Multi-Valued Decision Making for Transitional Stochastic Event: Determination of Sleep Stages Through EEG Record

Masatoshi Nakamura and Takenao Sugi

Abstract: Multi-valued decision making for transitional stochastic events was newly derived based on conditional probability of knowledge database which included experts' knowledge and experience. The proposed multi-valued decision making was successfully adopted to the determination of the five levels of the vigilance of a subject during the EEG (electroencephalogram) recording; awake stage (stage W), and sleep stages (stage REM (rapid eye movement), stage 1, stage 2, stage 3/4). Innovative feature of the proposed method is that the algorithm of decision making can be constructed only by use of the knowledge database, inspected by experts. The proposed multi-valued decision making with a mathematical background of the probability can also be applicable widely, in industries and in other medical fields for purposes of the multi-valued decision making.

Keywords: sleep stages, multi-valued decision making, transitional stochastic event

I. Introduction

Automatic realization of human decision making is required in many fields for a purpose of reducing and assisting human mental works. On-off decision making based on conditional probability of knowledge database was proposed by some of the authors [1] and was successfully applied to the problems of insulator washing timing in subpower stations and an another problem of spike detection on EEG records. The method, however, was restricted to the on-off (two valued) decision making for static events. A restriction of the number of values is required to be released for wide application of the method. This paper proposed a method of multi-valued decision making for transitional stochastic event based on conditional probability of knowledge database. The current method is an extension of the previous on-off decision making. Automatic determination of sleep stages of a subject through EEG record was successfully implemented by the use of the proposed method. Concerning the sleep stages of EEG, sleep spindle, K-complex, vertex sharp wave were the specific waves which characterized the sleep stages [2]. The automatic sleep stage determination methods of other researchers [3], [4] usually used those specific parameters. On the contrary, the proposed sleep stage determination followed the different approach. Innovative feature of the method is that the algorithm is developed only based the knowledge database which was inspected by a skillful medical doctor. The proposed multi-valued decision making with a mathematical background of the probability can also be applicable widely, in industries and in other medical fields for purposes of the multi-valued decision making.

Manuscript received: Jan. 1, 2001., Accepted: Feb. 23, 2001.

Masatoshi Nakamura: Department of Advanced Systems Control Engineering, Graduate School of Science and Engineering, Saga University, Honjohmachi, Saga 840-8502, Japan.(nakamura@cntl.ee.saga-u.ac.jp)

Takenao Sugi: Department of Electrical and Electronic Engineering, Faculty of Science and Engineering, Saga University, Honjohmachi, Saga 840-8502, Japan.(sugi@cntl.ee.saga-u.ac.jp)

II. Multi-valued decision making

1. Problem statement of decision making

The n-valued scalar variable at the step (time, k) is denoted by $x_k = \{\xi^1, \xi^2, \cdots, \xi^n\}$. The purpose of the multi-valued decision making for the transitional stochastic event x_k is to estimate the current stage ξ^* out of n stages ξ^i based on a conditional probability of the current measurement $\boldsymbol{y}_k$ (m-dimensional vector) and the knowledge database of the past measurements Y_{k-1}. The method is derived by using the Baysian rule and the transitional probability in the following sections.

2. Automatic decision making

2.1 Conditional probability

The probability density function (pdf) of x_k based on $Y_k = \{Y_0, \boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_k\} = \{Y_{k-1}, \boldsymbol{y}_k\}$ is derived by the use of Baysian rule as

$$\begin{aligned} f(x_k|Y_k) &= \frac{f(x_k, Y_k)}{f(Y_k)} \\ &= \frac{f(x_k, \boldsymbol{y}_k, Y_{k-1})}{f(\boldsymbol{y}_k, Y_{k-1})} \\ &= \frac{f(\boldsymbol{y}_k, x_k, Y_{k-1})}{f(x_k, Y_{k-1})} \frac{f(x_k, Y_{k-1})}{f(Y_{k-1})} \frac{f(Y_{k-1})}{f(\boldsymbol{y}_k, Y_{k-1})} \\ &= \frac{f(\boldsymbol{y}_k|x_k) f(x_k|Y_{k-1})}{f(\boldsymbol{y}_k|Y_{k-1})} \\ &= \frac{f(\boldsymbol{y}_k|x_k) f(x_k|Y_{k-1})}{f(\boldsymbol{y}_k|x_k) f(x_k|Y_{k-1}) dx_k} \end{aligned} \tag{1}$$

where the equality of the pdf $f(\boldsymbol{y}_k|x_k) = f(\boldsymbol{y}_k|x_k, Y_{k-1})$ is used, because x_k includes all information of Y_{k-1} for $\boldsymbol{y}_k$. The discrete pdf $f(x_k|Y_{k-1})$ in the right hand side of equation (1), as seen in Fig.1-1, is described by the summation of the n terms with Derac delta functions $\delta(\xi^j)$ as

$$f(x_k|Y_{k-1}) = \sum_{j=1}^{n} P_{k|k-1}(\xi^j)\delta(\xi^j) \tag{2}$$

where $P_{k|k-1}(\xi^j)$ is a predicted probability ξ^j at the k-th step based on the database Y_{k-1}, and summation of those probability $\sum_{j=1}^{n} P_{k|k-1}(\xi^j)$ being 1. Another pdf $f(\boldsymbol{y}_k|x_k)$ of a continuous function in the right hand side of equation (1),

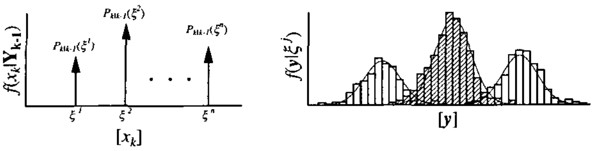


Fig. 1. Probability density functions : (1) discrete pdf $f(x_k|Y_{k-1})$ of the event, (2) continuos pdf $f(y|\xi^j)$ of the database.

as seen in Fig.1-2, is derived from the normalized histogram $h(\boldsymbol{y}|\xi^j)$ of the database for each stage ξ^j. By use of the Derac delta function, the probabilities for discrete random variable x_k (events), and continuous random variable $\boldsymbol{y}_k$ (events) were expressed in a uniform style. Usage of the Derac delta function is the crucial point of the following derivation. The parameters which characterize the pdf $f(\boldsymbol{y}_k|\xi^j)(= f(\boldsymbol{y}|\xi^j))$ is obtained by approximating the histogram $h(\boldsymbol{y}|\xi^j)$ by use of the least squares method which minimized the criteria $J = \min\{(\int f(\boldsymbol{y}|\xi^j) - h(\boldsymbol{y}|\xi^j))^2 dy\}$.

The conditional probability of ξ^i at the k-th step based on Y_k is obtained by integrating the impulsive pdf $f(x_k|Y_k)$ of equation (1) around the neighborhood of ξ^j as follows:

$$\begin{aligned} P_{k|k}(\xi^i) &= \int_{\xi^{i-\varepsilon}}^{\xi^{i+\varepsilon}} f(x_k|Y_k)dx_k \\ &= \frac{f(\boldsymbol{y}_k|\xi^i)P_{k|k-1}(\xi^i)}{\sum_{j=1}^{n} f(\boldsymbol{y}_k|\xi^j)P_{k|k-1}(\xi^j)} \end{aligned} \tag{3}$$

where the equality $f(\boldsymbol{y}_k|\xi^j) = \int f(\boldsymbol{y}_k|x_k)\delta(\xi^j)dx_k$ is used and ε is a small positive constant. The probability $P_{k|k}(\xi^i)$ is the key term for determining the current stage ξ^*. In the next subsection, the predicted probability $P_{k+1|k}(\xi^i)$ is derived to obtain the conditional probability, iteratively.

2.2 Transitional probability

As seen in Fig.2, the predicted probability of ξ^i at the $k+1$-th step based on the current information is determined by the following equation as

$$P_{k+1|k}(\xi^i) = \sum_{j=1}^{n} P_{k|k}(\xi^j)t^{ji} \tag{4}$$

where t^{ji} is a given transitional probability from j to i. Transitional probability t^{ji} is determined by the rate of the transition frequency from j to i in the knowledge database which are inspected by the expert (the qualified medical doctor) as

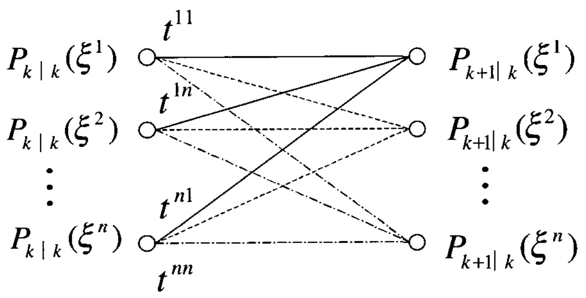


Fig. 2. Predicted probability $P_{k+1|k}(\xi^i)$.

$t^{ji} = N^{ji}/(N^{j1} + N^{j2} + \cdots + N^{jn})$ where the N^{ji} is the numbers (frequency) of the transition from j to i in the knowledge database. The summation of the transitional probabilities is restricted to

$$\sum_{i=1}^{n} t^{ji} = 1 \quad (j = 1, 2, \cdots, n) \tag{5}$$

The decision making algorithm is derived by combining the two probabilities in equations (3) and (4), iteratively.

2.3 Algorithm of the decision making

The decision making is executed by selecting the stage ξ^* at the k-th step such that the maximum value of the conditional probability $P_{k|k}(\xi^j)$ is attained as

$$\xi^* : \max_j \{P_{k|k}(\xi^j)\} \tag{6}$$

The iterative algorithm for the decision making, illustrated in Fig.3, is summarized in the following five procedures. Before starting the algorithm, pdf $f(\boldsymbol{y}|\xi^i)$ of the knowledge database, the transitional probabilities t^{ij} in equation (4), the *a priori* (predicted) probability $P_{1|0}(\xi^i) = 1/n$ should be given.

The algorithm starts for $k = 1$, by taking the following procedures as

1) Measurement of $\boldsymbol{y}_k$ 2) Calculation of the conditional probabilities $P_{k|k}(\xi^i)$ by equation (3) 3) Decision making ξ^* by equation (6) 4) Calculation of the predicted probabilities $P_{k+1|k}(\xi^i)$ by equation (4) 5) Go back to 1) by replacing $k+1$ by k.

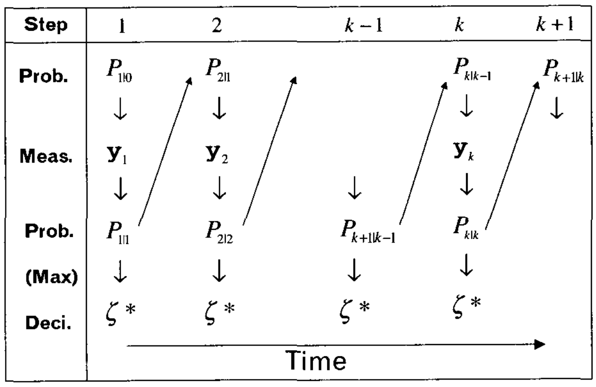


Fig. 3. Procedures of the decision making.

III. Determination of sleep stages

1. Sleep EEG and sleep stages

1.1 Acquisition of sleep EEG data

EEGs of three subjects were recorded at Kyoto University Hospital and Saga University. All EEGs, electromyogram (EMG) and electrooculogram (EOG) were recorded in a quiet room where the subject was placed in bed during a whole night. Exploring cup electrodes were fixed to the scalp at points of the international 10-20 system. The EEG were recorded by a digital electroencephalograph with a time constant of 0.3 s and a high frequency cut-off at 60 Hz (-3dB) with a sampling interval of 5 ms. The total lengths of the EEG for one subject, adopted in this analysis, were five hours long, which were segmented into 3647 segments (5 sec for one segment).

1.2 Sleep stages

Detection of sleep stages through sleep EEGs is an important task for inspecting neurophysiological and/or chicastic diseases of the subjects. The criteria for the sleep stages of EEG were summarized in Table 1 which was extracted from a well-known Rechtschaffen and Kales rule [2]. The sleep stages are mainly classified into REM (rapid eye movement) and non REM stages. The non REM is further classified into 1 to 4 stages. Each stages are characterized by characteristics of the EEG time series : dominant rhythm, fast wave, slow wave, sleep spindle, K-complex, eye movement and electromyogram (EMG) [5]. The automatic sleep stage determination methods of other researchers [3], [4] usually used those specific parameters. On the contrary, the proposed method of the sleep stage determination followed the different approach. The automatic determination of the sleep stages through EEG record was implemented by use of the method of multi-valued decision making explained in the previous section. Innovative feature of the method is that the algorithm is developed only based the knowledge database which was inspected by a skillful medical doctor.

Table 1. Criteria of the sleep stage determination.

Stage	Characteristics
W	Dominant rhythm, low voltage fast wave
1	Low voltage slow wave (2-7 Hz), Vertex sharp transients
2	Slow wave (less than 20 %), sleep spindle, K-complex
3	Slow wave (20-50 %), sleep spindle
4	Slow wave (more than 50 %), sleep spindle
REM	Low voltage slow wave (same as stage 1), rapid eye movement, low chin EMG

2. Automatic determination of sleep stages

2.1 Parameters for sleep EEG

Appropriate selection of the measurement variables y_k is crucial for the multi-valued decision making. A medical doctor (N. T) inspected the sleep EEG visually and made the sleep stage determination for the whole data. The knowledge database which was inspected by the medical doctor, was used to make the probability density functions for the parameters. The measurement variables y_k which characterized the sleep stages of subjects were selected as : duration and amplitude of delta wave (D_δ, A_δ), those of theta wave (D_θ, A_θ), those of alpha wave (D_α, A_α), total amount of EOG (electrooculogram, S_{EOG}), ratio of phase reversal of EOG ($R_{EOG} = S_{EOG}(left{+}right)/S_{EOG}(left{-}right)$) and total amount of chin-EMG (electromyogram, S_{EMG}). Those parameters were calculated from the periodogram for each 5 sec EEG time series.

The pdf $f(y|\xi^i)$ for each parameters were approximated by the nine dimensional normal distribution for the respective histogram of $h(y|\xi^i)$. Fig.4 shows the marginal pdf $f(y|\xi^i)$ for each parameter. The sleep stages were automatically determined by use of the algorithm of five procedures described in section III.1.2.

2.2 Sleep stages determination

The number of stages ξ^i for the sleep stage determination was five : ξ^1 waking stage W, ξ^2 stage REM, ξ^3 non REM stage 1, ξ^4 stage 2 and ξ^5 stage 3 or 4. The transitional probabilities were given as

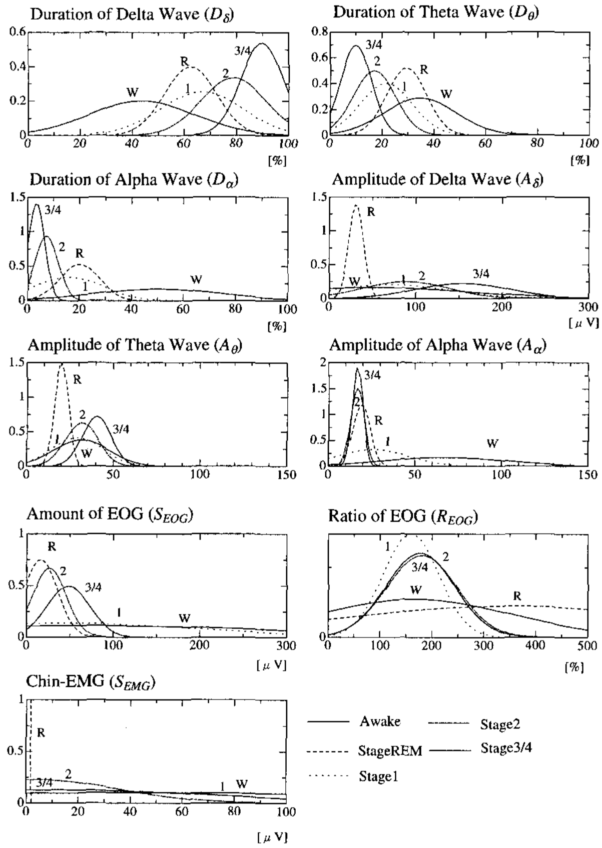


Fig. 4. The marginal pdf ($f(y|\xi^i)$) which characterized the sleep stages of the subject : duration and amplitude of delta wave (D_δ, A_δ), those of theta wave (D_θ, A_θ), those of alpha wave (D_α, A_α), total amount of EOG (electrooculogram S_{EOG}), ratio of phase reversal of EOG ($R_{EOG} = S_{EOG}(left + right)/S_{EOG}(left - right)$) and total amount of chin-EMG (electromyogram, S_{EMG}). Vertical axis shows the value each marginal pdf.

$$\{t^{ij}\} = \begin{bmatrix} 0.950 & 0.000 & 0.050 & 0.000 & 0.000 \\ 0.002 & 0.983 & 0.000 & 0.015 & 0.000 \\ 0.047 & 0.000 & 0.743 & 0.206 & 0.004 \\ 0.000 & 0.005 & 0.013 & 0.827 & 0.155 \\ 0.000 & 0.000 & 0.016 & 0.365 & 0.619 \end{bmatrix} \tag{7}$$

The value of each element t^{ij} was determined *a priori* based on the statistics of transitional frequency of the sleep stage for adjacent 5 sec EEG segments inspected by the qualified medical doctor.

The automatic determination of EEG sleep stages were executed by use of the procedure 1)-5) with equations (3), (4) and (6).

IV. Results and discussion

1. Results

Fig.5 shows a part of the automatic determination of sleep stages and corresponding visual inspection done by the qualified medical doctor (N.T) for a subject (female, 23 years old).

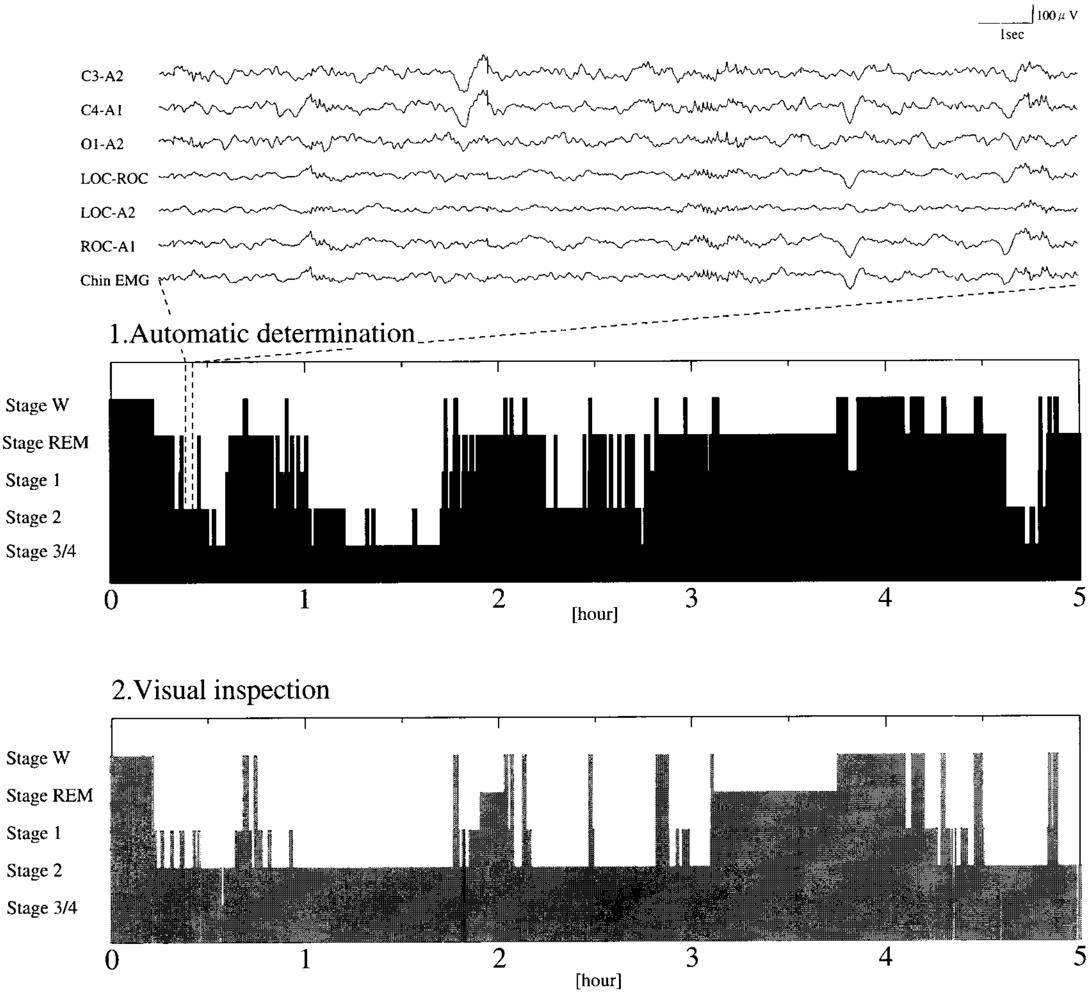


Fig. 5. Result of the sleep stage determination for a subject (female, 23 years old). The electrode C3-A2, C4-A1, O1-A2 are for electroencephalogram, LOC-ROC, LOC-A2 for electrooculogram and Chin EMG for electromyogram. The automatic determination (1) was executed by use of the procedure of the proposed method, and the visual inspection (2) was done by a qualified medical doctor (N. T) for each 5 sec record.

A slit of ten seconds record of the EEGs (C3-A2, C4-A1, O1-A2) and electrooculogram (LOC-ROC, LOC-A2, ROC-A1) and electromyogram (Chin EMG) out of 5 hours record is shown at the top of the figure. Results of the automatic determination and those of the visual inspection were compared. The percentage of the coincidence was over 70 %. Table 2 shows the statistics of the sleep stage determination for the subject. The results of automatic determination generally coincided with those done by the medical doctor in high accuracy except for the segments included in the stage 2 inspected by the medical doctor (the second row from the bottom in Table 2).

2. Discussion

This paper showed a possibility of applications of the multi-valued decision making to the determination of the automatic sleep stage. The accuracy of the automatic sleep stage determination can be improved if the additional conditions of the Rechtschaffen and Kales rule [2] are introduced in more exactly to the current proposed multi-valued decision making. In this study, artifacts in the electroencephalogram (EEG), such as electromyogram (EMG), electrooculogram (EOG) and other noises, defected slightly an accuracy of the automatic determination of the sleep stages of electroencephalogram (EEG). In the final goal of the automatic determination of the sleep stages, algorithms of the automatic elimination and/or exclusion of those artifacts should be included [6], and then the accuracy of the determination will be increased in more. The sleep stage determination by the multi-valued decision making can be improved and be powerful assistant tool for medical doctors clinically, because the method is not only based on the strict mathematical background but also based on the expert's knowledge and experience in the medical field.

The proposed multi-valued decision making with a mathe-

Table 2. Evaluation of the automatic determination of sleep stage.

		Automatic					Total
		W	REM	1	2	3/4	
E	W	367	16	94	14	5	496
E	REM	4	518	0	7	0	529
G	1	26	82	52	56	17	233
e	2	5	379	28	773	530	1715
r	3/4	1	17	8	108	540	674

matical background of the probability can also be applicable widely, in industries and in other medical fields.

References

[1] M. Nakamura, S. Goto and T. Sugi, "Artificial realization of human on-off decision making based on conditional probability of a database", *Journal of Artificial Life and Robotics*, vol. 4, no. 2, pp. 89-95, 2000.

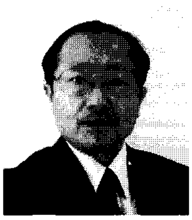
[2] A. Rechtschaffen and A. Kales, *A Manual of Standardized Terminology, Techniques and Scoring System for Sleep Stages of Human Subjects, California, Brain Information Service* 1968.

[3] J. R. Smith, I. Karakan, and M. Yang, "Automated analysis of the human sleep EEG", *Waking and Sleeping*, vol. 2, pp. 75-82, 1978.

[4] J. R. Smith and J. C. Principe, "SAMICOS - a sleep analyzing microcomputer system", *IEEE Trans. Biomed. Eng.*, vol. 33, pp. 935-941, 1988.

[5] G. Deuschl and A Eisen, *Recommendations for the Practice of Clinical Neurophysiology: Guidelines of the International Federation of Clinical Neurophysiology, Elsevier* 1999.

[6] M. Nakamura, T. Sugi, A. Ikeda, R. Kakigi and H. Shibasaki, "Clinical application of automatic integrative interpretation of awake background EEG : Quantitative interpretation, report making, and detection of artifacts and reduced vigilance level", *Electroenceph. clin. Neurophysiol.*, vol. 98, pp. 103-112, 1996.



Masatoshi Nakamura

He was born in Fukuoka, Japan in 1943. He received the B.S., M.S., and Ph.D. degrees in Electrical Engineering from Kyushu University, Fukuoka, Japan, in 1967, 1969, and 1974, respectively. From 1973 to 1974, he was a Research Associate with Kyushu University. Since 1974, he has been with the Faculty of Science and Engineering, Saga University, where he is currently a Professor of the Graduate School, Advanced Systems Control Engineering. His research interest include systems control theory and its applications, especially biomedical engineering, robotics, power system control, and thermal flow-control. Dr. Nakamura is a Fellow of the Society of Instrument and Control Engineers of Japan, and a member of the Institute of Control, Automation and Systems Engineers, the IEEE Control Systems Society, the Institute of Electrical Engineers of Japan, The Robotics Society of Japan, and the Institute of Systems, Control and Information of Japan.

Takenao Sugi

He was born in Fukuoka, Japan, in 1968. He received B.S., M.S. and Ph.D degrees, all in Electrical Engineering, from Saga University, Saga, Japan, in 1992, 1994 and 1999 respectively. Since 1995, he has been with the Faculty of Science and Engineering, Saga University, where he is currently a Lecturer of the Department of Electrical Engineering. His research interests include control theory and its applications to actual system, especially biomedical systems.