An Observer Design for MIMO Nonlinear Systems

Sungryul Lee, Yanghee Yee, and Mignon Park

Abstract: This paper presents a state observer design for a class of MIMO nonlinear systems that has a block triangular structure. For this, the extension of the existing design for SISO triangular systems to MIMO cases is provided. Since the gain of the proposed observer depends on a nonlinear part as well as a linear one of a system, it improves the transient performance of the high gain observer. Also, by using a generalized similarity transformation for the error dynamics, it is shown that under some boundedness condition, the proposed observer guarantees the global exponential convergence of the estimation error. Finally, we will give a simulation example to show the validity of our design methodology.

Keywords: MIMO nonlinear systems, state observer, triangular structure, high gain observer

I. Introduction

During the last decade, the observer design problem for MIMO nonlinear systems has received much attention in the literature. In general, the recent contributions to solve this problem can be classified into two main approaches. The first one is the error linearization based approach which allows us to construct a nonlinear observer with linear error dynamics. It originated from the efforts to extend Krener's work[1] for SISO systems to MIMO cases. In [2], the necessary and sufficient conditions under which a MIMO nonlinear time-variable system can be transformed into an observer canonical form have been obtained. The paper [3] has identified the class of nonlinear systems that can be changed into a linear observable form. While the previous studies have considered the nonlinear systems that can be changed into only the dual Brunovsky form, the result of [3] can be applied to a larger class of systems. On the other hand, most research along this direction requires a solution of some partial differential equation that is not easy to obtain. Moreover, since the conditions of error linearizability are quite restrictive, it is difficult to satisfy them in most cases. Due to these obstacles, the paper [4] has proposed the observer based on the approximate error linearization which minimizes the part of system that cannot be canceled by input output injection. Although the work of [4] relaxes very strong conditions of error linearizability, it guarantees only local stability of error dynamics.

The other is the high gain observer which is available for triangular nonlinear systems. In SISO nonlinear systems, it is well known that this structure leads to the design of the high gain observer with an exponential convergence.[5] Motivated by this fact, the block triangular structure has been characterized for an extension of the high gain observer to MIMO systems. In [6], the state observer of high gain type was proposed for block triangular multi output nonlinear systems. The result of [6] has an advantage in that an interconnection between blocks is allowed. But, in general, the high gain observer may show a large oscillation in a transient response and is sensitive to measurement noise. To overcome these drawbacks, the recent paper [7] has proposed the improved observer which exhibits less transient oscillation than the high gain observer.

In this paper, we extend the work of [7] to MIMO nonlinear systems. To the end, we consider a class of nonlinear systems which is composed of the linear observable part and the nonlinear part with a block triangular structure. It is first shown that the similarity transformation developed in [7] can easily be extended to MIMO cases. From this, we propose the state observer that can be seen as an interconnection of the observer of [7]. Also, by using similarity transformation on the error dynamics, it is shown that under the boundedness conditions, the proposed observer ensures the global exponential convergence of the estimation error. Finally, an illustrative example is given to verify the effectiveness of our design methodology. The remainder of this paper is organized as follows. In section II, we introduce the class of systems to be considered. Section III presents an nonlinear observer design. A simulation example is given in section IV and we conclude this paper in section V.

II. System descriptions

In this paper, we consider a multi input multi output nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $y \in R^p$ is the output, f, g and h are smooth vector fields. The control input $u : R \to R^m$ is assumed to be analytic time function. In particular, we will restrict our interest to the class of systems in the following form.

$$\begin{aligned} \dot{x}_i &= A_i x_i + g_i(x_1, \cdots, x_i; u; y_{i+1}, \cdots, y_p) \\ y_i &= C_i x_i \end{aligned} \quad ,1 \le i \le p \tag{2}$$

where

$$\begin{aligned} x &= [x_1\, x_2 \cdots x_p]^T \in R^n \\ x_i &= [x_{i1}\, x_{i2} \cdots x_{in_i}]^T \in R^{n_i} \\ y &= [y_1 \;\; \cdots \;\; y_p]^T \in R^p \\ x_{[1,i-1]} &= [x_1 \cdots x_{i-1}]^T \\ y_{[i+1,p]} &= [y_{i+1} \cdots y_p]^T \end{aligned}$$

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$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix},\ C_i = [1 \ \ 0 \ \ \cdots \ \ 0]$$

$$g_i = \begin{bmatrix} g_{i1}(x_{[1,i-1]}; x_{i1}; u; y_{[i+1,p]}) \\ g_{i2}(x_{[1,i-1]}; x_{i1}, x_{i2}; u; y_{[i+1,p]}) \\ \vdots \\ g_{in_i}(x_{[1,i-1]}; x_{[i1,in_i]}; u; y_{[i+1,p]}) \end{bmatrix}$$

Without the loss of generality, it is assumed that all n_i's are equal for the system (2). The sufficient and necessary conditions to transform the system (1) into (2) have been characterized in terms of the differential geometric method in [6]. As stated in several works, the above block triangular structure make it possible to extend the high gain observer design for SISO systems to MIMO cases. For the observer design, we assume the following condition on the system (2).

Assumption 1: The partial derivatives of g_i with respect to x and their respective time derivatives are bounded for all x and u.

Remark 1: In a practical viewpoint, the Assumption 1 may be somewhat a restrictive condition. However, if the state trajectory of the system (2) is bounded in a compact region, it can be relaxed outside a region of interest as in [6].

Remark 2: As mentioned in [7], the high gain observer of [6] for the system (2) can be rewritten as

$$\begin{aligned}\dot{\hat{x}}_i &= A_i\hat{x}_i + g_i(\hat{x}_1, \cdots, \hat{x}_i; u; y_{i+1}, \cdots, y_p) \\ &\quad + \Delta_{\theta_i}^{-1} K_i (y_i - C_i\hat{x}_i)\end{aligned} \quad 1 \le i \le p \quad (3)$$

where $\Delta_{\theta_i} = diag(1/\theta_i, 1/\theta_i^2, \cdots, 1/\theta_i^{n_i})$ and the gain matrices K_i are chosen such that all the eigenvalues of matrices $A_i - K_iC_i$ have negative real parts. The design parameters θ_i must be large enough to compensate for the effect of the system nonlinearity. However, as the value of θ_i grows, the observer (3) may cause larger transient oscillation. In section IV, we will show that the proposed observer improves the transient performance of the observer (3).

III. Nonlinear observer design

This section presents a nonlinear observer for the system (2) whose estimation error converges to zero exponentially. For this end, we first extend a similarity transformation developed in [7] to MIMO cases. It can be easily constructed through the following five steps as in [7].

Step 1: Define $\overline{M}_i(x,u,y)$, $1 \le i \le p$, as follows.

$$\overline{M}_i(x,u,y) = \begin{bmatrix} C_i \\ C_iF_i(x,u,y) \\ \vdots \\ C_iF_i^{n_i-1}(x,u,y) \end{bmatrix}$$

where $F_i(x,u,y) = A_i + G_{ii}(x,u,y)$ with $G_{ij} = \partial g_i / \partial x_j$. From the construction, it is very easy to show that the matrices $\overline{M}_i(x,u,y)$ are lower triangular and nonsingular for all x and u.

Step 2: Let $Q_i(x,u,y)$ be as follows.

$$Q_i(x,u,y) = \overline{M}_i(x,u,y)F_i(x,u,y)\overline{M}_i^{-1}(x,u,y) - A_i.$$

By definition, we can also show that the first $(n_i - 1)$ rows of the matrices $Q_i(x,u,y)$ have zero elements and so they are represented by

$$Q_i(x,u,y) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ q_{i1}(x,u,y) & q_{i2}(x,u,y) & \cdots & q_{in_i}(x,u,y) \end{bmatrix}$$

Step 3: Let $N_i(x,u,y) = R_iQ_i^T(x,u,y)R_i$ where $R_i = [B_i \ A_iB_i \cdots A_i^{n_i-1}B_i]$, $B_i = [0 \ \cdots \ 0 \ \ 1]^T$.

Therefore, the matrices $N_i(x,u,y)$ are given by

$$N_i(x,u,y) = \begin{bmatrix} q_{in_i}(x,u,y) & 0 & \cdots & 0 \\ q_{i(n_i-1)}(x,u,y) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ q_{i1}(x,u,y) & 0 & \cdots & 0 \end{bmatrix}$$

Furthermore, the matrix $N_i(x,u,y)$ can be decomposed into $N_i(x,u,y) = L_i(x,u,y)C_i$ where $L_i \in R^{n_i \times 1}$.

Step 4: Let us define $W_i(x,u,y)$ as

$$W_i(x,u,y) = \begin{bmatrix} C_i \\ C_i\overline{A}_i(x,u,y) \\ \vdots \\ C_i\overline{A}_i^{n_i-1}(x,u,y) \end{bmatrix} \quad (4)$$

where $\overline{A}_i(x,u,y) = A_i + N_i(x,u,y)$. Similarly, the matrices $W_i(x,u)$ are lower triangular and nonsingular for all x and u.

Step 5: Let $M_i(x,u,y) = W_i^{-1}(x,u,y)\overline{M}_i(x,u,y)$.

As a result, the matrices $M_i(x,u,y)$ play a role as similarity transformations for the error dynamics. From the above procedure, we can easily obtain the following lemma.

Lemma 1: For all matrices defined above, the following conditions always hold for $1 \le i \le p$.

$$\begin{aligned} M_i(x,u,y)F_i(x,u,y)M_i^{-1}(x,u,y) &= A_i + L_i(x,u,y)C_i \\ C_iM_i^{-1}(x,u,y) &= C_i \end{aligned} \quad (5)$$

Proof: By a construction, it follows that both (F_i, C_i) and $(\overline{A}_i, C_i)$ are observable pairs and similar matrices.[10] Thus, there exist similarity transformations T_i such that

$$\overline{A}_i = T_iF_iT_i^{-1},\ C_i = C_iT_i^{-1} \quad (6)$$

Substituting (6) into (4), we have

$$W_i(x,u,y) = \begin{bmatrix} C_iT_i^{-1} \\ C_iT_i^{-1}T_iF_iT_i^{-1} \\ \vdots \\ C_iT_i^{-1}T_iF_i^{n_i-1}T_i^{-1} \end{bmatrix} = \overline{M}_iT_i^{-1}.$$

which implies $T_i = W_i^{-1}\overline{M}_i$. This completes the proof of Lemma 1. ■

Now, we construct a nonlinear observer for the system (2) as follows.

$$\begin{aligned}\dot{\hat{x}}_i &= A_i\hat{x}_i + g_i(\hat{x}_1,\cdots,\hat{x}_i;u;y_{i+1},\cdots,y_p) \\ &\quad + M_i^{-1}(\hat{x},u,y)[L_i(\hat{x},u,y) + \Delta_{\theta_i}^{-1}K_i](y_i - C_i\hat{x}_i)\end{aligned} \tag{7}$$

where Δ_{θ_i} and K_i are chosen in the same way as Remark 2. In order to prove the main result of this paper, the following lemma is needed.

Lemma 2: Under the Assumption 1, there exist constants $c_{jk} > 0$ and $\rho_i > 0$ independent of θ_i such that for any θ_i satisfying $1 \le \theta_1$ and $\theta_{i-1}^{n_{i-1}} \le \theta_i$, $1 \le i \le p$,

i) $\left\|\Delta_{\theta_j} M_j(\hat{x},u,y)G_{jk}(\hat{x},u,y)M_k^{-1}(\hat{x},u,y)\Delta_{\theta_k}^{-1}\right\| \le c_{jk}$

for $j \ge k$ and $1 \le j,k \le p$

ii) $\left\|\Delta_{\theta_i}\dot{M}_i(\hat{x},u,y)M_i^{-1}(\hat{x},u,y)\Delta_{\theta_i}^{-1}\right\| \le \rho_i$

for $1 \le i \le p$

Proof: i) Let $H_{jk} = M_j(\hat{x},u,y)G_{jk}(\hat{x},u,y)M_k^{-1}(\hat{x},u,y)$. Then, from a simple computation, we have

$$(\Delta_{\theta_j} H_{jk}\Delta_{\theta_k}^{-1})_{l,m} = (H_{jk})_{l,m}\frac{\theta_k^m}{\theta_j^l}, \quad 1 \le l,m \le n_j = n_k$$

where $(\cdot)_{l,m}$ denotes the (l,m) th element of a matrix $(\cdot)$. Considering the Assumption 1, all entries of a matrix H_{jk} are bounded for all x and u. Moreover, taking $1 \le \theta_1$ and $\theta_{i-1}^{n_{i-1}} \le \theta_i$ into account, it is easy to prove the result of Lemma 1. ii) This case can be easily proved in the same way as that used in the case i) ■

Theorem 1: Suppose that the system (2) satisfies the Assumption 1. Then there exist θ_{i0}, $1 \le i \le p$ such that for all $\theta_i > \theta_{i0}$ satisfying $1 < \theta_i$ and $\theta_{i-1}^{n_{i-1}} < \theta_i$, the system (7) is a global exponential observer for the system (2).

Proof: Consider the first subsystem of the proposed observer (7). Let $e_1 = x_1 - \hat{x}_1$. Then we have

$$\begin{aligned}\dot{e}_1 &= A_1e_1 + g_1(x_1,u,y_{[2,p]}) - g_1(\hat{x}_1,u,y_{[2,p]}) \\ &\quad - M_1^{-1}(\hat{x},u,y)(L_1(\hat{x},u,y) + \Delta_{\theta_1}^{-1}K_1)C_1e_1\end{aligned} \tag{8}$$

By the mean value theorem[11], there always exists a vector η_{11} such that

$$\begin{aligned}&g_1(x_1,u,y_{[2,p]}) - g_1(\hat{x}_1,u,y_{[2,p]}) \\ &= G_{11}(\eta_{11},u,y_{[2,p]})e_1 \\ &= G_{11}(\hat{x},u,y_{[2,p]})e_1 + G_{11}(\eta_{11},u,y_{[2,p]})e_1 \\ &\quad - G_{11}(\hat{x},u,y_{[2,p]})e_1 \\ &= G_{11}(\hat{x},u,y_{[2,p]})e_1 + R_{11}\,e_1\end{aligned} \tag{9}$$

where $R_{11} = G_{11}(\eta_{11},u,y_{[2,p]}) - G_{11}(\hat{x},u,y_{[2,p]})$. Hereafter, for notational simplicity, we shall drop the arguments of all the matrices. Using the property (9), the error dynamics (8) becomes

$$\dot{e}_1 = [F_1 - M_1^{-1}(L_1 + \Delta_{\theta_1}^{-1}K_1)C_1]e_1 + R_{11}\,e_1$$

Considering the similarity transformation $\overline{e}_1 = M_1(\hat{x},u,y)e_1$, the error dynamics above leads to

$$\begin{aligned}\dot{\overline{e}}_1 &= M_1\dot{e}_1 + \dot{M}_1e_1 \\ &= M_1F_1M_1^{-1}\overline{e}_1 - (L_1 + \Delta_{\theta_1}^{-1}K_1)C_1M_1^{-1}\overline{e}_1 \\ &\quad + M_1R_{11}M_1^{-1}\overline{e}_1 + \dot{M}_1M_1^{-1}\overline{e}_1 \\ &= (\overline{A}_1 + L_1C_1)\overline{e}_1 - (L_1 + \Delta_{\theta_1}^{-1}K_1)C_1\overline{e}_1 \\ &\quad + M_1R_{11}M_1^{-1}\overline{e}_1 + \dot{M}_1M_1^{-1}\overline{e}_1 \\ &= (\overline{A}_1 - \Delta_{\theta_1}^{-1}K_1C_1)\overline{e}_1 + M_1R_{11}M_1^{-1}\overline{e}_1 + \dot{M}_1M_1^{-1}\overline{e}_1\end{aligned}$$

where the third equality follows from (5). Again, consider a scaling transformation $\tilde{e}_1 = \Delta_{\theta_1}\overline{e}_1$. Using the following properties

$$\begin{aligned}\Delta_{\theta_1}\overline{A}_1\Delta_{\theta_1}^{-1} &= \theta_1\overline{A}_1 \\ C_1\Delta_{\theta_1}^{-1} &= \theta_1C_1\end{aligned}$$

we obtain

$$\begin{aligned}\dot{\tilde{e}}_1 &= \theta_1(\overline{A}_1 - K_1C_1)\tilde{e}_1 + \Delta_{\theta_1}M_1R_{11}M_1^{-1}\Delta_{\theta_1}^{-1}\tilde{e}_1 \\ &\quad + \Delta_{\theta_1}\dot{M}_1M_1^{-1}\Delta_{\theta_1}^{-1}\tilde{e}_1\end{aligned}$$

Since $\overline{A}_1 - K_1C_1$ is Hurwitz, there exists a symmetric positive definite matrix P_1 such that

$$(\overline{A}_1 - K_1C_1)^TP_1 + P_1(\overline{A}_1 - K_1C_1) = -I_{n_1\times n_1}$$

where $I_{n_1\times n_1}$ is a ($n_1\times n_1$) identity matrix. Define $V_1(\tilde{e}_1) = \tilde{e}_1^TP_1\tilde{e}_1$ as the Lyapunov candidate function for the first subsystem of the observer (7). Then, its time derivative is

$$\begin{aligned}\dot{V}_1(\tilde{e}_1) &= \dot{\tilde{e}}_1^TP_1\tilde{e}_1 + \tilde{e}_1^TP_1\dot{\tilde{e}} \\ &= -\theta_1\|\tilde{e}_1\|^2 + 2\tilde{e}_1^TP_1\Delta_{\theta_1}M_1R_{11}M_1^{-1}\Delta_{\theta_1}^{-1}\tilde{e}_1 \\ &\quad + 2\tilde{e}_1^TP_1\Delta_{\theta_1}\dot{M}_1M_1^{-1}\Delta_{\theta_1}^{-1}\tilde{e}_1 \\ &\le -\theta_1\|\tilde{e}_1\|^2 + 2\|P_1\tilde{e}_1\|\left\|\Delta_{\theta_1}M_1R_{11}M_1^{-1}\Delta_{\theta_1}^{-1}\right\|\|\tilde{e}_1\| \\ &\quad + 2\|P_1\tilde{e}_1\|\left\|\Delta_{\theta_1}\dot{M}_1M_1^{-1}\Delta_{\theta_1}^{-1}\right\|\|\tilde{e}_1\|\end{aligned}$$

Selecting $\theta_1 \ge 1$ and considering Lemma 2, it follows that

$$\begin{aligned}\left\|\Delta_{\theta_1}M_1R_{11}M_1^{-1}\Delta_{\theta_1}^{-1}\right\| &\le c_{10} \\ \left\|\Delta_{\theta_1}\dot{M}_1M_1^{-1}\Delta_{\theta_1}^{-1}\right\| &\le c_{11}\end{aligned}$$

where c_{10} and c_{11} are positive constants independent of θ_1. Thus, it results that

$$\dot{V}_1(\tilde{e}_1) \le -\theta_1\|\tilde{e}_1\|^2 + \mu_{11}\|\tilde{e}_1\|^2$$

where $\mu_{11} = 2(c_{10} + c_{11})\lambda_M(P_1)$ and $\lambda_M(P_1)$ denotes the largest eigenvalue of P_1. Next, consider the 2nd subsystem of the proposed observer (7). Let $e_2 = x_2 - \hat{x}_2$. Then we have

$$\begin{aligned}\dot{e}_2 &= A_2e_2 + g_2(x_1,x_2,u,y_{[3,p]}) - g_2(\hat{x}_1,\hat{x}_2,u,y_{[3,p]}) \\ &\quad - M_2^{-1}(L_2 + \Delta_{\theta_2}^{-1}K_2)C_2e_2\end{aligned} \tag{10}$$

By the mean value theorem, there exist vectors η_{21} and η_{22} such that

$$\begin{aligned} & g_2(x_1,x_2,u,y_{[3,p]}) - g_2(\hat{x}_1,\hat{x}_2,u,y_{[3,p]}) \\ &= G_{21}(\eta_{21},\eta_{22},u,y_{[3,p]})e_1 + G_{22}(\eta_{21},\eta_{22},u,y_{[3,p]})e_2 \\ &= G_{22}(\hat{x}_1,\hat{x}_2,u,y_{[3,p]})e_2 + G_{22}(\eta_{21},\eta_{22},u,y_{[3,p]})e_2 \\ &\quad - G_{22}(\hat{x}_1,\hat{x}_2,u,y_{[3,p]})e_2 + G_{21}(\eta_{21},\eta_{22},u,y_{[3,p]})e_1 \\ &= G_{22}(\hat{x}_1,\hat{x}_2,u,y_{[3,p]})e_2 + R_{22}e_2 + G_{21}(\eta_{21},\eta_{22},u,y_{[3,p]})e_1 \end{aligned} \tag{11}$$

where $R_{22} = G_{22}(\eta_{21},\eta_{22},u,y_{[3,p]}) - G_{22}(\hat{x}_1,\hat{x}_2,u,y_{[3,p]})$. Using (11), the error dynamics (10) is

$$\begin{aligned} \dot{e}_2 &= F_2e_2 + R_{22}e_2 + G_{21}e_1 - M_2^{-1}(L_2 + \Delta_{\theta_2}^{-1}K_2)C_2e_2 \\ &= [F_2 - M_2^{-1}(L_2 + \Delta_{\theta_2}^{-1}K_2)C_2]e_2 + R_{22}e_2 + G_{21}e_1 \end{aligned} \tag{12}$$

Through a transformation $\tilde{e}_2 = \Delta_{\theta_2}M_2e_2$, (12) is transformed into

$$\begin{aligned} \dot{\tilde{e}}_2 &= \theta_2(\bar{A}_2 - K_2C_2)\tilde{e}_2 + \Delta_{\theta_2}M_2R_{22}M_2^{-1}\Delta_{\theta_2}^{-1}\tilde{e}_2 \\ &\quad + \Delta_{\theta_2}M_2G_{21}M_1^{-1}\Delta_{\theta_1}^{-1}\tilde{e}_1 + \Delta_{\theta_2}\dot{M}_2M_2^{-1}\Delta_{\theta_2}^{-1}\tilde{e}_2 \end{aligned}$$

Since $\bar{A}_2 - K_2C_2$ is Hurwitz, there exists a symmetric positive definite matrix P_2 such that

$$(\bar{A}_2 - K_2C_2)^TP_2 + P_2(\bar{A}_2 - K_2C_2) = -I_{n_2\times n_2}$$

where I_{n_2} is a ($n_2 \times n_2$) identity matrix. Again, consider the Lyapunov candidate function $V_2(\tilde{e}_2) = \tilde{e}_2^TP_2\tilde{e}_2$. Then, its time derivative becomes

$$\begin{aligned} \dot{V}_2(\tilde{e}_2) &= \dot{\tilde{e}}_2^TP_2\tilde{e}_2 + \tilde{e}_2^TP_2\dot{\tilde{e}}_2 \\ &= -\theta_2\|\tilde{e}_2\|^2 + 2\tilde{e}_2^TP_2\Delta_{\theta_2}M_2R_{22}M_2^{-1}\Delta_{\theta_2}^{-1}\tilde{e}_2 \\ &\quad + 2\tilde{e}_2^TP_2\Delta_{\theta_2}\dot{M}_2M_2^{-1}\Delta_{\theta_2}^{-1}\tilde{e}_2 \\ &\quad + 2\tilde{e}_2^TP_2\Delta_{\theta_2}M_2G_{21}M_1^{-1}\Delta_{\theta_1}^{-1}\tilde{e}_1 \\ &\le -\theta_2\|\tilde{e}_2\|^2 + 2\|P_2\tilde{e}_2\|\|\Delta_{\theta_2}M_2R_{22}M_2^{-1}\Delta_{\theta_2}^{-1}\|\|\tilde{e}_2\| \\ &\quad + 2\|P_2\tilde{e}_2\|\|\Delta_{\theta_2}\dot{M}_2M_2^{-1}\Delta_{\theta_2}^{-1}\|\|\tilde{e}_2\| \\ &\quad + 2\|P_2\tilde{e}_2\|\|\Delta_{\theta_2}M_2G_{21}M_1^{-1}\Delta_{\theta_1}^{-1}\|\|\tilde{e}_1\| \end{aligned}$$

Selecting $1 \le \theta_1^{n_1} \le \theta_2$, by the Lemma 2, it is easy to show that

$$\begin{aligned} \|\Delta_{\theta_2}M_2R_{22}M_2^{-1}\Delta_{\theta_2}^{-1}\| &\le c_{20} \\ \|\Delta_{\theta_2}\dot{M}_2M_2^{-1}\Delta_{\theta_2}^{-1}\| &\le c_{21} \end{aligned}$$

where c_{20}, c_{21} and c_{22} are positive constants independent of θ_1 and θ_2. Consequently, it follows that

$$\dot{V}_2(\tilde{e}_2) \le -\theta_2\|\tilde{e}_2\|^2 + \mu_{21}\|\tilde{e}_2\|^2 + \mu_{22}\|\tilde{e}_2\|^2 + \mu_{12}\|\tilde{e}_1\|^2$$

where

$$\begin{aligned} \mu_{21} &= 2(c_{20} + c_{21})\lambda_M(P_2) \\ \mu_{12} = \mu_{22} &= \lambda_M(P_2)c_{22} \end{aligned}$$

In the same way, for the i th subsystem of the observer (7), we obtain

$$\dot{V}_i(\tilde{e}_i) \le -\theta_i\|\tilde{e}_i\|^2 + \sum_{j=1}^{i}\mu_{ij}\|\tilde{e}_i\|^2 + \sum_{j=1}^{i-1}\mu_{ji}\|\tilde{e}_j\|^2 \tag{13}$$

where μ_{ij} are positive constants independent of $\theta_k, 1 \le k \le i$. For the overall observer system, consider the Lyapunov candidate function

$$V(\tilde{e}) = \sum_{i=1}^{p}V_i(\tilde{e}_i).$$

where $\tilde{e} = [\tilde{e}_1,\cdots,\tilde{e}_p]^T$. From (13), its time derivative becomes

$$\begin{aligned} \dot{V}(\tilde{e}) &\le (-\theta_1 + \sum_{j=1}^{p}\mu_{1j})\|\tilde{e}_1\|^2 + (-\theta_2 + \sum_{j=1}^{p}\mu_{2j})\|\tilde{e}_2\|^2 \\ &\quad + \cdots + (-\theta_p + \sum_{j=1}^{p}\mu_{pj})\|\tilde{e}_p\|^2 \end{aligned}$$

As a result, it is possible to choose $\theta_i, 1 \le i \le p$ such that for any given constant $\Sigma > 0$

$$\dot{V}(\tilde{e}) \le -\Sigma\|\tilde{e}\|^2$$

which completes the proof of Theorem 1. ■

Remark 3: In Theorem 1, the extension of the work of [7] to MIMO nonlinear systems has been achieved. The proposed observer design can also be applied to more general nonlinear systems of the form

$$\begin{aligned} \dot{x}_i &= f_i(x_1,\cdots,x_i,u,y_{i+1},\cdots,y_p) \\ y_i &= C_ix_i \end{aligned}, \quad 1 \le i \le p$$

where

$$f_i = \begin{bmatrix} f_{i1}(x_1,\cdots,x_{i-1};x_{i1},x_{i2};u;y_{i+1},\cdots,y_p) \\ f_{i2}(x_1,\cdots,x_{i-1};x_{i1},x_{i2},x_{i3};u;y_{i+1},\cdots,y_p) \\ \vdots \\ f_{in_i}(x_1,\cdots,x_i;u;y_{i+1},\cdots,y_p) \end{bmatrix}.$$

The generalization of our observer to the above systems can be made via the procedure similar to the foregoing arguments. But, it should be noted that an observer for the above systems is a local one.

Remark 4: In fact, an observer design for the class of systems considered in this paper was already dealt with in [6]. As will be shown in the next section, however, the proposed observer can be viewed as the improved one of the high gain observer given in [6]. By incorporating the additional term $M_i^{-1}L_i$ into the gain of the observer, the proposed design scheme minimizes the system nonlinearity to be suppressed by the design parameter θ_i, which leads to less transient oscillation.

IV. Illustrative example

To show the effectiveness of the proposed design scheme, we provide a simulation example borrowed from [6]. Consider a multi output nonlinear system in the form of (2).

$$\begin{aligned} \dot{x}_1 &= x_2 + 0.01x_1u \\ \dot{x}_2 &= -x_1 + (1 - x_1^2)x_2 + x_3u \\ \dot{x}_3 &= x_4 + 0.01x_2x_3\exp(u) \\ \dot{x}_4 &= -x_3 + (1 - x_3^2)x_4 + u \\ y_1 &= x_1 \\ y_2 &= x_3 \end{aligned} \tag{14}$$

As stated in [6], it can be shown that all the trajectories of the system (14) under $u = 2\sin 3t$ are bounded. Thus, in order to satisfy the Assumption 1, the nonlinear functions of system (14) can be replaced by their smooth bounded extensions outside a region of interest as in [6] First, through the five steps presented in Section III, the matrices M_i and L_i, $1 \le i \le 2$, can be easily computed as follows.

$$L_1 = \begin{bmatrix} 0.01u + 1 - \hat{x}_1^2 \\ -1 - 2\hat{x}_1\hat{x}_2 - 0.01u + 0.01\hat{x}_1^2 u \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0.01\hat{x}_2 \exp(u) + 1 - \hat{x}_3^2 \\ -1 - 2\hat{x}_3\hat{x}_4 - 0.01\hat{x}_2 \exp(u) + 0.01\hat{x}_2\hat{x}_3^2 \exp(u) \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 1 & 0 \\ -1 + \hat{x}_1^2 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 \\ -1 + \hat{x}_3^2 & 1 \end{bmatrix}$$

As a result, the proposed observer (7) is given by the following equation.

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} \hat{x}_2 + 0.01\hat{x}_1 u \\ -\hat{x}_1 + (1 - \hat{x}_1^2)\hat{x}_2 + y_2 u \end{bmatrix} + \begin{bmatrix} 0.01u + 1 - \hat{x}_1^2 + \theta_1 k_{11} \\ -2\hat{x}_1^2 + \theta_1 k_{11} + \hat{x}_1^4 - \theta_1 k_{11}\hat{x}_1^2 - 2x_1x_2 + \theta_1^2 k_{12} \end{bmatrix}(y_1 - \hat{x}_1)$$

$$\begin{bmatrix} \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{bmatrix} = \begin{bmatrix} \hat{x}_4 + 0.01\hat{x}_2\hat{x}_3 \exp(u) \\ -\hat{x}_3 + (1 - \hat{x}_3^2)\hat{x}_4 + u \end{bmatrix} + \begin{bmatrix} 0.01\hat{x}_2 \exp(u) + 1 - \hat{x}_3^2 + \theta_2 k_{21} \\ -2\hat{x}_3^2 + \theta_2 k_{21} + \hat{x}_3^4 - \theta_2 k_{21}\hat{x}_3^2 - 2x_3x_4 + \theta_2^2 k_{22} \end{bmatrix}(y_2 - \hat{x}_3) \quad (15)$$

The simulation was carried out with $u = 2\sin 3t$ and the following settings.

$$k_{11} = 2, k_{12} = 1, k_{21} = 2, k_{22} = 1, \theta_1 = 3, \theta_2 = 9$$
$$x_1(0) = 2, x_2(0) = 1, x_3(0) = 3, x_4(0) = 3.5$$
$$\hat{x}_1(0) = 0.5, \hat{x}_2(0) = 0, \hat{x}_3(0) = 0.5, \hat{x}_4(0) = 0.5$$

The gain matrices K_1 and K_2 were chosen such that all the eigenvalues of $A_i - K_iC_i$ are located at –1. Fig. 1-4 shows the simulation results for the proposed observer (15). It illustrates that the estimation error of our observer converges to zero exponentially. Moreover, the performance of the proposed observer was compared with the observer (3) . With the same settings as above, our observer exhibits less transient oscillation than the observer (3) without increasing a settling time.

V. Conclusions

In this paper, an observer design problem for multi input multi output nonlinear systems has been tackled. To be precise, the extension of the work of [7] to MIMO nonlinear systems has been achieved. To guarantee the stability of the high gain observer, its gain must be large enough to compensate for the effect of the system nonlinearity. But, this may cause large transient oscillation. Since the proposed design scheme minimizes the nonlinearity of a system to be suppressed by the design parameters θ_i, it improves the transient performance of the high gain observer. Furthermore, using the similarity transformation on the error, it was shown that in spite of the interconnection between subsystems, the proposed observer guarantees the global exponential convergence to zero of the estimation error. Finally, the simulation results demonstrate that the proposed observer improves the result of the high gain observer.

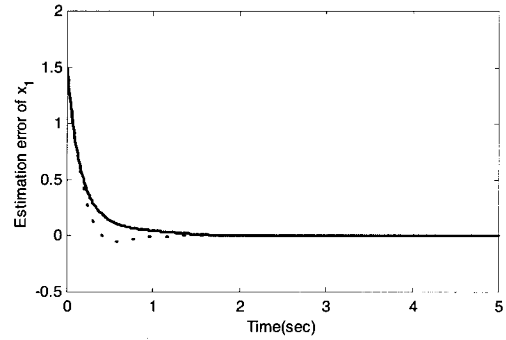

Fig. 1. The estimation error of x_1.
(solid : proposed, dashed : observer (3))

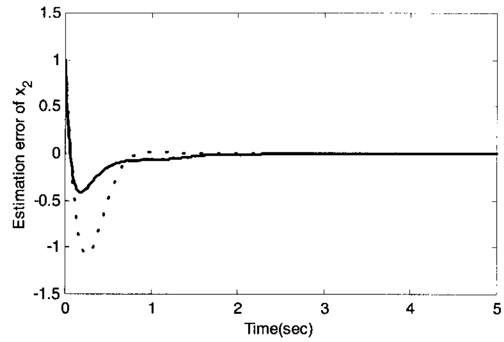

Fig. 2.The estimation error of x_2.
(solid : proposed, dashed : observer (3))

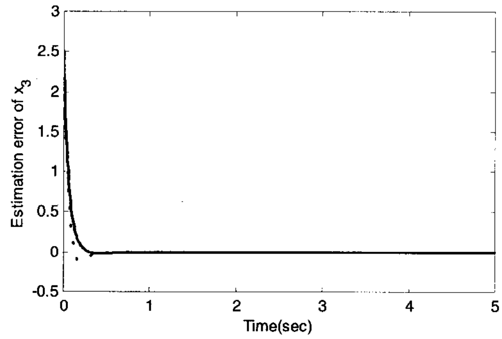

Fig. 3. The estimation error of x_3.
(solid : proposed, dashed : observer (3))

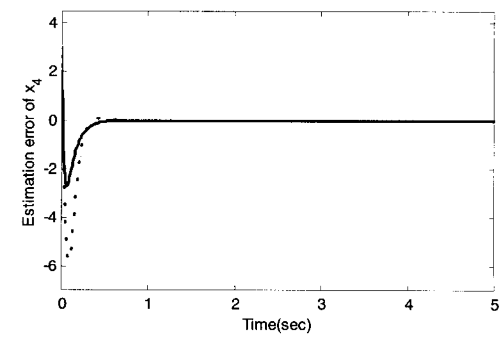

Fig. 4. The estimation error of x_4.
(solid : proposed, dashed : observer (3))

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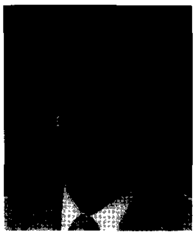

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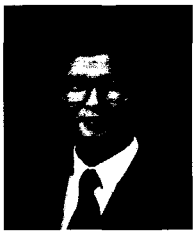

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