied to the surface, repeat steps a, b and c.

In the above algorithm,  $z_1$  in the consequent part determines the deformed depth when the surface is pressed. That is, a larger value for  $z_1$  means larger deformation. Furthermore, users are able to control the region and shape of deformation by adjusting  $d_0$  and  $z_1$ . Therefore, the new fuzzy rule allows users to manipulate surface geometries, such as curvature and depth of deformation at any point on the surface.

## III. Simulation and Considerations

To illustrate the approaches proposed in this paper, we show the effect of the deformation, according to the change of the parameters in the fuzzy model through the simulation of a plane surface. In this case, a plane surface is represented by only one rule whose consequent part is a constant expression as follows:

$$R^1$$
: If  $x$  is  $A_x(20, 100)$  and  $y$  is  $A_y(20, 100)$ ,  
then  $z = f_1(x, y) = 30$  (9)

where, 
$$A_x(p_x, q_x) = \exp\left\{-\left(\frac{x - p_x}{q_x}\right)^2\right\}$$
,

 $A_y(p_y,q_y) = \exp\left\{-(\frac{x-p_y}{q_y})^2\right\}$  and the other parameters are defined as follows :

$$x_0=20$$
,  $y_0=20$ ,  $z_0=30$ ,  $z_1=30$  and  $d_0=10$  (10)

If a force is applied to the above plane surface, a new rule set  $R^{i^{****}}$  for surface deformation is added. Assume that a following new rule set is added for surface deformation:

$$R^{i^{\text{new}}}$$
: If  $x$  is  $A_x^{i^{\text{new}}}$  and  $y$  is  $A_y^{i^{\text{new}}}$ ,  
then  $z^i = a^i x + b^i y + c^i$  (11)

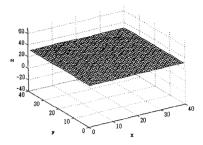
where  $A_x^{i^{\text{now}}}$ ,  $A_y^{i^{\text{now}}}$ ,  $a^i$ ,  $b^i$  and  $c^i$  are defined in equations (4), (5) and (6) and determined by the parameters  $x_0$ ,  $y_0$ ,  $z_0$ ,  $z_1$  and  $d_0$ . To show the validity of the proposed method, we simulate the effect of deformation according to the change of parameters  $x_0$ ,  $y_0$ ,  $z_0$ ,  $z_1$  and  $d_0$ . First of all, we change the parameters  $z_1$  for the fixed values of  $z_0$ ,  $z_0$ ,  $z_0$  and  $z_0$ .

Assume that each new rule set of following two cases is added for surface deformation respectively:

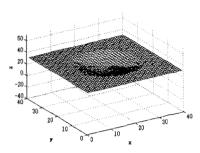
Case1: 
$$x_0=20$$
,  $y_0=20$ ,  $z_0=30$ ,  $z_1=20$ ,  $d_0=10$  (12)

Case2: 
$$x_0=20, y_0=20, z_0=30, z_1=10, d_0=10$$
 (13)

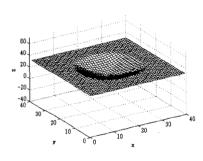
Fig. 4(a) shows a plane surface represented by equation(9) and Fig. 4(b) and (c) show surfaces deformed according to each case. As shown in Fig. 4, as the value for the  $z_1$  decreases, the depth of deformation increases because the depth of deformation depends on the value of  $z_1$ . That is, the value for the  $z_1$  describes the depth of deformation.



### (a) Before deformation( $z_1$ =30)



(b) After deformation( $z_1=20$ )



(c) After deformation( $z_1$ =10)

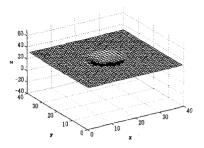
Fig. 4. Deformation according to the change of the parameter  $z_1$ 

On the other hand, the parameter  $d_0$  has its own deformation characteristic, which is defined by the region it affects and the shape it creates. Now we will show the deformation characteristic according to the change of the parameter  $d_0$ . Consider the following cases that show the deforming effect according to the change of the  $d_0$  for the fixed values of  $x_0$ ,  $y_0$ ,  $z_0$ , and  $z_1$ .

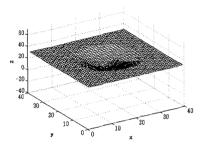
Case1: 
$$x_0=20, y_0=20, z_0=30, z_1=20, d_0=7$$
 (14)

Case2: 
$$x_0=20, y_0=20, z_0=30, z_1=20, d_0=10$$
 (15)

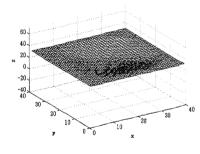
Case3: 
$$x_0 = 20$$
,  $y_0 = 20$ ,  $z_0 = 30$ ,  $z_1 = 20$ ,  $d_0 = 13$  (16)



(a) After deformation( $d_0$ =7)



(b) After deformation( $d_0=10$ )



(c) After deformation( $d_0$ =13)

Fig. 5. Deformation according to the change of the parameter

For each case, the graphical results are shown in Fig. 5(a), (b) and (c) respectively. As shown in Fig. 5, for a small value of  $d_0$ , the deformation occurs in a small neighborhood of the point pressed. A large value for  $d_0$  results in deformation of a wide area and a curve more smoothly connected. Thus, users are able to control the region of deformation and the properties of the shape being deformed by adjusting the parameter of  $d_0$  that defines the membership functions.

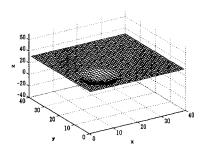
Next, we will show the effects of the deformation according to the change of  $x_0$  and  $y_0$ . Consider the following three cases, where only the values of  $x_0$  and  $y_0$  are changed.

Case1: 
$$x_0=10, y_0=10, z_0=30, z_1=20, d_0=7$$
 (17)

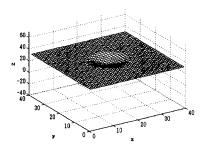
Case2: 
$$x_0=20, y_0=20, z_0=30, z_1=20, d_0=7$$
 (18)

Case3: 
$$x_0=30, y_0=30, z_0=30, z_1=20, d_0=7$$
 (19)

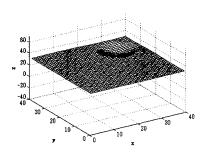
For each case, the graphical results are shown Fig. 6(a), (b), and (c) respectively. If the values for  $x_0$  and  $y_0$  are changed, the pressed point is also changed to a position corresponding to the coordinates  $(x_0, y_0)$  and we can say that the values of  $x_0$  and  $y_0$  corresponds to a point pressed on the surface of an object.



(a) After deformation  $(x_0=10, y_0=10)$ 



(b) After deformation  $(x_0=20, y_0=20)$ 



(c) After deformation  $(x_0=30, y_0=30)$ 

Fig. 7. Deformation according to the change of the parameters  $x_0$  and  $y_0$ 

We have shown that the proposed method forces designers to translate surface deformations into geometrical expressions and provide for various kinds of deformation. In the proposed method, the specified region of the deformation is described by the parameters of the membership function and the depth of deformation is controlled by the consequent parameters of the fuzzy model. This means that the designer can easily determine which parameters should be used and how they should be changed in order to obtain the required shapes. Therefore, the proposed method provides a novel interface that

allows users to touch and push anywhere on the surface to alter forms, even if these forms are defined only by simple fuzzy rules. Obviously, we are dealing here with an easy and well defined object, however, the proposed method should be improved in different ways for more complex deformation tasks. If we increase the number of fuzzy rules, the shape deformed may be seem to be more precise and the proposed method can be easily applied to much more complex shape representation. However, there are still many challenging problems when confronted with more complex shape representation and practical application.

As a future work, thus, more research will be conducted on various cases. As another future work, we have to mention some practical applications in order to utilize the proposed method properly. The proposed method will be combined with a teleoperated nano-manipulator using visual feedback to increase touch and feel in the three dimensional form deforming process. This application will demonstrate the possibility of using the proposed method as a industrial application.

#### IV. Conclusion

We have suggested a new modeling method for a 3D surface deformation based on the fuzzy model and it is shown that the proposed method is useful in displaying a 3D surface deformation through computer simulation. The proposed method is simple and efficient compared to the conventional methods because it is sufficient that only one additional rule set be added to the fuzzy model to model a surface deformation.

For convenience, the proposed algorithm is applied to an easy and well defined object, but it is also applicable more complex objects. However there are still many challenge problems such as much more complex shape representation and practical applications.

As a future work, thus, more researches will be conducted on more complex shape representation and practical applications.

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