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CYCLOTOMIC UNITS AND DIVISIBILITY OF THE CLASS NUMBER OF FUNCTION FIELDS

JAEHYUN AHN AND HWANYUP JUNG

ABSTRACT. Let $k = \mathbb{F}_q(T)$ be a rational function field. Let ℓ be a prime number with $(\ell, q-1) = 1$. Let K/k be an elementary abelian ℓ-extension which is contained in some cyclotomic function field. In this paper, we study the ℓ-divisibility of ideal class number h_K of K by using cyclotomic units.

1. Introduction

Let K be a finite abelian number field. When K is a real extension, the class number h_K of K is difficult to calculate. If the conductor of K is divisible by many primes $q_1, \dots, q_s$, where each q_i is congruent to 1 modulo a prime ℓ, then h_K has a tendency to be divisible by a higher power of ℓ. There are several ways to show results of this type. Cornell and Rosen([3], [4]) showed this result using unramified ℓ-extensions and central extensions of K. There is another way to show similar results using the group of cyclotomic units and its index in the full unit group $\mathcal{O}_K^*$ of K. It is carried out by Kučera ([9]) for $p = 2$ and compositum K of quadratic fields and by Greither, Hachami and Kučera ([5]) for odd prime p. In function field case, Bae and Jung ([1]) obtained some results of ℓ-rank of ideal class groups of real cyclotomic function fields following Cornell and Rosen. In this paper, we show similar results following the idea in [5] and compare our results with those in ([1]). Now we state our result precisely.

Let $\mathbb{A} = \mathbb{F}_q[T]$ be the ring of polynomials over a finite field $\mathbb{F}_q$ with q elements and $k = \mathbb{F}_q(T)$. Let $q = p^f$ with $p = \mathrm{char}(k)$ and $f > 0$. For each $M \in \mathbb{A}$, one uses the Carlitz module to construct a field extension k_M, called the M-th cyclotomic function field and its maximal real subfield k_M^+. For a finite abelian extension F of k which is contained

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in some cyclotomic function field, we call M the conductor of F if k_M is the smallest cyclotomic function field which contains F. Also we call F *real* if F is contained in k_M^+ where M is the conductor of F.

Let ℓ be a prime rational number with $(\ell, q-1) = 1$. For $s \in \mathbb{Z}$, $s > 0$, we write $S = \{1, \dots, s\}$. Let $Q_1, \dots, Q_s$ be distinct monic irreducible polynomials in $\mathbb{A}$. In addition, if $\ell \neq p$, we require that each Q_i satisfies $q^{\deg(Q_i)} \equiv 1 \mod \ell$. We choose $e_1, \dots, e_s \in \mathbb{Z}$ so that $e_i = 1$ (resp. $e_i \geq 2$) for $\ell \neq p$ (resp. $\ell = p$) for each $i \in S$. Let K_i be any elementary abelian ℓ-extension of k in $k_{Q_i^{e_i}}$ for each $i \in S$ and δ_i be the ℓ-rank of $G_i = \mathrm{Gal}(K_i/k)$. Since $(\ell, q-1) = 1$, each K_i is real, i.e., $K_i \subseteq k^+_{Q_i^{e_i}}$. It is known ([1, Section 5]) that $\delta_i = 1$ if $\ell \neq p$ and $\delta_i \leq f \times \deg(Q_i) \times (e_i - 1 - [(e_i-1)/p])$ if $\ell = p$.

Let K be the compositum of all K_i, $i \in S$ and $G = \mathrm{Gal}(K/k)$. Since $G \simeq \prod_{i=1}^s G_i$, G is an elementary abelian ℓ-group of rank $\delta = \sum_{i=1}^s \delta_i$. Let $\mathcal{O}_K$ be the integral closure of $\mathbb{A}$ in K. Let h_K be the ideal class number of $\mathcal{O}_K$. Our main result is the following theorem.

MAIN THEOREM. *With notations as above, the ideal class number h_K is divisible by $\ell^{\prod_{i=1}^s(1+\delta_i) - s\,\delta - 1}$.*

Now we assume that K_i is the maximal elementary abelian ℓ-extension of k in $K_{Q_i^{e_i}}$. Then δ_i equals to $f \times \deg(Q_i) \times (e_i - 1 - [(e_i-1)/p])$ for $\ell = p$. In [1], Bae and Jung showed that h_K is divisible by $\ell^{s(s-3)/2}$ (resp. $\ell^{\sum_{i<j}\delta_i\delta_j - \sum_i \delta_i}$) for $l \neq p$ (resp. for $\ell = p$). For $\ell \neq p$, our MAIN THEOREM says that h_K is divisible by $\ell^{2^s - s^2 - 1}$. Note that $2^s - s^2 - 1 > s(s-3)/2$ for $s > 4$. For $\ell = p$, elementary calculations show that

$$\prod_{i=1}^s (1+\delta_i) \geq 1 + \sum_i \delta_i + \sum_{i<j} \delta_i\delta_j + (\sum_{i=3}^s \binom{s}{i}/s) \sum_i \delta_i,$$

and so $\prod_{i=1}^s(1+\delta_i) - s\delta - 1$ is greater than $\sum_{i<j}\delta_i\delta_j - \sum_i \delta_i$ for $s > 4$. Thus our result gives larger ℓ-factor of h_K than the result in [1] for $s > 4$. In [1], authors also give results for the case that ℓ divides $q-1$. These cases corresponds to the case $p = 2$ in the number field case. Thus it will be interesting to consider the function field analogue of Kučera's result ([9]).

2. Cyclotomic units

We keep all the notations of the preceding section. In this section, we give some basic facts of the cyclotomic function fields and cyclotomic units which are needed in the proof of MAIN THEOREM. Let k^{ac} be an algebraic closure of k. Then k^{ac} becomes an $\mathbb{A}$-module (called Carlitz module) under the following action: For $u \in k^{ac}$ and $M \in \mathbb{A}$, define

$$u^M = M(\varphi + \mu_T)(u),$$

where the map φ is defined as $\varphi(u) = u^q$ and μ_T is defined as $\mu_T(u) = Tu$. It is known that the set Λ_M of roots of $u^M = 0$ generates an abelian extension $k(\Lambda_M)$ of k, called the *M-th cyclotomic function field* and we denote it by k_M for simplicity. By a *primitive M-th torsion point*, we mean a generator of Λ_M as an $\mathbb{A}$-module. The following facts are basic in the cyclotomic function field theory.

PROPOSITION 2.1 ([8]). (i) $\mathrm{Gal}(k_M/k) \cong (\mathbb{A}/M)^*$.

(ii) *k_M^+ is the fixed field of $\mathbb{F}_q^*$ in k_M, and it is the maximal subfield of k_M, where $\infty = (1/T)$ splits completely.*

(iii) *If $(M, N) = 1$, then k_M and k_N are linearly disjoint over k.*

(iv) *If $M = P^n$, a power of monic irreducible P, then $N_{k_M/k}(\lambda_M) = P$. Here λ_M is a primitive M-torsion point.*

Recall that the group D of cyclotomic numbers of K is defined as the subgroup of K^* generated by $\mathbb{F}_q^*$ and all elements $N_{k_N/K_N}(\lambda_N^A)$ with $N, A \in \mathbb{A}$ where $K_N = k_N \cap K$. Let $C = D \cap \mathcal{O}_K^*$, called the group of cyclotomic units of K (cf. [7, Section 3]). For $\emptyset \neq I \subset S$, we set $M_I = \prod_{i \in I} Q_i^{e_i}$, $\delta_I = \sum_{i \in I} \delta_i$, K_I the compositum of $K_i, i \in I$ and $x_I = N_{k_{M_I}/K_I}(\lambda_I)$, where λ_I is a fixed primitive M_I-th torsion point. It is known that D is generated by $\mathbb{F}_q^* \cup \{x_I^\sigma : \sigma \in G, \emptyset \neq I \subset S\}$, and D (resp. C) has rank $\ell^\delta + s - 1$ (resp. $\ell^\delta - 1$). The index of C in the full unit group $\mathcal{O}_K^*$ is related to the ideal class number h_K.

PROPOSITION 2.2 ([2], Corollary 3.11).

$$[\mathcal{O}_K^* : C] = (q-1)^{\ell^\delta - 1} h_K.$$

As the classical case, we have the following.

LEMMA 2.3. *Let $N, P, Q \in \mathbb{A}$ with P monic irreducible, $N = PQ$. If $P \nmid Q$, we let $\sigma = (P, k_Q/k)^{-1} \in \mathrm{Gal}(k_Q/k)$. Then we have*

$$N_{k_N/k_Q}(\lambda_N) = \begin{cases} \lambda_N^P & \text{if } P \mid Q\,, \\ (\lambda_N^P)^{(1-\sigma)} & \text{if } P \nmid Q, Q \notin \mathbb{F}_q^*. \end{cases}$$

Note that λ_N^P is also primitive Q-torsion point and so $\lambda_N^P = \lambda_Q^\tau$ for some $\tau \in \mathrm{Gal}(k_Q/k)$.

Proof. First we note that

$$(k_N : k_Q) = \begin{cases} q^{\deg(P)} & \text{if } P|Q, \\ q^{\deg(P)} - 1 & \text{if } P \nmid Q. \end{cases}$$

Let $\mathcal{W} = \mathcal{W}(Q\mathbb{A}/N\mathbb{A})$ be a complete set of representatives of $Q\mathbb{A}/N\mathbb{A}$ consisting of monic polynomials. If $P|Q$, then

$$\mathrm{Gal}(k_N/k_Q) \simeq \{(1+X) + N\mathbb{A} \in (\mathbb{A}/N\mathbb{A})^* : X \in \mathcal{W}\}$$

and so we have

$$N_{k_N/k_Q}(\lambda_N) = \prod_{X \in \mathcal{W}} \lambda_N^{1+X} = \prod_{X \in \mathcal{W}} (\lambda_N + \lambda_N^X).$$

We claim that $\Lambda_P = \{\lambda_N^X : X \in \mathcal{W}\}$. Clearly $\lambda_N^X \neq \lambda_N^Y$ for any distinct $X, Y \in \mathcal{W}$ and $(\lambda_N^X)^P = 0$. Since $|\mathcal{W}| = |Q\mathbb{A}/N\mathbb{A}| = |\mathbb{A}/P\mathbb{A}| = |\Lambda_P|$, we get the claim. Since

$$u^P = \prod_{\lambda \in \Lambda_P} (u + \lambda) = \prod_{X \in \mathcal{W}} (u + \lambda_N^X),$$

we have

$$\lambda_N^P = \prod_{X \in \mathcal{W}} (\lambda_N + \lambda_N^X) = N_{k_N/k_Q}(\lambda_N).$$

Now, suppose that $P \nmid Q$. There exists $X_0 \in Q\mathbb{A}$ such that $X_0 \equiv -1 \mod P$. Without loss of generality, we may assume that $X_0 \in \mathcal{W}$. Then $P|(X_0+1)$; so $X_0 + 1$ is not prime to N. Hence we have

$$\mathrm{Gal}(k_N/k_Q) \simeq \{(1+X) + N\mathbb{A} \in (\mathbb{A}/N\mathbb{A})^* : X \in \mathcal{W}, X \neq X_0\}$$

and

$$N_{k_N/k_Q}(\lambda_N) = \frac{\prod_{X \in \mathcal{W}} (\lambda_N + \lambda_N^X)}{\lambda_N + \lambda_N^{X_0}}.$$

Let $1 + X_0 = PB$. Since $X_0 \in Q\mathbb{A}$, $(P, k_Q/k)^{-1} = (B, k_Q/k) = \sigma$ and so

$$\lambda_N + \lambda_N^{X_0} = \lambda_N^{PB} = (\lambda_N^P)^B = (\lambda_N^P)^{(B, k_Q/k)} = (\lambda_N^P)^\sigma.$$

Therefore, we get $N_{k_N/k_Q}(\lambda_N) = (\lambda_N^P)^{1-\sigma}$. □

Let $\sigma_{i1}, \ldots, \sigma_{i\delta_i}$ be the generators of $\mathrm{Gal}(K_S/K_{S-\{i\}})$. For $i \in S, 1 \leq j \leq \delta_i$, define $N_{i,j} = 1 + \sigma_{ij} + \cdots + \sigma_{ij}^{\ell-1}$, the norm corresponding to the group $\langle \sigma_{ij} \rangle$. For $\emptyset \neq I \subset S$, let $\widetilde{I} = \{(i,j) : i \in I, 1 \leq j \leq \delta_i\}$. For $J \subset \widetilde{I}$, we define

$$x_{I,J} = x_I^{\prod_{(i,j) \in J} N_{i,j}}.$$

For any subset J of $\widetilde{I}$, we say that J satisfies the condition $(*)$ for I if $|\{j : (i,j) \in J\}| < \delta_i$ for each $i \in I$. Note that, if $\ell \neq p$, then $J = \emptyset$ is the only one which satisfies $(*)$ and so $x_{I,J}$ is just x_I. For $\emptyset \neq I \subset S$, $J \subset \widetilde{I}$, define

$$S_{I,J} = \{ \prod_{i \in I, 1 \leq j \leq \delta_i, (i,j) \notin J} \sigma_{ij}^{a_{ij}} : 0 \leq a_{ij} \leq \ell - 2 \}.$$

Let $B = \{x_{\{i\}}^{\prod_{1 \leq j \leq \delta_i} \sigma_{ij}^{\ell-1}} : 1 \leq i \leq s\} \cup \{x_{I,J}^{\sigma} : \emptyset \neq I \subset S, J \subset \widetilde{I}, J \text{ satisfies}(*), \sigma \in S_{I,J}\}$.

PROPOSITION 2.4. *B is a $\mathbb{Z}$-basis of $D/\mathbb{F}_q^*$.*

Proof. First we calculate the cardinality of B. We have

$$|B| = \sum_{\emptyset \neq I \subset S} \sum_{J \subset \widetilde{I}, J \text{ satisfies } (*)} (\ell - 1)^{\delta_I - |J|} + s.$$

Then,

$$\begin{aligned} \sum_{\substack{J \subset \widetilde{I} \\ J \text{ satisfies } (*)}} (\ell-1)^{\delta_I - |J|} &= \sum_{J \subset \widetilde{I}} (\ell-1)^{\delta_I - |J|} - \sum_{i \in I} \sum_{J \subset \widetilde{I}, \widetilde{\{i\}} \subset J} (\ell-1)^{\delta_I - |J|} \\ &\quad + \sum_{\substack{i,j \in I \\ i<j}} \sum_{J \subset \widetilde{I}, \widetilde{\{i,j\}} \subset J} (\ell-1)^{\delta_I - |J|} - \cdots \\ &= \sum_{I' \subset I} \mu(I', I) g(I'), \end{aligned}$$

where $\mu(I', I) = (-1)^{|I| - |I'|}$, the Möbius function on the subsets of S and $g(I') = \sum_{J \subset \widetilde{I'}} (\ell - 1)^{\delta_{I'} - |J|} = \ell^{\delta_{I'}}$. So, by the Möbius inversion formula, we have

$$\ell^{\delta} = g(S) = \sum_{I \subset S} \sum_{\substack{J \subset \widetilde{I} \\ J \text{ satisfies } (*)}} (\ell - 1)^{\delta_I - |J|}.$$

Thus, $|B| = \ell^{\delta} + s - 1$. Therefore, the cardinality of B is equal to the $\mathbb{Z}$-rank of D. It remains to show that B generate D modulo $\mathbb{F}_q^*$. The remaining part of the proof is so directly analogous to Greither, Hachami and Kučera's proof in the number field case that the reader should refer to ([5, Proposition 1.2]). □

For $\ell \neq p$, since $J = \emptyset$ is the only subset of $\widetilde{I}$ which satisfies $(*)$, we have

$$B = \{x_I^{\sigma} : \emptyset \neq I \subset S, \sigma \in S_I\} \cup \{x_{\{i\}}^{\sigma_{i1}^{\ell-1}} : i = 1, \ldots, s\},$$

where $S_I = \{\prod_{i\in I} \sigma_{i1}^{a_i} : 0 \le a_i \le \ell - 2\}$.

3. Proof of the main theorem

For $\emptyset \ne I \subset S$ and $J \subset \widetilde{I}$, we define

$$y_{I,J} = x_{I,J}^{e_{I,J}} \text{ with } e_{I,J} = \prod_{i\in I} \prod_{1\le j\le \delta_i, (i,j)\notin J} (1-\sigma_{ij})^{\ell-2}.$$

Let L be the subgroup of k^* generated by $Q_1, \dots, Q_s$. For $J \subset \widetilde{I}$, we call J satisfy the condition $(**)$ for I if, for each $i \in I$, $|\{j \in \{1, \dots, \delta_i\} : (i,j) \in J\}| = \delta_i - 1$. Clearly, the condition $(**)$ is a sufficient condition for the condition $(*)$ and so, if $\ell \ne p$, only $J = \emptyset$ satisfies the condition $(**)$. We define the subgroup U of C generated by $\{y_{I,J}^{\sigma} : \emptyset \ne I \subset S, J \subset \widetilde{I}, J \text{ satisfies } (**), \sigma \in G\}$.

PROPOSITION 3.1. *For any $u \in U$ and $\sigma \in G$, there are $\psi_u(\sigma) \in L$ and $f_u(\sigma) \in D$ such that $u^{1-\sigma} = \psi_u(\sigma) f_u(\sigma)^{\ell}$ and $\psi_u(\sigma)$ is uniquely determined modulo L^{ℓ}.*

Proof. Since $(\ell, q-1) = 1$, $K^{\ell} \cap L = L^{\ell}$. From this, the uniqueness of $\psi_u(\sigma)$ modulo L^{ℓ} follows. As in [5], it suffices to prove the proposition for $u = y_{I,J}$ with $\emptyset \ne I \subset S$, $J \subset \widetilde{I}$, J satisfies $(**)$ and $\sigma = \sigma_{ij}$ with $i \in I$, $(i,j) \notin J$.

We use induction on $|I|$. Suppose that the proposition is true for all u generated by $y_{I',J'}$ with $I' \subsetneq I$ and $J' \subseteq \widetilde{I'}$ with J' satisfying $(**)$. Note that $(1-\sigma_{ij})^{\ell-1}(1-\sigma_{ij}) = (1-\sigma_{ij})^{\ell-1} = N_{i,j} + \ell\alpha$ for some $\alpha \in \mathbb{Z}[\sigma_{ij}]$. Thus

$$\begin{aligned} y_{I,J}^{1-\sigma_{ij}} &= x_{I,J}^{(N_{i,j}+\ell\alpha)\prod_{i'\in I-\{i\}}\prod_{1\le j'\le\delta_{i'},(i',j')\notin J}(1-\sigma_{i'j'})^{\ell-2}} \\ &= (x_{I,J}^{N_{i,j}})^{\prod_{i'}\prod_{j'}(1-\sigma_{i'j'})^{\ell-2}} w^{\ell} \end{aligned}$$

for some $w \in D$. Note that $x_{I,J}^{N_{i,j}} = x_{I,J-\widetilde{\{i\}}}^{\prod_{1\le j\le\delta_i} N_{i,j}}$ and that $\prod_{1\le j\le\delta_i} N_{i,j}$ is the norm corresponding to $\mathrm{Gal}(K_I/K_{I-\{i\}})$.

If $I - \{i\} \ne \emptyset$, by Lemma 2.3, we have $x_{I,J}^{N_{i,j}} = (x_{I-\{i\},J-\widetilde{\{i\}}}^{1-\tau})^{\tau'}$ where $\tau, \tau' \in G$ with $\tau = (Q_i, K_{I-\{i\}}/k)$. Note that $J - \widetilde{\{i\}} \subset \widetilde{I - \{i\}}$ and satisfy $(**)$ for $I - \{i\}$. From the induction hypothesis, we have that

$$y_{I,J}^{1-\sigma_{ij}} = (y_{I-\{i\},J-\widetilde{\{i\}}}^{1-\tau})^{\tau'} w^{\ell}$$

can be written in the desired form and so it remains to prove the case $I = \{i\}$. But when $I = \{i\}$, we have

$$x_{I,J}^{N_{i,j}} = x_I^{\prod_{1\le j\le \delta_i} N_{i,j}} = N_{k_{Q_i^{e_i}}/k}(\lambda_{Q_i^{e_i}}) = Q_i \in L.$$

It finishes the proof of the proposition. □

From the proposition 3.1, it is obvious that for each $u \in U$, $\psi_u : G \to L/L^\ell$ is a homomorphism. Now we define a map $\Psi : U \to \mathrm{Hom}(G, L/L^\ell)$ defined by $\Psi(u) = \psi_u$. Then Ψ is a homomorphism. So we consider $u \in \ker(\Psi)$. From proposition 3.1, we have $u^{1-\sigma} = f_u(\sigma)^\ell$ with $f_u(\sigma) \in D$ for all $\sigma \in G$. Since

$$u^{1-\sigma\tau} = u^{1-\sigma}u^{\sigma(1-\tau)} = f_u(\sigma)^\ell f_u(\tau)^{\ell\sigma} = (f_u(\sigma)f_u(\tau)^\sigma)^\ell,$$

we see that $f_u : G \to D$ is a 1-cocycle for $u \in \ker(\Psi)$. Therefore, we have the following proposition as the classical case ([5, Proposition 2.5]).

PROPOSITION 3.2. *For any $u \in \ker(\Psi)$, there exists $\alpha(u) \in K^*$ and $\varphi(u) \in L$ such that*

$$u = \varphi(u)\alpha(u)^\ell.$$

Note that $\varphi(u)$ is unique modulo L^ℓ, and so $\varphi : \ker(\Psi) \to L/L^\ell$ is a homomorphism. We denote $\ker(\varphi)$ by N. From Proposition 3.2, any element of N is the ℓ-th power in K. Since $N \subset U \subset \mathcal{O}_K^*$, it becomes the ℓ-th power in $\mathcal{O}_K^*$ and so $N \subset C \cap (O_K^*)^\ell$.

As Greither, Hachami and Kučera ([5, Lemma 2.7-2.8]), we have

LEMMA 3.3. (i) *$[U : N]$ divides $\ell^s|\mathrm{Im}(\Psi)|$ and $|\mathrm{Im}(\Psi)|$ divides $\ell^{\delta s}$.*
(ii) *$\{y_{I,J}C^\ell : \emptyset \neq I \subset S$ and $J \subset \widetilde{I}, J$ satisfies $(**)\} \cup \{y_{\{i\},J}^{\sigma_{i,1}} : i = 1,\ldots,s$ and $J = \widetilde{\{i\}}\backslash\{(i,1)\}\}$ is free over $\mathbb{Z}/\ell\mathbb{Z}$ in the vector space C/C^ℓ.*

Now we are ready to prove MAIN THEOREM.

Proof of the main theorem. By Proposition 2.2 and Lemma 3.3 (i), it suffices to prove that $\ell^{\prod_{i=1}^s(1+\delta_i)-t-1}$ divides $[\mathcal{O}_K^* : C]$, where $|\mathrm{Im}(\Psi)| = \ell^t, t \geq 0$. We denote $\mathcal{O}_K^*$ by E for simplicity. We consider the short exact sequence

$$0 \to C/C\cap E^\ell \simeq E^\ell C/E^\ell \to E/E^\ell \to E/E^\ell C \to 0.$$

Since E/E^ℓ is $(\mathbb{Z}/\ell\mathbb{Z})$-vector space of dimension $[K : k] - 1 = \ell^\delta - 1$, we have

$$\mathrm{rk}_\ell(E/C) = \dim(E/E^\ell C) = \ell^\delta - 1 - \dim(C/C\cap E^\ell), \tag{3.1}$$

where $\mathrm{rk}_\ell(E/C)$ denotes ℓ-rank of E/C and dim denotes $\dim_{\mathbb{Z}/\ell\mathbb{Z}}$ for simplicity. We denote the image of N and U in C/C^ℓ by $\bar{N}$ and $\bar{U}$, respectively. Since $N \subset C \cap E^\ell$,

$$\dim(C/C \cap E^\ell) \leq \dim((C/C^\ell)/\bar{N}) = \ell^\delta - 1 - \dim(\bar{N}). \tag{3.2}$$

From (3.1) and (3.2), we get

$$\mathrm{rk}_\ell(E/C) \geq \dim \bar{N}.$$

But from Lemma 3.3 (i), (ii), we have

$$\dim \bar{U} \geq \prod_{i=1}^{s}(1+\delta_i) - 1 + s \quad \text{and} \quad \dim \bar{U}/N \leq s + t.$$

Therefore,

$$\mathrm{rk}_\ell(E/C) \geq \dim \bar{N} \geq \prod_{i=1}^{s}(1+\delta_i) - t - 1,$$

which completes the proof of the theorem. □

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Department of Mathematics
KAIST
Daejon 305-701, Korea
E-mail: jaehyun@mathx.kaist.ac.kr
hyjung@mathx.kaist.ac.kr