# Transition of Rivulet Flow from Linear to Droplet Stream

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**Key words**: *Rivulet, Necking, Retraction, Contact angle hysteresis*

## Abstract

When a liquid is supplied through a nozzle onto a relatively non-wetting inclined solid surface, a narrow rivulet forms. There exist several regimes of rivulet flow depending on various flow conditions. In this paper, the fundamental mechanism behind the transition of a linear rivulet to a droplet flow is investigated. The experiments show that the droplet flow emerges due to the necking of a liquid thread near the nozzle. Based on the observation, it is argued that when the retraction velocity of a liquid thread exceeds its axial velocity, the bifurcation of the liquid thread occurs, and this argument is experimentally verified.

## Nomenclature

$\mathrm{Bo}_T$ : tangential Bond number, $\rho g d^2 \sin\alpha/\sigma$
$C_A$ : contact angle hysteresis, $\cos\theta_R - \cos\theta_A$
$d$ : tube inner diameter [m]
$g$ : gravity [m/s$^2$]
$h$ : height [m]
$U$ : velocity [m/s]
$w$ : rivulet width [m]
We : Weber number, $\rho U_f^2/\sigma$

## Greek symbols

$\alpha$ : inclination angle [degree]
$\theta$ : contact angle [degree]
$\rho$ : density [kg/m$^3$]
$\sigma$ : surface tension [N/m]

## Subscripts

$A$ : advancing
$f$ : axial
$R$ : receding
$r$ : retraction

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## 1. Introduction

A narrow stream of liquid flowing down a solid surface, while surrounded by an atmospheric gas, is called a rivulet. Rivulets on an inclined plane are frequently observed in many industrial applications, such as ground-water permeation, oil recovery, coating process, evaporative heat exchangers, fill packings in cooling towers, absorption coolers, etc. When a liquid is supplied through a nozzle onto a highly hydrophilic solid surface, a film flow appears. On a relatively non-wetting surface, however, it is known that a narrow rivulet flow forms instead of film and Kern's(1,2) earlier experimental study classified the rivulet flow into four regimes. Schmuki & Laso(3) later conducted an experimental study to reproduce the four flow regimes and named them droplet flow (a series of droplets sliding down), linear rivulet, meanders and oscillating rivulet following the order they appear as the flow rate increases at a given inclination. It was also revealed that as the liquid viscosity increases, the flow rates causing the transition between each regime tend to increase, indicating that the flow instability is suppressed. The surface tension was shown to have a similar effect to that of the viscosity when its value exceeds a certain limit, but for a low surface tension, the linear rivulet dominates while the droplet and oscillating flows hardly appear.

Although the flow regimes of rivulet have been identified clearly, the studies to elucidate the mechanisms that induce transition between each regime have been relatively sparse to date. Davis and his colleagues(4-6) and Schiaffino & Sonin(7) performed analytical studies on the instability of a rivulet tending toward a droplet flow. Those studies basically assumed that the transition of a straight rivulet to the droplet flow occurs due to a mechanism closely related to the Rayleigh instability.(8) Schiaffino & Sonin(7) verified that their theoretical wavelengths agree with the experimental results for the beads with fixed contact lines. Schmuki & Laso(3) conducted the instability analysis for the transition of a straight rivulet to the pendulum rivulet based on the energy minimum principle. In spite of the aforementioned works, the complete understanding of the transition mechanisms still require further investigations and we address one of those issues in this work.

Thus a mechanism that governs the transition to the droplet flow is investigated in this work. In the present experiments where the liquid is supplied through a nozzle as will be described in detail below, we show that the retraction of a liquid thread on a solid surface plays a dominant role in forming the droplet flow rather than the Rayleigh instability of a liquid jet.

## 2. Experimental Set-up and Procedure

A schematic of the experimental set-up employed in this study is shown in Fig. 1. Liquid is supplied through a glass tube onto an inclined plane by the hydrostatic pressure in the tank. The inner diameter of the tubes is varied as 4, 5, and 6 mm and the inclination of the plane varies from 15° to 70° with respect to the horizontal plane. The images of the flow configuration are acquired by a CCD camera (Pulnix TM-200) equipped with a zoom lens (Moritex MV-Z07545), and they are recorded by an S-VHS VCR (Philips VR988). The in-

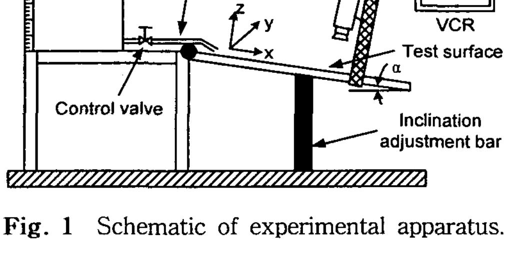

**Fig. 1** Schematic of experimental apparatus.

**Table 1** Contact angle hysteresis and equilibrium contact angles

| Surface | $C_A$, $\cos\theta_R - \cos\theta_A$ | Contact angle, $\theta$ |
|---|---|---|
| Bakelite | 0.136 | 59.7° |
| PP | 0.426 | 94.3° |
| PC | 0.387 | 87.4° |
| Acrylic | 0.184 | 80.8° |
| Teflon | 0.274 | 105.9° |
| PF | 0.073 | 95.9° |

stantaneous flow rate of the liquid is obtained by measuring the liquid level in the tank and using the calibration table. It should be noted that since the hydrostatic head in the supply tank decreases during the experiment, the flow rate decreases as well. Consequently, the flow starts as the oscillating rivulet and finally reaches the droplet flow. The cross sectional area of the tank is much larger than the tube area, thus the liquid level decreases in the tank exerts negligible disturbance on the flow at the tube outlet.

The liquid used in the present study is distilled water. Six different solids are used to evaluate the effects of the wetting on the rivulet dynamics, and these are acrylic, bakelite, polycarbonate (PC), para-film (PF: thermoplastic paraffin), polypropylene (PP) and teflon. The surfaces are cleaned prior to each experiment using distilled water and ethanol.

To obtain the fundamental data related to wetting, contact angles and contact angle hysteresis ( $C_A$ ) of the distilled water on each surface are measured. Methods of measuring contact angle and contact angle hysteresis are similar to those suggested by Schiaffino(9) and Dussan and Chow,(10) respectively. The results are shown in Table 1, where $\theta$, $\theta_R$ and $\theta_A$ are contact angle, receding and advancing contact angles, respectively. As the values suggest, most solid surfaces used in this work have a rather high contact angle, ensuring the partial wetting of the liquid.

## 3. Results and discussions

As discussed above, the analytical efforts to elucidate the transition mechanism of linear rivulet to droplet flow(3-6) have been largely focused on the break-up of a liquid thread associated with the Rayleigh instability.(8) Although the flow field is complicated due to the presence of moving contact lines, the fundamental feature of the Rayleigh instability is that when a liquid thread is subject to perturbations with sufficiently long wavelengths, the fluid is pushed out of the neck amplifying the disturbance to lead to the break-up of the

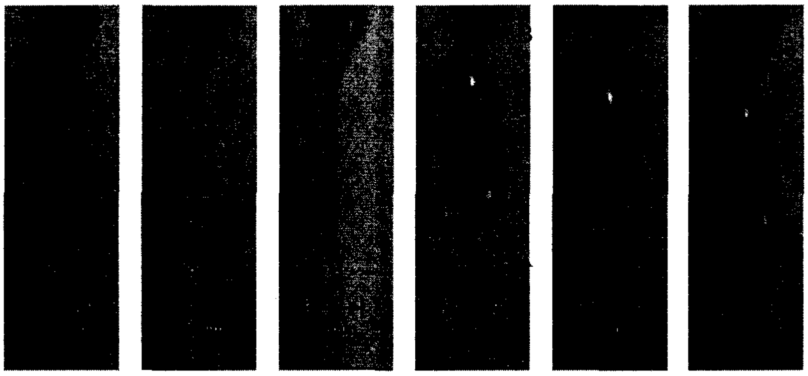
(a) 0 s (b) 0.1 s (c) 0.2 s (d) 1.9 s (e) 2.03 s (f) 2.13 s

**Fig. 2** Transition of linear rivulet to droplet flow on teflon surface. The tube inner diameter is 5 mm, the inclination 15°, and the mass flow rate 2 g/s.

thread. The relevant theories well describe the break-up of liquid jets in air(8) and the instability of a molten bead.(7) However, those theories do not provide on explanation on why the droplet flow emerges as the flow rate decreases in a situation where liquid is supplied onto the solid surface as in our experimental set-up.

The experimental results in this study suggest that the occurrence of the droplet flow be directly affected by the flow configuration at the tube exit rather than the break-up of a linear rivulet downstream. Figure 2 shows the images of the flow at the tube exit when the droplet flow first emerges. In a stable droplet flow regime, the process (d) through (f) shown in Fig. 2 repeats itself. That is, the liquid accumulates near the tube exit until the bulge becomes heavy enough to overcome the resistance at the frontal contact line and at the bridge on its upper side. The large drop thus formed is finally released to slide down the inclined surface by gravity. The present experiments suggest that once the linear rivulet is broken up as shown in Fig. 2 (a) through (c), the droplet flow always emerge without returning to the linear continuous rivulet. Therefore, it can be stated that the fundamental mechanism that induces transition of the linear rivulet to the droplet flow is associated with the bursting or necking of the liquid thread near the tube exit.

Based on the observation of Fig. 2, we argue that the bifurcation of a liquid thread arises when the retraction velocity at the thread's neck exceeds a low axial flow velocity. Taylor(12) showed that the rim of a thin liquid sheet having the thickness, $h$, retracts due to the surface tension at the velocity, $U_r$.

$$U_r = \left(\frac{2\sigma}{\rho h}\right)^{1/2} \tag{1}$$

where, $\sigma$ and $\rho$ denote the surface tension and the density of the liquid, respectively.

This formula neglects the effects of viscosity, which can be sizable when the film or thread becomes very thin and narrow. However, assuming that it provides an approximate scale of the retraction velocity of our rivulet, we suggest that the relative magnitude of the axial flow velocity, $U_f$, to the retraction velocity, $U_r$, should play a major role in determining the transition from linear rivulet to droplet flow. Then we find that the ratio of $U_f$ to $U_r$ can be simply scaled by the Weber number, $\rho U_f^2 d/\sigma$, provided that $h$ is scaled as the tube inner diameter, $d$.

$$\frac{U_f}{U_r} \sim \mathrm{We}^{1/2} \tag{2}$$

The experimental results verify this argument as shown in Fig. 3. $\mathrm{Bo}_T$ is the tangential Bond number defined as $\rho g d^2 \sin\alpha/\sigma$ and it corresponds to the ratio of the tangential gravitational force to the surface tension thus representing the effect of inclination. In Fig. 3, the symbol × denotes Kern's experiments,(1) and

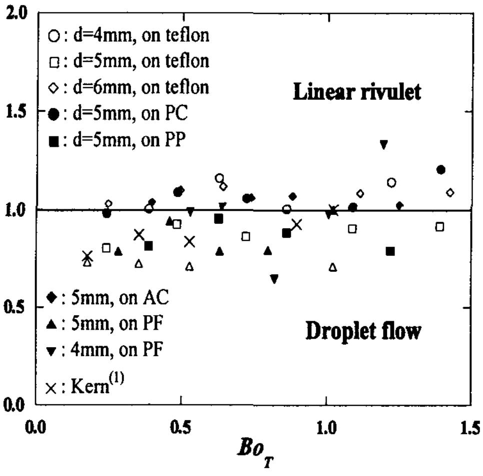

**Fig. 3** The critical Weber number versus the tangential Bond number.

it is shown that the current experimental results agree well with the previous study. Each point in the figure corresponds to the transition condition of linear rivulet to droplet flow in various tube sizes, solid surfaces, and inclination angles. Regardless of experimental conditions, all the points in Fig. 3 are scattered around $We^{1/2} \sim 1$, indicating that $U_f / U_r$ indeed determines the transition. Summarizing the foregoing results, the linear rivulet continues while the axial flow velocity exceeds the retraction velocity, but the droplet flow emerges as soon as the retraction becomes fast enough to cause the complete necking or bursting of the liquid thread.

To verify the use of the tube inner diameter instead of rivulet height in determining both Weber number and tangential Bond number, the non-dimensional widths versus contact angles are shown in Fig. 4. In this figure, $w$ is the width of rivulet. The non-dimensional width ranges between 0.5 and 1.3, thus we conclude that the rivulet width has the same scale as the tube inner diameter. Fig. 4 also shows that the non-dimensional width decreases as the contact angle increases as can be easily explained. However, it is noteworthy that for each contact angle, the variation of the non-dimensional width with the tube diameter is insignificant.

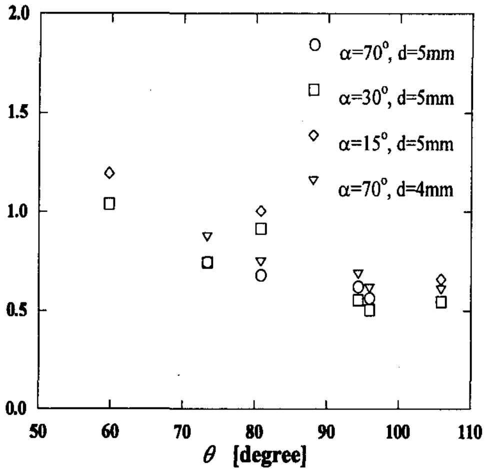

**Fig. 4** Non-dimensional width along contact angle on PF.

## 4. Conclusion

In the present study, it was investigated the fundamental mechanism of the transition of a linear rivulet to a droplet flow. It was experimentally shown that the droplet flow emerges due to the necking of a liquid thread near the nozzle. Based on the observations, it was argued that when the retraction velocity of a liquid thread exceeds its axial velocity, the necking occurs. The argument was verified by showing that the critical value of $We^{1/2}$ is nearly constant regardless of experimental conditions.

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