Robust H_∞ Control of Uncertain Descriptor Systems With Time-Varying Delays

Jong Hae Kim

Abstract: This paper is concerned with H_∞ controller design methods for descriptor systems with and without time-varying delays in state and control input. The sufficient condition for the existence of an H_∞ controller and the controller design method are presented by linear matrix inequality (LMI), singular value decomposition, Schur complements, and changes of variables. Since the obtained sufficient condition can be changed to an LMI form by proper manipulations, all solutions including controller gain can be obtained at the same time. Moreover, it is shown that robust H_∞ controller design problem for parameter uncertain descriptor systems with time-varying delays in state and control input can be solvable using the proposed method.

Keywords: descriptor systems, H_∞ control, time delays, LMI

I. Introduction

H_∞ control has been one of the most important notions in the field of automatic control theory. Although H_∞ control theory has been perfectly developed over the last decade, most of works have been developed based on state space equations. State space models are very useful, but the state variables thus introduced do not provide a physical meaning. Hence, the descriptor form is a natural representation of linear dynamical systems, and makes it possible to analyze a larger class of systems than state space equations do[1,2], because state space equations cannot represent algebraic restrictions between state variables and some physical phenomena, like impulse and hysterisis which are important in circuit theory, cannot be treated properly. The special characteristics for descriptor systems have drawn considerable attention due to extensive applications of descriptor systems in large scale systems, singular perturbation theory, and in particular, constraint mechanical systems. Many essential notions and results in control theory based on the state space form have been generalized for the descriptor form[3-11].

Recently, the descriptor H_∞ control problem has been considered by many researchers. Especially, Masubuchi *et al.*[1] considered the H_∞ control problem for descriptor systems that possibly have impulsive modes and/or jw axis zeros in order to eliminate the assumptions. Rehm and Allgower [7] treated the H_∞ control problem of high index or even non-regular linear descriptor systems with norm-bounded uncertainties in the system matrices. Also, Takaba *et al.*[9] treated robust H_2 performance of uncertain descriptor systems. However, there are no papers considering robust H_∞ controller design methods for descriptor systems with time-varying delays by LMI technique. Although some papers treated H_∞ control problem of singular systems with no time delay, it was not easy to calculate the solutions because of non-convexity of sufficient conditions.

Since the stability analysis and control of dynamic systems with time delay are problems of recurring interest as time delay often are the causes for instability and poor performance of control systems, the study of time delay systems has received considerable attention over the past years[12,13, and references therein]. This fact motivates the author to develop robust H_∞ control of descriptor stems with and without time-varying delays in state and control input by strict LMI sufficient condition.

The objective of this paper is to present not only the LMI sufficient condition for the existence of H_∞ controller but also the H_∞ controller design algorithm for delayed descriptor systems using singular value decomposition, changes of variables, and LMI approach. Therefore, all solutions can be obtained at the same time, because the presented sufficient condition is an LMI form regarding all variables. Moreover, it is shown that robust H_∞ controller design problem for norm-bounded parameter uncertain descriptor systems with time-varying delays in state and control input can be solvable using the proposed method. Finally, a numerical example is given to check the validity of the proposed method. The following notations will be used in this paper. $(\cdot)^T$, $(\cdot)^{-1}$, $deg(\cdot)$, $det(\cdot)$, and $rank(\cdot)$ denote the transpose, inverse, degree, determinant, and rank of a matrix. An identity matrix with proper dimensions is denoted as I. I_r, $x_r(t)$, and $\mathbf{R}^r$ denote an identity matrix with $r \times r$ dimension, $r \times 1$ dimensional vector, and $r \times 1$ dimensional real vector, respectively. $*$ represents the elements below the main diagonal of a symmetric matrix. In the following, we summarize some definitions and useful properties for the system $E\dot{x}(t) = Ax(t)$. If $det(sE - A)$ is not identically zero, a pencil $sE - A$ (or a pair (E, A)) is regular. The property of regularity guarantees the existence and uniqueness of solution for any specified initial condition. The singular system has no impulsive mode (or impulse free) if and only if $rank(E) = deg\ det(sE - A)$. The condition of impulse free ensures that singular system has no infinite poles.

II. Main results

Let us a linear time invariant descriptor system with time-varying delays in state and control input

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_d x(t - d_1(t)) + B_1 u(t) \\ &\quad + B_d u(t - d_2(t)) + B_2 w(t) \\ z(t) &= Cx(t) + C_d x(t - d_1(t)) + D_1 u(t) \end{aligned}$$

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$$\begin{array}{rcl} & & +D_d u(t-d_2(t)) + D_2 w(t) \\ x(t) &=& \phi_1(t),\ t\in[-d_1(t),0] \\ u(t) &=& \phi_2(t),\ t\in[-d_2(t),0] \end{array} \quad (1)$$

where $x(t) \in \mathbf{R}^n$ is the descriptor variable, $z(t) \in \mathbf{R}^q$ is the controlled output variable, $u(t) \in \mathbf{R}^m$ is the control input variable, $w(t) \in \mathbf{R}^p$ is the disturbance input variable, E is singular matrix with $rank(E) = r \leq n$, $\phi_i(t), (i = 1,2)$, are continuous vector valued initial functions, and all matrices have proper dimensions. Here, time-varying delay is satisfied with

$$0 \leq d_i(t) < \infty,\ \dot{d}_i(t) \leq \eta_i < 1,\ (i = 1,2). \quad (2)$$

Associated with the system (1), we propose the following control law

$$u(t) = Kx(t). \quad (3)$$

When we apply the control (3) to the system (1), the resulting closed loop system is given by

$$\begin{array}{rcl} E\dot{x}(t) &=& A_K x(t) + A_d x(t-d_1(t)) \\ & & +B_d K x(t-d_2(t)) + B_2 w(t) \\ z(t) &=& C_K x(t) + C_d x(t-d_1(t)) \\ & & +D_d K x(t-d_2(t)) + D_2 w(t) \end{array} \quad (4)$$

where, $A_K = A + B_1 K$ and $C_K = C + D_1 K$. Also, we introduce H_∞ performance measure as follows:

$$J = \int_0^\infty [z(t)^T z(t) - \gamma^2 w(t)^T w(t)]dt. \quad (5)$$

In other words, the objective of this paper is to determine the stabilizing H_∞ controller gain within the upper bound, i.e., $\sup_{w(t)\in L_2[0,\infty)\neq 0} \frac{||z(t)||_2}{||w(t)||_2} < \gamma$ satisfying regularity and property of impulse free in the closed loop system (4).

Definition 1: Consider the delayed descriptor system (1). If there exist a control law (3) such that the closed loop system is regular, impulse free, and asymptotically stable in the closed loop system, (3) is said to be an H_∞ control law for the system (1).

Lemma 1: Consider the delayed singular system (1). For a given positive real number γ, if there exist an invertible symmetric matrix P, positive definite matrices $R_i, (i = 1,2)$, and a controller gain K satisfying

$$E^T P = PE \geq 0 \quad (6)$$

$$\begin{bmatrix} \Psi & PA_d & PB_d & PB_2 & C_K^T \\ * & -(1-\eta_1)R_1 & 0 & 0 & C_d^T \\ * & * & -(1-\eta_2)R_2 & 0 & D_d^T \\ * & * & * & -\gamma^2 I & D_2^T \\ * & * & * & * & -I \end{bmatrix} < 0 \quad (7)$$

then, (3) is an H_∞ controller such that the closed loop system (4) is regular, impulse free, and asymptotically stable. Also, (4) is satisfied with γ bound. Here, $\Psi = A_K^T P + PA_K + R_1 + K^T R_2 K$.

Proof : Firstly, we define a Lyapunov functional candidate as

$$\begin{array}{l} V(x(t)) = x(t)^T E^T P x(t) + \int_{t-d_1(t)}^t x(\tau)^T R_1 x(\tau) d\tau \\ \quad + \int_{t-d_2(t)}^t x(\tau)^T K^T R_2 K x(\tau) d\tau \end{array} \quad (8)$$

with (6). Taking the derivative of (8) along the solution of the closed loop system (4) yields

$$\begin{array}{l} \dot{V}(x(t)) = \dot{x}(t)^T E^T P x(t) + x(t)^T P E \dot{x}(t) \\ \quad + x(t)^T R_1 x(t) + x(t)^T K^T R_2 K x(t) \\ \quad - (1-\dot{d}_1(t)) x(t-d_1(t))^T R_1 x(t-d_1(t)) \\ \quad - (1-\dot{d}_2(t)) x(t-d_2(t))^T K^T R_2 K x(t-d_2(t)), \end{array} \quad (9)$$

which is negative definite when the following matrix is negative definite as follows:

$$\begin{array}{l} \dot{V}_a(x(t)) = \dot{x}(t)^T E^T P x(t) + x(t)^T P E \dot{x}(t) \\ \quad + x(t)^T R_1 x(t) + x(t)^T K^T R_2 K x(t) \\ \quad - (1-\eta_1) x(t-d_1(t))^T R_1 x(t-d_1(t)) \\ \quad - (1-\eta_2) x(t-d_2(t))^T K^T R_2 K x(t-d_2(t)). \end{array} \quad (10)$$

Therefore, $\dot{V}_a(x(t)) < 0$ with zero initial condition, $w(t) = 0$, means asymptotic stability of the closed loop system. From (5), (9), and (10), the matrix inequality (7) implies

$$\begin{array}{l} \dot{V}((x(t))) \leq \dot{V}_a((x(t))) \\ \quad < -z(t)^T z(t) + \gamma^2 w(t)^T w(t) < 0. \end{array} \quad (11)$$

Therefore, we have

$$\begin{bmatrix} x(t) \\ x(t-d_1(t)) \\ Kx(t-d_2(t)) \\ w(t) \end{bmatrix}^T \times \begin{bmatrix} \Pi & PA_d + C_K^T C_d & PB_d + C_K^T D_d & PB_2 + C_K^T D_2 \\ * & -\tilde{R}_1 + C_d^T C_d & C_d^T D_d & C_d^T D_2 \\ * & * & -\tilde{R}_2 + D_d^T D_d & D_d^T D_2 \\ * & * & * & -\gamma^2 I + D_2^T D_2 \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-d_1(t)) \\ Kx(t-d_2(t)) \\ w(t) \end{bmatrix} = \eta(t)^T Z \eta(t) < 0 \quad (12)$$

which ensures the asymptotic stability and H_∞ norm bound within prescribed level γ of the closed loop system (4). Here,

$$\begin{array}{l} \Pi = A_K^T P + PA_K + R_1 + K^T R_2 K + C_K^T C_K, \\ \tilde{R}_i = (1-\eta_i) R_i,\ (i = 1,2), \\ \eta(t) = [x(t)^T\ x(t-d_1(t))^T\ x(t-d_2(t))^T K^T\ w(t)^T]^T. \end{array}$$

Hence, the matrix inequality $Z < 0$ can be transformed into (7) using Schur complements. ∎

In the following Theorem 1, the H_∞ controller design method for the descriptor system (1) is presented.

Theorem 1: For a given positive real number γ, if there exist positive definite matrices X_1, S_{11}, S_{14}, S_2, an invertible symmetric matrix X_4, and matrices S_{12}, M_1, M_2 satisfying

$$\begin{bmatrix} \Sigma_1 & \Sigma_2 & B_{21} & \Sigma_4 & M_1^T & X_1 & 0 \\ * & \Sigma_3 & B_{22} & \Sigma_5 & M_2^T & 0 & X_4 \\ * & * & -\gamma^2 I & D_2^T & 0 & 0 & 0 \\ * & * & * & \Sigma_6 & 0 & 0 & 0 \\ * & * & * & * & -S_2 & 0 & 0 \\ * & * & * & * & * & -S_{11} & -S_{12} \\ * & * & * & * & * & * & -S_{14} \end{bmatrix} < 0 \quad (13)$$

then, the matrix expressed by

$$K = \begin{bmatrix} M_1P_1 & M_2P_4 \end{bmatrix} \quad (14)$$

is an H_∞ controller gain satisfying Definition 1. Here,

$$\begin{aligned}
&\Sigma_1 = A_1X_1 + X_1A_1^T + M_1^TB_{11}^T + B_{11}M_1 + A_{d1}\tilde{S}_{11}A_{d1}^T \\
&\quad + A_{d2}\tilde{S}_{12}^TA_{d1}^T + A_{d1}\tilde{S}_{12}A_{d2}^T + A_{d2}\tilde{S}_{14}A_{d2}^T + B_{d1}\tilde{S}_2B_{d1}^T, \\
&\Sigma_2 = M_1^TB_{12}^T + B_{11}M_2 + A_{d1}\tilde{S}_{11}A_{d3}^T \\
&\quad + A_{d2}\tilde{S}_{12}^TA_{d3}^T + A_{d1}\tilde{S}_{12}A_{d4}^T + A_{d2}\tilde{S}_{14}A_{d4}^T + B_{d1}\tilde{S}_2B_{d2}^T, \\
&\Sigma_3 = A_4X_4 + X_4A_4^T + M_2^TB_{12}^T + B_{12}M_2 + A_{d3}\tilde{S}_{11}A_{d3}^T \\
&\quad + A_{d4}\tilde{S}_{12}^TA_{d3}^T + A_{d3}\tilde{S}_{12}A_{d4}^T + A_{d4}\tilde{S}_{14}A_{d4}^T + B_{d2}\tilde{S}_2B_{d2}^T, \\
&\Sigma_4 = M_1^TD_1^T + X_1C_1^T + A_{d1}\tilde{S}_{11}C_{d1}^T \\
&\quad + A_{d2}\tilde{S}_{12}^TC_{d1}^T + A_{d1}\tilde{S}_{12}C_{d2}^T + A_{d2}\tilde{S}_{14}C_{d2}^T + B_{d1}\tilde{S}_2D_d^T, \\
&\Sigma_5 = M_2^TD_1^T + X_4C_2^T + A_{d3}\tilde{S}_{11}C_{d1}^T \\
&\quad + A_{d4}\tilde{S}_{12}^TC_{d1}^T + A_{d3}\tilde{S}_{12}C_{d2}^T + A_{d4}\tilde{S}_{14}C_{d2}^T + B_{d2}\tilde{S}_2D_d^T, \\
&\Sigma_6 = -I + C_{d1}\tilde{S}_{11}C_{d1}^T + C_{d2}\tilde{S}_{12}^TC_{d1}^T + C_{d1}\tilde{S}_{12}C_{d2}^T \\
&\quad + C_{d2}\tilde{S}_{14}C_{d2}^T + D_d\tilde{S}_2D_d^T, \\
&\tilde{S}_{1i} = (1-\eta_1)^{-1}S_{1i}, (i = 1,2,4), \\
&\tilde{S}_2 = (1-\eta_2)^{-1}S_2, \\
&P_1 = X_1^{-1},\ P_4 = X_4^{-1}.
\end{aligned}$$

Proof : Using Schur complements[13,14] and changes of variables, $X = P^{-1}$, $R_i = S_i^{-1}, (i = 1,2)$, $M = KP^{-1} = KX$, (7) is equivalent to

$$\begin{bmatrix} \Gamma_1 & B_2 & \Gamma_2 & M^T & X \\ * & -\gamma^2 I & D_2^T & 0 & 0 \\ * & * & \Gamma_3 & 0 & 0 \\ * & * & * & -S_2 & 0 \\ * & * & * & * & -S_1 \end{bmatrix} < 0, \quad (15)$$

where,

$$\begin{aligned}
&\Gamma_1 = AX + XA^T + M^TB_1^T \\
&\quad + B_1M + (1-\eta_1)^{-1}A_dS_1A_d^T + (1-\eta_2)^{-1}B_dS_2B_d^T, \\
&\Gamma_2 = M^TD_1^T + XC^T + (1-\eta_1)^{-1}A_dS_1C_d^T \\
&\quad + (1-\eta_2)^{-1}B_dS_2D_d^T, \\
&\Gamma_3 = -I + (1-\eta_1)^{-1}C_dS_1C_d^T + (1-\eta_2)^{-1}D_dS_2D_d^T.
\end{aligned}$$

To obtain an LMI sufficient condition in terms of finding all variables and eliminate the equality in (6), we make use of singular value decomposition and changes of variables. Without loss of generality, we assume that the system matrices (1) have the following singular value decomposition form[1,3]:

$$\begin{aligned}
&E = \left[\begin{array}{c|c} I_r & 0 \\ \hline 0 & 0 \end{array}\right],\ A = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_4 \end{array}\right], \\
&A_d = \left[\begin{array}{c|c} A_{d1} & A_{d2} \\ \hline A_{d3} & A_{d4} \end{array}\right],\ B_1 = \left[\begin{array}{c} B_{11} \\ \hline B_{12} \end{array}\right], \\
&B_d = \left[\begin{array}{c} B_{d1} \\ \hline B_{d2} \end{array}\right],\ B_2 = \left[\begin{array}{c} B_{21} \\ \hline B_{22} \end{array}\right], \\
&C = \left[\begin{array}{c|c} C_1 & C_2 \end{array}\right],\ C_d = \left[\begin{array}{c|c} C_{d1} & C_{d2} \end{array}\right], \\
&D_1 = D_1,\ D_d = D_d,\ D_2 = D_2.
\end{aligned} \quad (16)$$

where all decomposed matrices have appropriate dimensions. Also, if we set

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_4 \end{bmatrix} \quad (17)$$

in order to satisfy the condition (6), and if other solutions have the following structure

$$\begin{aligned}
M &= \begin{bmatrix} M_1 & M_2 \end{bmatrix}, \\
X &= P^{-1} = \begin{bmatrix} X_1 & 0 \\ 0 & X_4 \end{bmatrix}, \\
S_1 &= \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{14} \end{bmatrix}
\end{aligned} \quad (18)$$

and we apply (16) and (18) to (15) then (15) is equivalent to (13). ■

The proposed controller design algorithm can be directly applied to the descriptor systems without time delay by simple manipulations in the following Theorem 2.

Theorem 2: Consider a descriptor system without time-varying delays in (1), in other words, $A_d = 0$, $B_d = 0$, $C_d = 0$, and $D_d = 0$. For a given positive real number γ, if there exist a positive definite matrix X_1, an invertible symmetric matrix X_4, and matrices M_1, M_2 satisfying

$$\begin{bmatrix} \Delta_1 & \Delta_2 & B_{21} & X_1C_1^T + M_1^TD_1^T \\ * & \Delta_3 & B_{22} & X_4C_2^T + M_2^TD_1^T \\ * & * & -\gamma^2 I & D_2^T \\ * & * & * & -I \end{bmatrix} < 0 \quad (19)$$

then, (14) is an H_∞ controller gain satisfying Definition 1. Here,

$$\begin{aligned}
\Delta_1 &= A_1X_1 + X_1A_1^T + M_1^TB_{11}^T + B_{11}M_1, \\
\Delta_2 &= M_1^TB_{12}^T + B_{11}M_2, \\
\Delta_3 &= A_4X_4 + X_4A_4^T + M_2^TB_{12}^T + B_{12}M_2.
\end{aligned}$$

Proof : Using the Lyapunov functional, $V(x(t)) = x(t)^TE^TPx(t)$ with $E^TP = PE \geq 0$, the H_∞ performance measure (5), and the proof procedures of Lemma 1 and Theorem 1, the result can be straightforwardly obtained. ■

Remark 1: In the case of $E = I$, the problem can be solvable from LMI (15) directly. In other words, the problem is reduced to the result of [13]. Thus, the result is generalization of [13].

Remark 2: In the case of delayed singular systems with norm-bounded parameter uncertainties, the system can be transformed into delayed descriptor systems without parameter uncertainties in the following Corollary 1. Therefore, the presented algorithm can be easily extended to the robust H_∞ controller design method for norm-bounded parameter uncertain descriptor systems with time-varying delays.

Lemma 2: Consider a parameter uncertain descriptor system with time-varying delays in state and control input

$$\begin{aligned}
&E\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t - d_1(t)) \\
&\quad + [B_1 + \Delta B_1(t)]u(t) + [B_d + \Delta B_d(t)]u(t - d_2(t)) \\
&\quad + [B_2 + \Delta B_2(t)]w(t) \\
&z(t) = [C + \Delta C(t)]x(t) + [C_d + \Delta C_d(t)]x(t - d_1(t)) \\
&\quad + [D_1 + \Delta D_1(t)]u(t) + [D_d + \Delta D_d(t)](t - d_2(t)) \\
&\quad + [D_2 + \Delta D_2(t)]w(t)
\end{aligned} \quad (20)$$

with parameter uncertainties

$$\begin{bmatrix} \Delta A(t) & \Delta A_d(t) & \Delta B_1(t) & \Delta B_d(t) & \Delta B_2(t) \\ \Delta C(t) & \Delta C_d(t) & \Delta D_1(t) & \Delta D_d(t) & \Delta D_2(t) \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F(t) \begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 \end{bmatrix},$$
$$F(t)^T F(t) \leq I \tag{21}$$

where, H_i $(i = 1, 2)$ and E_j $(j = 1, 2, 3, 4, 5)$ are known matrices. This system (20) is stabilizable and has an H_∞ performance $\gamma > 0$ by a state feedback controller if and only if there exist a $\lambda > 0$ such that delayed descriptor systems without parameter uncertainties

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_d x(t - d_1(t)) + B_1 u(t) \\ &\quad + B_d u(t - d_2(t)) + [B_2 \ \gamma\lambda H_1] \begin{bmatrix} w(t) \\ \hat{w}(t) \end{bmatrix} \\ \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} &= [C_1 \ \tfrac{1}{\lambda}E_1]x(t) + [C_d \ \tfrac{1}{\lambda}E_2]x(t - d_1(t)) \\ &\quad + [D_1 \ \tfrac{1}{\lambda}E_3]u(t) + [D_d \ \tfrac{1}{\lambda}E_4]u(t - d_2(t)) \\ &\quad + \begin{bmatrix} D_2 & \gamma\lambda H_2 \\ \frac{1}{\lambda}E_5 & 0 \end{bmatrix} \begin{bmatrix} w(t) \\ \hat{w}(t) \end{bmatrix} \end{aligned} \tag{22}$$

with additional disturbance input variable $\hat{w}(t)$ and additional controlled output variable $\hat{z}(t)$, is stabilizable and has an H_∞ performance $\gamma > 0$ by the same state feedback controller.

Proof : Using the results of [7,15], the proof can be straightforwardly derived. ■

Therefore, the problem of robust H_∞ control for uncertain descriptor systems with time-varying delays in both state and control input can be solvable using the proposed method.

Example : In order to check the validity of the proposed method, a descriptor system is considered as follows:

$$\begin{aligned} &\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 0 \end{array}\right] \dot{x}(t) = \left[\begin{array}{cc|c} -2 & 1 & 0 \\ 1 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right] x(t) \\ &+ \left[\begin{array}{cc|c} 0.5 & 0.4 & 0.5 \\ 0.3 & 0.6 & 0.4 \\ \hline 0.5 & 0.3 & 0.4 \end{array}\right] x(t - d_1(t)) + \left[\begin{array}{c} 0.5 \\ 1.5 \\ \hline 1 \end{array}\right] u(t) \\ &+ \left[\begin{array}{c} 0.2 \\ 0.3 \\ \hline 0.1 \end{array}\right] u(t - d_2(t)) + \left[\begin{array}{c} 1 \\ 1 \\ \hline 0 \end{array}\right] w(t) \\ &z(t) = \left[\begin{array}{cc|c} 1 & 2 & 0 \end{array}\right] x(t) + \left[\begin{array}{cc|c} 0.2 & 0.1 & 0 \end{array}\right] x(t - d_1(t)) \\ &\quad + u(t) + 0.1u(t - d_2(t)) + 0.1w(t) \\ &d_1(t) = 2 + 0.6\sin t, \ d_2(t) = 5 + 0.3\cos t, \ \gamma = 1. \end{aligned} \tag{23}$$

All solutions can be calculated at the same time from the LMI Toolbox[14] because the proposed optimization problem of Theorem 1 is an LMI form in terms of finding all variables. The solutions satisfying Theorem 1 are as follows:

$$\begin{aligned} X_1 &= \begin{bmatrix} 1.7900 & -0.1494 \\ -0.1494 & 1.1282 \end{bmatrix}, \ X_4 = -3.4990, \\ S_{11} &= \begin{bmatrix} 5.5490 & -0.1031 \\ -0.1031 & 5.2490 \end{bmatrix}, \ S_{12} = \begin{bmatrix} 0.2604 \\ -0.0644 \end{bmatrix}, \\ S_{14} &= 6.6112, \ S_2 = 7.6150, \\ M_1 &= \begin{bmatrix} -2.0085 & -2.7930 \end{bmatrix}, \ M_2 = 0.0285. \end{aligned} \tag{24}$$

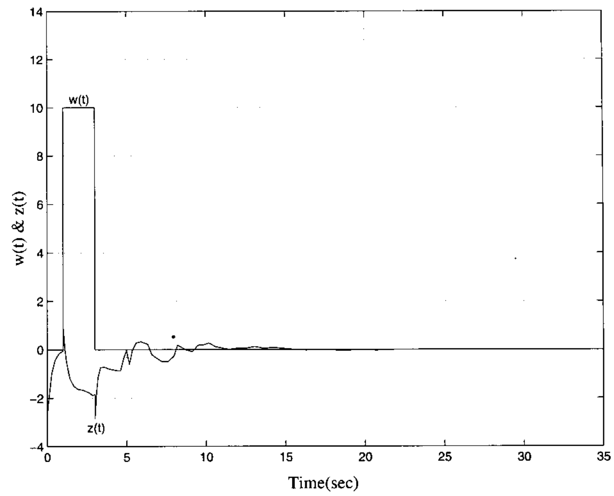

Fig. 1. The trajectories of $w(t)$ and $z(t)$.

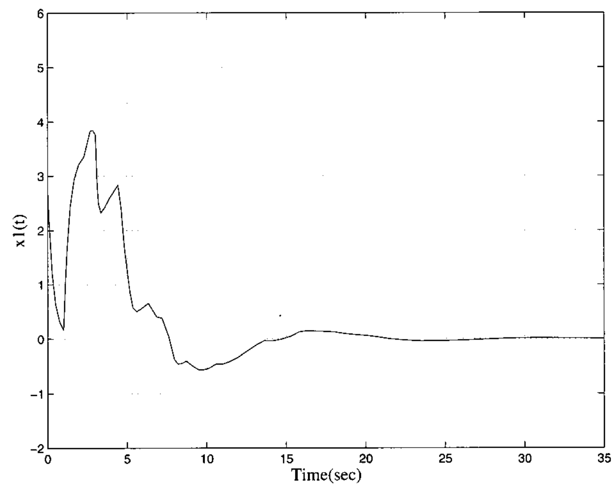

Fig. 2. The trajectory of $x_1(t)$.

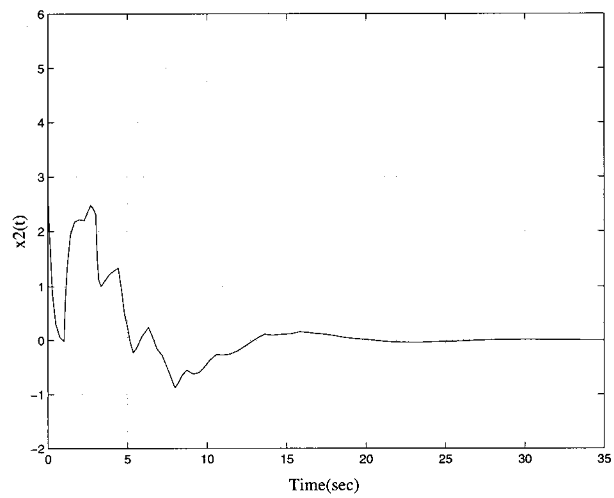

Fig. 3. The trajectory of $x_2(t)$.

Therefore, the H_∞ control law by (14) is

$$u(t) = \begin{bmatrix} -1.3436 & -2.6537 & -0.0081 \end{bmatrix} x(t). \tag{25}$$

For computer simulation, if we take disturbance input like Fig. 1, then the trajectories of descriptor variables($x_1(t)$, $x_2(t)$,

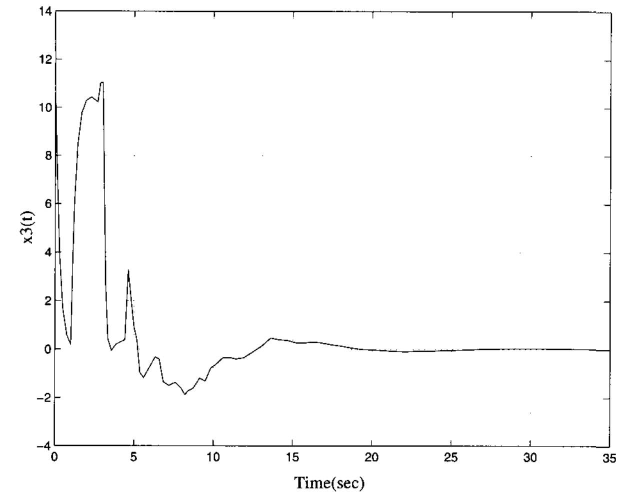

Fig. 4. The trajectory of $x_3(t)$.

$x_3(t)$) and the controlled output($z(t)$) are shown in Fig.1-Fig.4. From the simulation results of Fig.2-Fig.4, the obtained controller stabilizes the descriptor system with time-varying delays in both state and control input because the values of descriptor variables converge to zero as time goes to infinity. Also, H_∞ norm bound can be calculated as $0.2932(\gamma < 1)$ from the L_2 induced norm property between $w(t)$ and $z(t)$ in Fig. 1. Thus, the obtained controller guarantees not only asymptotic stability including regularity and the property of impulse free but also the H_∞ norm bound, $(\gamma < 1)$, of the closed loop system.

III. Conclusions

This paper considered the design problem of H_∞ controller for descriptor systems with time-varying delays in state and control input by LMI approach. The presented H_∞ controller guaranteed asymptotic stability, regularity, the property of impulse free, and H_∞ norm bound in the closed loop system. Also, the H_∞ controller design method for descriptor systems without time delay was discussed. Using the proposed method and system transformation, robust H_∞ control problem for uncertain descriptor systems with time-varying delays could be solvable. Finally, the validity of the controller design method was checked by a numerical example.

References

[1] I. Masubuchi, Y. Kamitane, A. Ohara, and N. Suda, "H_∞ control for descriptor systems: A matrix inequalities approach," *Automatica*, vol. 33, pp. 669-673, 1997.

[2] H. S. Wang, C. F. Yung, and F. R. Chang, "Bounded real lemma and H_∞ control for descriptor systems," *IEE Proc.-Control Theory Appl.*, vol. 145, pp. 316-322, 1998.

[3] D. J. Bender and A. J. Laub, "The linear-quadratic optimal regulator for descriptor systems," *IEEE Trans. Automat. Control*, vol. 32, pp. 672-688, 1987.

[4] J. D. Cobb, "Controllability, observability, and duality in singular systems," *IEEE Trans. Automat. Control*, vol. 29, pp. 1076-1082, 1984.

[5] C. H. Fang, L. Lee, and F. R. Chang, "Robust control analysis and design for discrete-time singular systems," *Automatica*, vol. 30, pp. 1741-1750, 1994.

[6] F. L. Lewis, "Preliminary notes on optimal control for singular systems," *Proc. 24th Conf. on Decision and Control*, pp. 262-272, 1985.

[7] A. Rehm and F. Allgower, "H_∞ control of descriptor systems with norm-bounded uncertainties in the system matrices," *Proc. Amreican Control conference*, pp. 3244-3248, 2000.

[8] V. L. Syrmos, P. Misra, and R. Aripirala, "On the discrete generalized Lyapunov equations," *Automatica*, vol. 31, pp. 297-301, 1995.

[9] K. Takaba and T. Katayama, "Robust H_2 performance of uncertain descriptor systems," *Proc. European Control Conf.*, WE-E-B-2, 1997.

[10] K. Takaba, N. Morihara, and T. Katayama, "A generalized Lyapunov theorem for descriptor system," *Systems and Control Lett.*, vol. 24, pp. 49-51, 1995.

[11] L. Zhang, J. Lam, and Q. Zhang, "Lyapunov and Riccati equations of discrete-time descriptor systems," *IEEE Trans. Automat. Control*, vol. 44, pp. 2134-2139, 1999.

[12] H. H. Choi and M. J. Chung, "Memoryless H_∞ controller design for linear systems with delayed state and control," *Automatica*, vol. 31, pp. 917-919, 1995.

[13] J. H. Kim and H. B. Park, "H_∞ state feedback control for generalized continuous/discrete time-delay system," *Automatica*, vol. 35, pp. 1443-1451, 1999.

[14] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox*, The Math Works Inc., 1995.

[15] K. Gu, "H_∞ control of systems under norm bounded uncertainties in all system matrices," *IEEE Trans. Automat. Control*, vol. 39, pp. 1320-1322, 1994.

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