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Nonlinear Localized Modes in Photonic Crystals

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Abstract: We give a brief overview of nonlinear localized modes in photonic crystals. We explain how photonic crystals can potentially be important in making small scale active devices which operate in an all optical way. Two models to approach nonlinear photonic crystals, the coupled mode theory and the discrete lattice theory using a Green's function, are explained.

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I. INTRODUCTION

One of the major challenges in the study of photonic crystals [1] is to make them tunable. Tunable photonic crystals operating in an all-optically controlled way hold great promise in the future as a possible candidate for replacing electronic, semiconductor based devices. All optical control is highly desirable for faster operational speed and lower manufacturing cost. Such promise may be realized through the utilization of the nonlinear optical properties of materials in designing photonic crystals. This produced a rapidly growing research activities recently in the area of nonlinear optics of photonic crystals, or the study of nonlinear photonic crystals. It combines photonic crystals with numerous earlier ideas and works in nonlinear optics in the wake of newly advancing technologies. Various topics, such as harmonic generation and wave mixing, optical switching and limiting, ultrafast optics, solitons, lasing in photonic bandgap structures, etc., have received a large amount of attention recently. Among them, the study of nonlinear localized modes such as solitons in photonic crystals deserve particular attention. Recent works in this area reveal interesting novel properties of nonlinear photonic crystals which may find a direct application in all-optical signal processing or logic operations.

But why is a photonic crystal more distinguished than the other systems which support nonlinear localized modes, e.g., an optical fiber possessing solitons or crystals supporting spatial solitons? How can we understand the characteristics of such nonlinear localized modes in photonic crystals? These are the two issues we will try to answer in this brief overview of nonlinear photonic crystals in the light of optical signal processing.

In linear wave systems, a wavepacket can be of any shape, either large or small, since it can always be expressed as a superposition of elementary component solutions (usually plane waves). Thus, in order to be a digitized information carrier, a wavepacket should take a certain form, e.g., either Gaussian or square shape, and retain its form during propagation. The arbitrariness in shape also implies that there are no definite localized modes in a linear wave system when the system possesses the translational symmetry. The situation changes, however, if the symmetry is broken by the presence of defects or an external potential. In general, a confining potential gives rise to discrete, localized modes of waves. For instance, the vanishing boundary condition of an infinite potential well results in discrete wavevectors (eigenvalues) and the corresponding wave modes (eigenfunctions). In a photonic crystal, defects can play the role of a potential such that localized modes can exist in an otherwise forbidden bandgap region. However, the amplitude of each discrete mode can still be of any value unless we go through the quantization of energy that leads to photons. In practice, the photon number is too big in a realistic photonic crystal so that the procedure of quantization is not usually necessary. Thus once again one has to set the unit of amplitude arbitrarily if one wants to take the localized mode as an information carrier. It is desirable to have a small value of amplitude for the information unit, especially for the use of small scale, compact integrated photonic devices. But it can not be too small since the small amplitude mode can be easily lost by any mechanism which degrades the quality of the unit mode.

In nonlinear systems, things are quite different. Certain nonlinear systems can support localized modes even without breaking translational symmetry. More surprisingly, localized modes can be automatically quantized. In order to illustrate these properties, we consider the well-known nonlinear Schrödinger(NLS) equation, which describes the propagation of optical pulses in an optical fiber.

$$i\partial_x \psi = -\beta \partial_{tt}\psi - \gamma|\psi|^2\psi. \quad (1)$$

Here, x and t denote space and the retarded time variable, ψ the slowly varying amplitude of an electric field, β the group velocity dispersion and γ a nonlinearity coefficient coming from the third order susceptibility. However, the NLS equation also appears in many other situations where the variable and coefficients have a different meaning, and our discussion serves for the general case. One way to understand the nonlinear localized mode in the NLS equation is to compare it with the quantum mechanical Schrödinger equation

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\partial_{xx}\psi + V\psi. \quad (2)$$

Note that the NLS equation can be regarded as the usual Schrödinger equation if we identify the potential V with minus the intensity, $V = -\gamma|\psi|^2$. Of course, this identification is a formal one since the intensity itself is made of a wave function ψ. Nevertheless, it provides a qualitative understanding of nonlinear localized modes. Assume that the amplitude ψ has a shape of a localized pulse. This introduces an effective potential well according to the identification $V = -\gamma|\psi|^2$. If the well is too narrow, that is the pulse is too sharp, then the experience from quantum mechanics tells us that the wave function tends to spread. For a wider well, the opposite focusing behavior can be observed. This adiabatic, self adjusting mechanism can lead to a stable, stationary shape of a pulse known as soliton. Such a shape has the form of a hyperbolic secant function and the wave function can be obtained analytically with the result,

$$\psi = \sqrt{\frac{2}{\gamma}} A\text{sech}\Big(\frac{At}{\sqrt{\beta}}\Big) exp(iA^2x). \quad (3)$$

Note that the amplitude is proportional to an arbitrary constant A, which in turn is inversely proportional to the pulse width. Thus the time area of a pulse $\int|\psi|dt$ becomes $\pi\sqrt{2\beta/\gamma}$, which depends only on material parameters β and γ. Moreover, according to the nonlinear dynamics of the NLS equation, any arbitrary shape input pulse breaks up into several fundamental solitons. The soliton itself is robust against external disturbances by automatically self-adjusting its form into a soliton again. Thus a soliton can be thought of as a quantized, localized mode appearing in classical nonlinear systems. This makes the soliton a wonderful candidate for an information carrier. Indeed, it is this property of solitons that brought recent active development of the optical soliton communication system. However, there is a serious drawback in the use of optical solitons for small scale photonic devices. The pulse area is simply too big! For a fast operational speed and also for a large capacity, the soliton pulse width should be small, usually in the range of picoseconds, but that requires a large peak power of the soliton pulse in the optical fiber case. This would be fine with a macroscopic system such as the optical transmission system. But for small size integrated photonic devices, the peak power, or the pulse area, has to be significantly smaller. This requires in general a large nonlinear effect (large γ). A few materials can possess a large value of the third order nonlinear susceptibility, e.g. the chalcogenide glass possesses several hundred times larger χ_3 value than the usual fiber glass. But even with these large χ_3 values, since the area is inversely proportional to the square root of the nonlinear coefficient, one still needs a way to reduce the area to reach the desirable operating regime such as where e.g. the operating speed is 1 THz and each signal bit carries energy of 1 pico joule.

This is where the photonic crystal structure comes in. Nonlinearity can not only be enhanced by using new materials, but it can also be greatly enhanced through the artificial structure of materials. One such example is a microresonator structure which can confine light with a high quality factor. Such a resonator structure was shown to have an enhanced nonlinearity which is proportional to the finesse squared [2,3]. Confined light in photonic crystals which is trapped by defects can also experience enhanced nonlinearities. On the other hand, the localized mode which exists inside the bandgap can depend critically on the intensity of light when nonlinearity is present. In nonlinear photonic crystals having periodic modulation of the nonlinear refractive index [4], photonic band gap indeed becomes dynamically tunable. This raises the possibility of nonlinearity induced self trapped light and nonlinear localized modes in photonic crystals. Known examples of nonlinear localized modes in the forbidden gaps are gap solitons in one- [5] or two- [6] dimensional structures. It was shown that the total electromagnetic energy of a gap soliton depends on the dimension d of the system. For $d = 1$, there is no threshold for creating a soliton while for $d = 2$, the energy is some finite number throughout the gap [6]. In the case of a photonic crystal doped with resonant atoms, self induced transparency(SIT) type solitons can exist and these solitons can even be created at extremely low intensities (few photons) unlike the ordinary SIT solitons [7]. The underlying reason for this reduced intensity, or area, is that the Bragg reflector can enhance by multiple reflections the field coupling to the dopant atoms so as to make the pulse area effectively 2π of the SIT soliton. This shows that photonic crystal is a wonderful structure for enhancing nonlinearity thereby presenting a good potential for realistic applications. However, explicit nonlinear crystal and defect structures which can perform all optical operations are still largely unknown and they are important issues of a future research.

Now, we move to the second part of the question as to how we model such nonlinear localized modes. The modeling of gap solitons is based on the coupled mode theory where modes generated by the difference in periodic refractive indices are treated as independent variables which again are coupled through Bragg reflections. In order to illustrate the idea of a coupled mode theory, we consider for example the Maxwell's equation in the form of a scalar wave equation,

$$\nabla^2 E(\vec{r},t) - \frac{\epsilon(\vec{r})}{c^2}\frac{\partial^2 E(\vec{r},t)}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial^2 P_{NL}(\vec{r},t)}{\partial t^2} = 0, \quad (4)$$

where the nonlinear polarization $P_{NL} = \chi^{(3)}|E|^2E$. For the square lattice, we also assume the dielectric constant of the form,

$$\epsilon(\vec{r}) = \epsilon_0 + \Delta\epsilon[\cos(\vec{G}_1\cdot\vec{r}) + \cos(\vec{G}_2\cdot\vec{r})], \quad (5)$$

where $\vec{G}_1 = (2\pi/a_0)\hat{x}$ and $\vec{G}_2 = (2\pi/a_0)\hat{y}$. This model exhibits an indirect photonic band gap. By expanding the electric field amplitude about the band edges, adopting the slowly varying envelop approximation,

$$E(\vec{r},t) = (E_1(\vec{r},t)e^{i\pi\hat{x}\cdot\vec{r}/a} + E_2(\vec{r},t)e^{-i\pi\hat{x}\cdot\vec{r}/a})e^{-iwt} + c.c \quad (6)$$

and neglecting higher order harmonic generations, the Maxwell's equation simplifies to a set of coupled equations

$$\begin{aligned} i\frac{\partial E_1}{\partial t} + i\frac{\partial E_1}{\partial x} + \frac{\partial^2 E_1}{\partial y^2} + \delta E_1 + \beta E_2 + \frac{2}{3}\alpha[|E_1|^2 + 2|E_2|^2]E_1 = 0 \\ i\frac{\partial E_2}{\partial t} - i\frac{\partial E_2}{\partial x} + \frac{\partial^2 E_2}{\partial y^2} + \delta E_2 + \beta E_1 + \frac{2}{3}\alpha[|E_2|^2 + 2|E_1|^2]E_2 = 0, \end{aligned} \quad (7)$$

for certain constants δ, α, β [6]. This is a coupled NLS equation which is known to possess solitary wave solutions. This modeling however has a serious drawback since the contrast in dielectric constants of photonic crystals is usually big while the above model works only for the small constrast, i.e. the difference in refractive indices are relatively small. Also, the coupled model assumes that the added nonlinear materials also have a periodic structure which is not true with defects. Though the first example of a two-dimensional nonlinear photonic crystal has the refractive index constant but the second order nonlinear susceptibility is spatially periodic [8,9], defect modes, particularly associated with nonlinearity, hold a good promise for future applications. These symmetry breaking defects can not be modeled in terms of a conventional coupled mode theory.

An alternative to the coupled mode theory is the Green's function approach. The Green's function method is widely used in the linear case of photonic crystals [10] and in computing defect modes. An application of the Green's function method to the nonlinear photonic crystals was made by assuming that the electric field inside defects is constant and by considering fields only at defects locations [11]. This leads to an effective discrete nonlinear lattice equation where coupling constants are determined by the value of Green's function at each defect points. In order to understand the method, we consider the scalar wave equation for two dimensional photonic crystals,

$$\nabla^2 E(x,y,t) - \frac{1}{c^2}\partial_t^2[\epsilon(x,y)E] = 0. \quad (8)$$

By splitting the dielectric constant into the periodic linear part and the nonlinear part located at defects,

$$\epsilon(x,y) = \epsilon_p(x,y) + \epsilon_d(x,y), \quad (9)$$

we can rewrite the scalar wave equation in the form

$$\nabla^2 E(x,y,t) - \frac{1}{c^2}\partial_t^2[\epsilon_p(x,y)E] = \frac{1}{c^2}\partial_t^2[\epsilon_d(x,y)E]. \quad (10)$$

Regarding the r.h.s. of the above equation formally as the source term, we may transform the equation into an integral equation,

$$E(\vec{r},t) = \int G(\vec{r},\vec{r}',t,t')\frac{1}{c^2}\partial_{t'}^2[\epsilon_d(\vec{r}')E], \quad (11)$$

where the Green's function G satisfies

$$(\nabla^2 - \frac{1}{c^2}\epsilon_p(x,y)\partial_t^2)G(\vec{r},\vec{r}',t,t') = \delta(\vec{r}-\vec{r}')\delta(t-t'). \quad (12)$$

We require that only defect rods possess nonlinearity such that

$$\epsilon_d(\vec{r}) = [\epsilon_d^{(0)} + |E(\vec{r},t)|^2] \sum_m \theta(\vec{r} - \vec{r}_m), \qquad (13)$$

where θ is one at defect locations and zero otherwise. Assuming that the electric field inside a defect is constant and defining that $E_m(t) = E(\vec{r}_m, t)$, we obtain a coupled lattice model

$$E_m(t) = \sum_n \int dt' J_{mn}(t,t') \frac{1}{c^2} \partial_{t'}^2 [(\epsilon_d^{(0)} + |E_n(t')|^2 E_n(t')], \qquad (14)$$

where $J_{mn}(t,t') = G(\vec{r}_m, \vec{r}_n', t, t')$. This equation in the static limit and with an assumption of nearest neighbor interactions have been studied by McGurn who showed that the intrinsic localized modes, both even and odd type parity and kink type modes are possible [11]. Another static case analysis including long-range interaction has been also made [12]. Considering the case of a two dimensional photonic crystal with embedded nonlinear rods, these works demonstrated that the effective interaction in such a structure is nonlocal, so that the nonlinear effects can be described by a nonlinear lattice model that include the long range coupling and nonlocal nonlinearity. This approach, however, require a large number of defect modes to simulate the whole system via lattice models, and the time dynamics seems to possess only a restricted validity. In view of application of photonic crystals utilizing only a few point defects and defect lines, this approach obviously has some difficulties. Presumably, the best way to handle the problem is to solve Maxwell's equation numerically without making any assumption. This can be done e.g., by using the Finite Difference Time Domain (FDTD) method. However such a numerical work can be followed only after the guidance provided by simple effective models of nonlinear photonic crystals.

In conclusion, we may safely say that the research in nonlinear photonic crystals is still in a very early stage. As we have argued in this paper, presently known approaches reveal novel properties of nonlinear photonic crystals but at the same time, these approaches have very much restricted validity. In view of promising applications of photonic crystals, particularly as a nonlinearity-controlled active device, we believe that the study of nonlinear photonic crystals is an interesting, even aside from its important application, and still widely open area of research.

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