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Blind Neural Equalizer using Higher-Order Statistics

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Abstract

This paper discusses a blind equalization technique for FIR channel system, that might be minimum phase or not, in digital communication. The proposed techniques consist of two parts. One is to estimate the original channel coefficients based on fourth-order cumulants of the channel output, the other is to employ RBF neural network to model an inverse system for the original channel. Here, the estimated channel is used as a reference system to train the RBF. The proposed RBF equalizer provides fast and easy learning, due to the structural efficiency and excellent recognition-capability of RBF neural network. Throughout the simulation studies, it was found that the proposed blind RBF equalizer performed favorably better than the blind MLP equalizer, while requiring the relatively smaller computation steps in tranining.

Key Words : Neural network, RBF, Blind equalizer, HOS(higher-order-statistics)

Ⅰ. Introduction

Intersymbol Interferences (ISI) arises in pulse modulation systems whenever the energy of one received pulse does not die away completely before the beginning of the next. ISI may be caused by band limiting (as, for example, in telephone channels) or frequency selectivity (fading or multipath propagation) as in digital microwave radio and in mobile communication systems. The purpose of an equalizer is to combat the ISI problem. The most widely known equalizer is an adaptive transversal equalizer, in which output signal is compared to the expected signal and FIR filter coefficients are adjusted in accordance with the error between the desired and actual filter output.

For the last three decades, many of blind equalizers that do not use the known training sequence have been proposed in the literature beginning with Sato[1-3], because there are some practical situations when the conventional adaptive algorithms are not suitable for wireless communications during an outage(caused by severe fading).

Most current blind equalization techniques use higher order statistics (HOS) of the received sequences, directly or indirectly, because they are efficient tools for identifying that may be the nonminimum phase[4,5]. The HOS based techniques have the capability to identify a nonminimum phase system simply from its output because of the property of polyspectra to preserve not only the magnitude but also the phase information of the signal.

This paper develops a new method to solve the problems of blind equalization, by combining the advantages of HOS and a RBF neural network. The main purpose of the proposed blind RBF equalizer is to solve the obstacles of long time training and complexity that are often encountered in the blind MLP equalizers[6-8]. The RBF equalizer provides fast and easy learning, while requiring the relatively smaller computation steps in tranining[9-12]. The proposed techniques firstly estimates the order and coefficients of the original channel based on the autocorrelation and the fourth-order cumulants of the received signals. Then, an equalizer system using a RBF neural network is trained with input sequences from the estimated channel models. The nonlinear structure of the neural network equalizer makes the method superior to linear equalizers.

In the Section Ⅱ, a brief summary of RBF network is presented. Section Ⅲ presents the cumulant-based channel estimation algorithms. Section IV gives the learning procedure for the blind RBF equalizer. Simulation results are provided in Section V and Section VI gives the conclusions.

Ⅱ. Radial Basis Functions

A RBF network is a three-layer network whose output nodes form a linear combination of the basis functions computed by the hidden layer nodes. The basis functions in the hidden layer produce a localized response to input stimuli. The most common choice of basis functions for the hidden nodes in the network is the Gaussian function. A diagram of a radial basis function network is shown in Fig. 1.

Due to the radial nature of the basis functions, a hidden node produces a significant response only when the input falls within a small localized region of the input space; the output response of the Gaussian basis function depends only on the Euclidean norm of the difference between the center vector and an input vector is very small. On the other hand, if the difference of these two is large, the reponse is weak(low value). The Gaussian basis function and the network output are described as follows:

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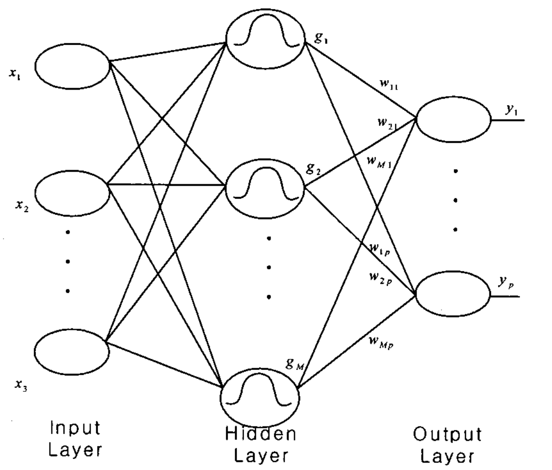

Fig. 1 The architecture of radial basis function network

$$g_j = \exp\left[-\frac{\|X-C_j\|^2}{2\sigma_j^2}\right], j=1,2,\cdots,M \quad (1)$$

and

$$y_k = \sum_{j=1}^{M} w_{jk} g_j, \quad k=1,2,\cdots,P, \quad (2)$$

where

$X = (x_1, x_2, \ldots, x_N)^T$= the input vector,

$C_j = (C_{j1}, C_{j2}, \ldots, C_{jN})^T$

= the mean vector (center) of the jth node,

σ_j^2 : the normalization parameter for the jth node,

g_j : the output of the jth node in the hidden layer

w_{jk} : the weight between the jth hidden node and kth output node,

y_k : the network outputs of kth node.

III. Channel Estimation

The block diagram of a base-band communication system subject to intersymbol interference (ISI) and additive white Gaussian noise (AWGN) is shown in Fig. 2. Assume that the received signal $\{y_k\}$ is generated by an FIR system described by

$$y_k = \sum_{i=0}^{L} h_i s_{k-i} + n_k = \hat{y}_k + n_k \quad (3)$$

Where $\{h_i\}, 0 \le i \le L$ is the impulse response of the channel and $\{s_k\}$ is i.i.d., nonGaussian with mean $E\{s_k\}=0$, second-order moment $E\{s_k^2\} = \sigma_s \neq 0$,

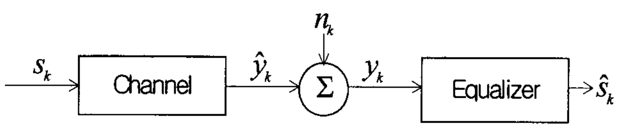

Fig. 2 Equalizer system

skewness $E\{s_k^3\} = 0$,and $E\{s_k^4\} - 3[E\{s_k^2\}]^2 = \gamma_s \neq 0$. For example, $\{s_k\}$could be a two-level symmetric PAM sequence. The additive noise $\{n_k\}$ is zero mean, Gaussian, and statistically independent of $\{s_k\}$.

It is also assumed that the FIR system can be either minimum phase or nonminimum phase, i.e., the polynomial has zeros inside and outside the unit circle. The task of the blind equalizer is to recover the input sequence $\{s_k\}$ by processing $\{y_k\}$ only, with a constant delay and possibly a constant phase shift, as shown in Fig. 2.

1. Estimation of channel order

The autocorrelation technique is used to estimate the channel order that is required to specify the number of centers in an RBF equalizer. Consider the autocorrelation

$$\rho_l = \frac{Cov(y_k, y_{k+l})}{\sqrt{Var(y_k)}\sqrt{Var(y_{k+l})}} = \frac{E[(y_k - E[y_k]) \cdot (y_{k+l} - E[y_{k+l}])]}{\sqrt{E[(y_k - E[y_k])^2]} \cdot \sqrt{E[(y_{k+l} - E[y_{k+l}])^2]}} \quad (4)$$

where $Cov(\cdot,\cdot)$, $Var(\cdot)$, and $E[\cdot]$ denote the covariance, variance, and expectation operator respectively and l is correlation lag. From (3), $E[y_k]$ is represented as

$$E[y_k] = h_0 E[s_k] + h_1 E[s_{k-1}] + h_2 E[s_{k-2}] + \ldots + h_p E[s_{k-p}] + E[n_k] \quad (5)$$

Assuming that s_k is a binary independent sequence and n_k is an additive white Gaussian (AWGN) sequence, $E[y_k]$ and $E[y_{k+l}]$ go to zero. Thus (4) reduces to

$$\rho_l = \frac{E[y_k \cdot y_{k+l}]}{E[y_k^2]} \quad (6)$$

Then, (6) can be rewritten as

$$\rho_l = \begin{cases} 1, & l=0 \\ \dfrac{\sum_{i=0}^{p-l} h_i h_{i+l}}{\left(\sum_{i=0}^{p} h^2 + \sigma_n^2\right)}, & 1 \le l \le p \\ 0, & l > p \end{cases} \quad (7)$$

where $\sigma_n^2 = E[n_k^2]$ is noise variance. As shown in (7), the basic idea of the autocorrelation technique for channel order estimation is that when the lag l exceeds the correct order of the channel model, the autocorrelation becomes zero. In practical cases, the autocorrelation in (6) is not available. Thus, we consider the normalized sample autocorrelation, using the information in the smaller set of measurements (the received signal) to estimate the channel order. The normalized sample autocorrelation is

$$\hat{\rho}_l = \sum_{k=1}^{N-l}(y_k - \bar{y})(y_{k+l} - \bar{y}) \Big/ \sum_{k=1}^{N}(y_k - \bar{y})^2, \; l \ge 0 \quad (8)$$

where N is the number of samples, and $\bar{y}$ is the sample mean of y_k

$$\bar{y}=\frac{\sum_{k=1}^{N} y_k}{N} \tag{9}$$

A sample autocorrelation, $\hat{\rho}_l$ is regarded as meaningful if it is outside the 95 percent confidence interval

$$-1.96\frac{1}{\sqrt{N}} \le \hat{\rho}_l \le 1.96\frac{1}{\sqrt{N}} \tag{10}$$

The technique selects the last meaningful sample autocorrelation as an estimate $\hat{\rho}_l$ for the channel order.

2. Channel coefficient estimation

Using the properties of higher order cumulants and the problem assumptions, the following expressions can be written for the channel described by (3)

$$C_y(l,m,n)=C_{\tilde{y}}(l,m,n)+C_n(l,m,n) \tag{11}$$

$C_y(l,m,n)$ is the fourth-order cumulant sequence of $\{y_k\}$, which is defined as

$$\begin{aligned} C_y(l,m,n)=&M_y(l,m,n)-M_y(l)M_y(n-m)\\ &-M_y(m)M_y(n-l)-M_y(n)M_y(m-l)\end{aligned} \tag{12}$$

where the second-order moments $M_y(j)$ and the fourth-order moments $M_y(l,m,n)$ of y_k are defined as

$$\begin{aligned} &M_y(j)=E\,[y_k, y_{k+j}] ,\\ &M_y(l,m,n)=E\,[y_k, y_{k+l}, y_{k+m}, y_{k+n}]\\ &\qquad j,l,m,n\ =\ 0,\pm1,\pm2,\cdots\end{aligned} \tag{13}$$

since $\{n_k\}$ is Gaussian, its fourth-order cumulants has zeros, which means

$$C_y(l,m,n)=C_{\tilde{y}}(l,m,n) \tag{14}$$

with knowing this fact, it is easy to show that,

$$C_y(l,m,n)=\gamma_s \sum_{t=0}^{p-\max(l,m,n)} h_t\cdot h_{t+l}\cdot h_{t+m}\cdot h_{t+n}\cdots \tag{15}$$

(15) can be represented as

$$\boldsymbol{CH}=\boldsymbol{c} \tag{16}$$

where $\boldsymbol{C}$ and $\boldsymbol{c}$ are the matrix and vector consisting of the estimated fourth-order cumulants, and $\boldsymbol{H}$ is the unknown coefficient vector. The solution of (16) can be given in an explicit form as

$$\boldsymbol{H}=(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H\boldsymbol{c} \tag{17}$$

IV. Implementation of the Blind RBF Equalizer

A blind RBF equalizer is implemented in this section. It uses higher-order cumulants and a RBF neural network. Fig. 3 shows the block diagram of the blind RBF equalizer system.

In order to train a neural network to serve as a channel equalizer, it is necessary to generate appropriate training data. The following is the training input to RBF network, which is generated by the receiver instead of being provided by the transmitter

$$r_k=\sum_i^p h_i x_{k-i} \tag{18}$$

where x_k and r_k are input signal to the estimated channel model $\{h_i\}$ and input signal to the RBF equalizer respectively. Therefore, input patterns for the network is the r_k, and the corresponding target is x_k. In this study, it is assumed that the network is trained to reconstruct the originally transmitted binary signal (1 or -1).

The estimated channel is characterized by its transfer function, which in general has the form

$$H(Z)=\sum_{i=0}^{p} h_i Z^{-i} \tag{19}$$

where p is the estimated channel order. If q denotes the equalizer order (number of tap delay elements in the equalizer), then there are $M=2^{p+q+1}$ different sequences

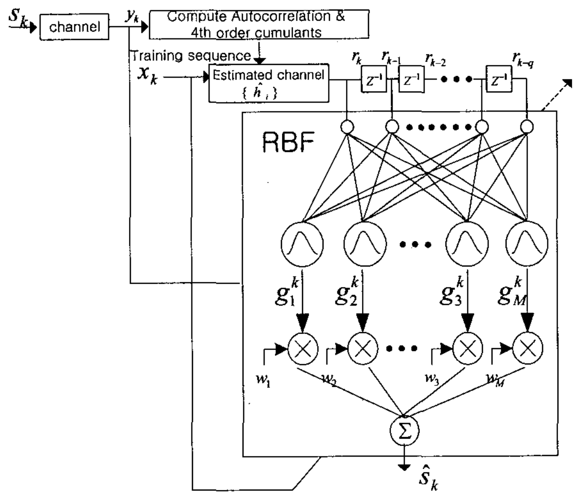

Fig. 3 The structure of blind RBF equalizer system

$$\boldsymbol{x}_k=[x_k, x_{k-1},\cdots,x_{k-p-q}]^T \tag{20}$$

and the corresponding received signal vectors, $\boldsymbol{r}_k$, are represented as

$$\boldsymbol{r}_k=[r_k, r_{k-1},\cdots,r_{k-q}]^T \tag{21}$$

where

$$r_k=\sum_{i=0}^{p} h_i x_{k-i} \tag{22}$$

The training input patterns, $\{\boldsymbol{R}_i\}$, can be obtained by the following procedures

```
if( x_k = x_i ) {
    R_i = r_k;
}
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where $\boldsymbol{x}_i$ and $\boldsymbol{R}_i$, $i=1,2,\cdots,M$, are the combination of $\boldsymbol{x}_k$, and training input pattern, respectively

$$\boldsymbol{R}_i=[R_{i,0},R_{i,1},\cdots,R_{i,q}]^T \tag{23}$$

These patterns can be trained by the following RBF LMS(least mean square)

$$\begin{aligned} g_i^k &= e^{-\frac{\|r_k - R_i\|^2}{2\sigma_i^2}} \\ e_k &= x_{k-d} - \sum_{i=0}^{M} w_i^k g_i^k \\ w_i^k &= w_i^{k-1} + \beta e_k g_i^k \end{aligned} \tag{24}$$

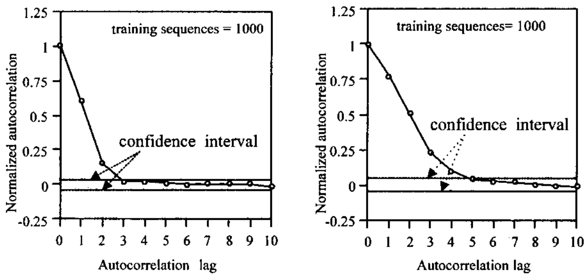
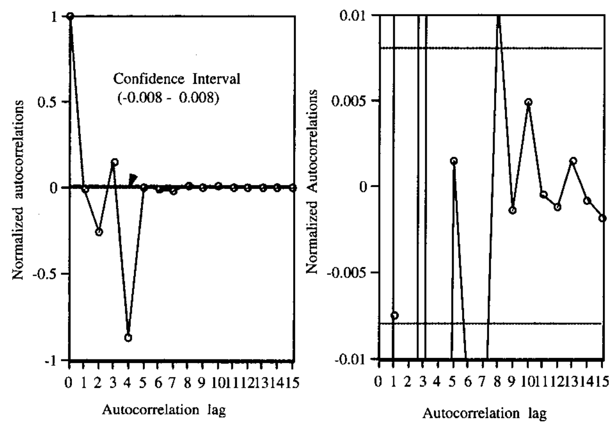


Fig.4 Channel order estimation
(a) $H(z) = 0.348 + 0.87z^{-1} + 0.348z^{-2}$
(b) $H(z)=0.227+0.46z^{-1}+0.688z^{-2}+0.46z^{-3}+0.227z^{-4}$



(a) (b)

Fig.5 Channel order estimation noise variance = 0.01 , training sequence = 60000
(a) original graph
(b) partially enlarged graph

$$\begin{aligned} H(Z)=&0.04-0.05Z^{-1}+0.07Z^{-2}-0.21Z^{-3} \\ &-0.5Z^{-4}+0.72Z^{-5}+0.36Z^{-6}+0.0Z^{-7} \\ &+0.21Z^{-8}+0.03Z^{-9}+0.07Z^{-10} \end{aligned}$$

where e_k is error , β is the learning rate(step sizw) for weights.

V. Simulation Results

In this section, some results of computer simulations are presented to demonstrate how higher order cumulants and layered feed forward networks can be utilized to form a blind adaptive equalizer. For the computational convenience, it is assumed that the binary signals (+1 or -1) are generated at random with an additive white Gaussian noise. Firstly, the channel order is estimated with three different channel models. Autocorrelations of channel observations were computed using (8) and the results of them are illustrated in Fig.4-5. As shown in the Figures, channel orders were correctly revealed from their normalized sample autocorrelations. For the estimates of the channel coefficients, 5 different realizations with the training sequences equal to 512 are performed with SNR equal to 10 db. The mean value of the estimates is shown in the Table 1.

Table 1. Channel coefficient estimation

Original channel model	estimated channel coefficient
$H(Z)=0.5+1.0Z^{-1}$	h_0 = 0.50854 h_1 = 1.00076
$H(z)=0.348+0.87z^{-1}+0.348z^{-2}$	h_0 = 0.35706 h_1 = 0.87566 h_2 = 0.34682

The results show that the channel is almost correctly estimated from the channel output observations. Finally, the RBF equalizer is trained with the estimated channel model. Fig.6 shows the error rate comparison of linear equalizer and MLP and RBF neural network equalizers.

As shown in the graph, the performance of the blind RBF equalizer is superior to that of the blind MLP equalizer and linear equalizer.

VI. Conclusion

In this paper, a blind equalization technique is discussed based on higher-order statistics and a RBF. The main procedures of the proposed blind equalizer consist of two parts. One is to estimate the order and coefficients of original channel using higher-order-cumulants; the estimated channel is used to generate the reference signal. The other part is to reconstruct the originally transmitted symbols (signals) after training the RBF neural network. The main purpose of the blind RBF equalize is to solve the obstacles of long time training and complexity that are often encountered in the MLP equalizers. The proposed RBF equalizer provides fast and easy learning, due to the structural efficiency and excellent recognition-capability of RBF neural network. Throughout the simulation studies, it was found that the proposed blind RBF equalizer performed favorably better than the blind MLP equalizer,

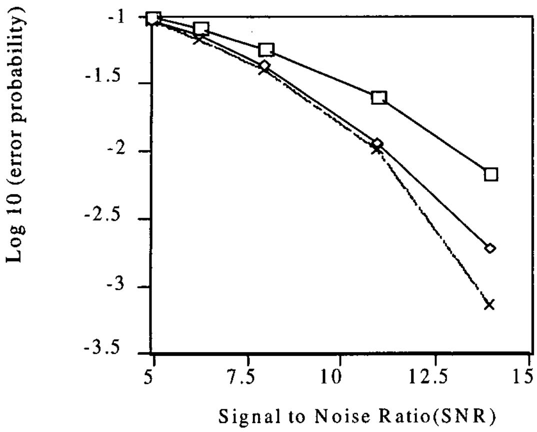


Fig.6 Error rate comparison : $H(Z)=0.5+1.0Z^{-1}$
□ : blind linear equalizer
◇ : blind MLP equalizer
× : blind RBF equalizer

while requiring the relatively smaller computation steps in tranining.

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