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Design and Application of a New Sliding Mode Controller with Disturbance Estimator

Seung-Bok Choi, Joon-Ho Ham and Jong-Sung Park

Department of Mechanical Engineering, Inha University, Incheon, South Korea

ABSTRACT

The conventional sliding mode control (SMC) technique requires a priori knowledge of the upperbounds of disturbances and/or modeling uncertainties to assure robustness. This, however, may not be easy to obtain in practical situation. This paper presents a new methodology, a sliding mode control with disturbance estimator (SMCDE), which offers a robust control performance without a priori knowledge about the disturbance. The proposed technique is featured by an average value of the imposed disturbance over a certain period. A nonlinear spring-mass-damper system and a two-link robot system are adopted as illustrative application examples. Control performances such as estimation error and tracking error are compared between the proposed methodology and conventional scheme.

Keywords : Sliding Mode Control, Disturbance Estimation, Sampling Time, Regulating Control, Tracking Control

1. Introduction

The problem of controlling nonlinear dynamical systems subjected to external disturbances has been studied for a long time. One deterministic approach to this problem is by means of a variable structure system (VSS). The VSS is a special class of nonlinear control mechanism characterized by a discontinuous control action which changes the structure upon reaching a set of sliding surfaces. During the motion on the sliding surface (sliding phase), the system has invariance properties yielding motion which is independent of certain disturbances or perturbations (Utkin, 1978). Therefore, many approaches to reduce or eliminate the reaching phase in which the VSS may be sensitive to the disturbances have been developed. One may use a high feedback gain to shorten the reaching phase. However, it causes higher chattering which is undesirable in practice [1]. Another approach to reduce or eliminate the reaching phase is to use an optimal sliding surface [2] or a moving sliding surface [3][4]. However, the design of over-conservative high feedback gains is inevitable in this approach since the upperbounds of the disturbances are normally used to formulate the sliding mode controller.

Recently, various techniques for the disturbances or perturbation estimation in VSS, which offer robust control performance without a priori knowledge about the perturbations, have been proposed to avoid the over-conservative design. Kozek et al [5] proposed a sliding mode controller associated with linear disturbance observer and proved its effectiveness by applying it to the levitation system of high speed electro-magnetic vehicles. Lu and Chen [6] proposed a perturbation estimator using the theory of VSS to enhance the robustness of a pole placement controller design. Liu and Peng [7] developed a disturbance observer by treating plant nonlinearities and parameter variations as a lumped disturbance, and showed its superior performance to the standard adaptive control scheme. Elmali and Olgac [8] proposed a very effective methodology, called a sliding mode control with perturbations estimation (SMCPE), which offers a robust feedback control with much lower gains than its conventional counterparts. This method has been successfully implemented on a two-axis planar SCARA robot [9].

The disturbance estimator proposed in this work has a similar form to the SMCPE. However, the proposed one does not include state derivative terms (included in the SMCPE) which may cause undesirable noise and chattering in the estimation process. Instead, the integrated average value of the imposed perturbations is used over a certain sampling period to avoid the noise and chattering phenomena. We call this method a sliding mode control with disturbance estimator (SMCDE). In order to demonstrate the effectiveness of the proposed methodology, two examples are adopted : a nonlinear spring-mass-damper system and a planar two-link robotic manipulator. A regulating control and a tracking control are performed for the former system and the later system, respectively. Control performances of the proposed SMCDE are compared with those of conventional scheme, SMCPE, proposed by Elmali and Olgac [8].

2. Design of SMCDE

Consider a typical second-order system subjected to an external disturbance :

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= f(x_1(t), x_2(t)) + u(t) + d(t) \end{aligned} \tag{1}$$

In the above equation, u(t) is the control input and d(t) is the external disturbance. The form of Eq.(1) is simple, but many physical systems including robotic manipulator are expressed in this form. In order to formulate a conventional sliding mode controller (SMC), we first define a sliding surface (in fact, a line in this case) by

$$s(t) = cx_1(t) + x_2(t) = 0, \quad c > 0 \tag{2}$$

Thus, s(t) dynamics can be expressed as follows :

$$\dot{s}(t) = cx_2(t) + f(x_1(t), x_2(t)) + u(t) + d(t) \tag{3}$$

Now, we can design the following SMC so that a sliding mode condition $s(t)\dot{s}(t) < 0$ can be satisfied [3] :

$$\begin{aligned} u(t) &= u_{equ}(t) - k\,\mathrm{sgn}(s(t)),\ k > |d(t)| \\ u_{equ}(t) &= -cx_2(t) - f(x_1(t), x_2(t)) \end{aligned} \tag{4}$$

In the above equation, k is a discontinuous control gain, sgn(·) is the signum function and $|\cdot|$ represents absolute value. As clearly seen in Eq.(4), the upperbound (or variation limit) of the external disturbance should be known to guarantee robust and stable control performance. However, an accurate knowledge of the upperbound may not be easy to obtain in practice. This may yield the over-conservative high feedback gains which result in undesirable control performances such as high chattering. Consequently, an accurate estimation of the disturbance is necessary to enhance the control performance.

We first design the SMCPE proposed by Elmali and Olgac [8] for the system(1) as follows :

$$\hat{u}(t) = u_{equ}(t) - k\,\mathrm{sgn}(s(t)) - d_{estimated}(t) \tag{5}$$

where

$$\begin{aligned} d_{estimated}(t) &= \dot{x}_{2(calculated)}(t) - f(x_1(t), x_2(t)) - \\ &\qquad \hat{u}(t-\delta) \\ \dot{x}_{2(calculated)}(t) &= \{x_2(t) - x_2(t-\delta)\}/\delta \end{aligned} \tag{6}$$

In the above equations, δ is the sampling time(small time step) for the estimation. Both estimation and control performances, of course, depend upon the sampling time. It is observed from Eq.(6) that since the SMCPE uses the derivative term of $x_2(t)$ in the estimation process, undesirable noise and chattering may occur. In order to resolve the drawback of the SMCPE, we propose a new methodology for the disturbance estimation.

We arrange s(t) dynamics, which includes the disturbance estimator as follows :

$$\begin{aligned} \dot{s}(t) &= cx_2(t) + f(x_1(t), x_2(t)) + u_{equ}(t) - \\ &\quad k\,\mathrm{sgn}(s(t)) - d_{estimated}(t) + d(t) \end{aligned} \tag{7}$$

Integration of the above equation from $T-\delta$ to T yields

$$\begin{aligned} s(T) &= s(T-\delta) + \\ &\int_{T-\delta}^{T} (cx_2(t) + f_1(x_1(t), x_2(t)) + u_{eq}(T-\delta) - \\ &\quad k\,\mathrm{sgn}(s(T-\delta)) - d_{estimated}(T-\delta) + d(t))dt \end{aligned} \tag{8}$$

In the above equation, the third, fourth, and fifth terms of the right-hand side remain as control input component during the integration. Thus, Eq.(8) can be rearranged by

$$\begin{aligned} &\int_{T-\delta}^{T} d(t)dt = \\ &\quad s(T) - s(T-\delta) + \delta \cdot k\,\mathrm{sgn}(s(T-\delta)) + \\ &\quad \delta \cdot d_{estimated}(T-\delta) - \\ &\quad \int_{T-\delta}^{T} (cx_2(t) + f(x_1(t), x_2(t)) + u_{equ}(T-\delta))dt \end{aligned} \tag{9}$$

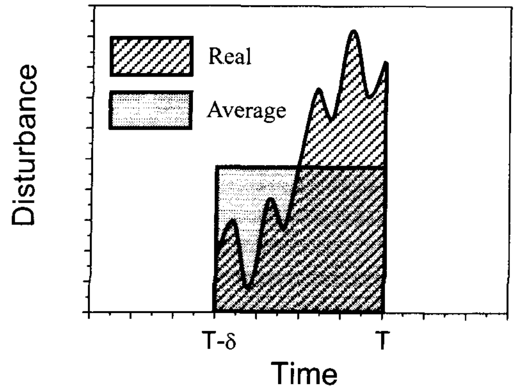


Fig. 1 Average value of the disturbance over a certain period

We now define a constant which is the same value as the left-hand side of Eq.(9) :

$$\int_{T-\delta}^{T} d_{average}(T)dt = \int_{T-\delta}^{T} d(t)dt \tag{10}$$

The graphical representation of Eq.(10) is shown in Fig.1. During the integration time, the integrated value of d(t) is equal to the integrated value of $d_{average}(T)$. Thus, the average value of the disturbance is given by

$$d_{average}(T) = \int_{T-\delta}^{T} d(t)dt \Big/ \delta \tag{11}$$

Substituting Eqs.(10) and (11) into Eq.(9) yields

$$\begin{aligned} d_{average}(T) = & \\ & \{s(T) - s(T-\delta)\}/\delta \\ & + k\,\mathrm{sgn}(s(T-\delta)) + d_{estimated}(T-\delta) \\ & - \int_{T-\delta}^{T} \left(cx_2(t) + f(x_1(t), x_2(t)) + u_{equ}(T-\delta)\right)dt \Big/ \delta \end{aligned} \tag{12}$$

The last term of the right-hand side in Eq.(12) is hard to calculate accurately. Thus, it is approximated by

$$\begin{aligned} X_c(T) = & \\ & \{cx_2(T) + f(x_1(T), x_2(T)) + u_{equ}(T-\delta)\}/2 \\ & = -\left(u_{equ}(T) - u_{equ}(T-\delta)\right)/2 \end{aligned} \tag{13}$$

Now, by substituting Eq.(13) into Eq.(12) gives the final form of the $d_{average}(T)$ as follows.

$$\begin{aligned} d_{average}(T) = & \{s(T) - s(T-\delta)\}/\delta - X_c(T) \\ & + k\,\mathrm{sgn}(s(T-\delta)) + d_{estimated}(T-\delta) \end{aligned} \tag{14}$$

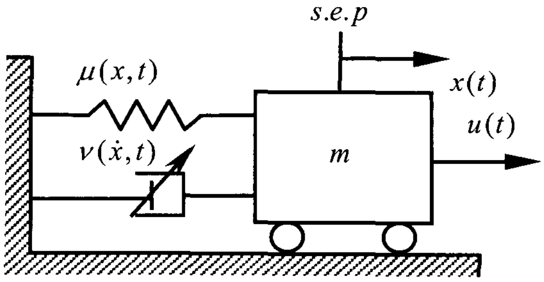


Fig. 2 A nonlinear spring-damper

Finally, $d_{average}(T)$ can be realized by the Taylor series :

$$d_{estimated}(t) = \sum_{i=0}^{n} \delta^i \cdot d_{average}^{(i)}(T)/i! \tag{15}$$

Consequently, the combination of Eqs.(5), (14) and (15) consists of the sliding mode controller with the disturbance estimator proposed in this work. It is expected that the proposed SMCDE can offer more accurate estimation than the conventional SMCPE since it does not include the derivative term of $x_2(t)$ and also it uses the integrated average value instead of a discontinuous gain.

3. Control Applications

3.1 Nonlinear Spring-Mass-Damper System

Firstly, we consider a nonlinear spring-mass-damper system shown in Fig. 2. The governing equation of motion of the system is given by [10].

$$\begin{aligned} & \dot{x}_1(t) = x_2(t) \\ & \dot{x}_2(t) = \sum_{i=1}^{4} f_i(x,t) + bu(t) + hd(t) \\ & x_1(t_0) = x_{10},\ x_2(t_0) = x_{20} \end{aligned} \tag{16}$$

where

$$\begin{aligned} & b = h = 1/m \\ & f_1(x,t) = -\mu_0 x_1(t)/m,\ f_2(x,t) = -u_1 x_1^3(t)/m \\ & f_3(x,t) = -v_0 x_2(t)/m,\ f_4(x,t) = -v_1 x_2(t)|x_2(t)|/m \end{aligned} \tag{17}$$

Now, we design three types sliding mode controllers : without disturbance estimator (SMC), the SMCPE proposed by Elmali and Olgac [8], and the SMCDE proposed in this work. The three controllers are designed as follows :

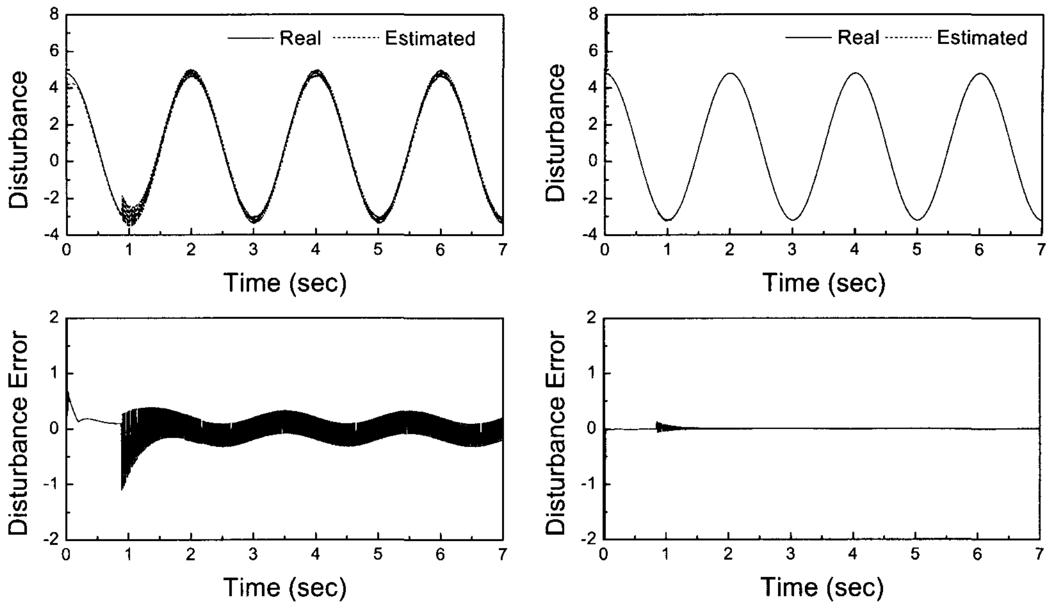


Fig. 3 Comparison of disturbance estimation result

1) SMC

$$u(t) = -\sum_{i=1}^{4} f_i(x,t) - cx_2(t) - k\,\mathrm{sgn}(s(t)) = u_{equ}(t) - k\,\mathrm{sgn}(s(t)) \tag{18}$$

2) SMCPE

$$\hat{u}(t) = u_{equ}(t) - k\,\mathrm{sgn}(s(t)) - d_{estimated}(t)$$
$$d_{estimated}(t) = \dot{x}_{2(calculated)}(t) - \sum_{i=1}^{4} f_i(x,t) - \hat{u}(t-\delta) \tag{19}$$

3) SMCDE

$$\hat{u}(t) = u_{equ}(t) - k\,\mathrm{sgn}(s(t)) - d_{estimated}(t)$$
$$d_{estimated}(T) = d_{average}(T) + \delta \cdot \dot{d}_{average}(T) = 2d_{average}(T) - d_{average}(T-\delta) \tag{20}$$
$$d_{average}(T) = \{s(T) - s(T-\delta)\}/\delta + k\,\mathrm{sgn}(s(T-\delta)) + \{u_{equ}(T) - u_{equ}(T-\delta)\}/2 + d_{estimated}(T-\delta)$$

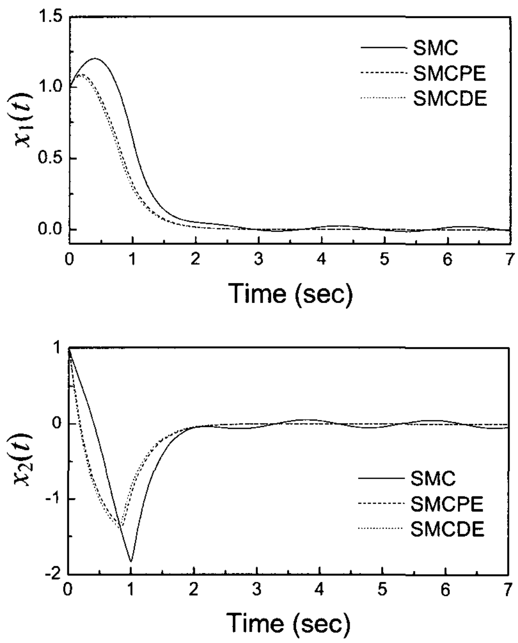


Fig. 4 Comparison of regulating responses

For computer simulation, the following parameters are used : $m=1, \mu_0=\mu_1=4, v_0=v_1=8, k=5, c=2.928$, $\delta=0.01$, $x_{10}(0)=x_{20}(0)=1.0, d(t)=0.25+4\sin(\pi \cdot t)$ Fig. 3 presents the disturbance estimation result obtained by the SMCPE and SMCDE. It is clearly observed that the SMCPE exhibits undesirable chattering due to the existence of the derivative term of $x_2(t)$. On the other hand, there is no significant estimation error in the proposed method. Fig. 4 compares the regulating control responses among the three controllers. We clearly see that the SMC, which does not have the disturbance estimator,

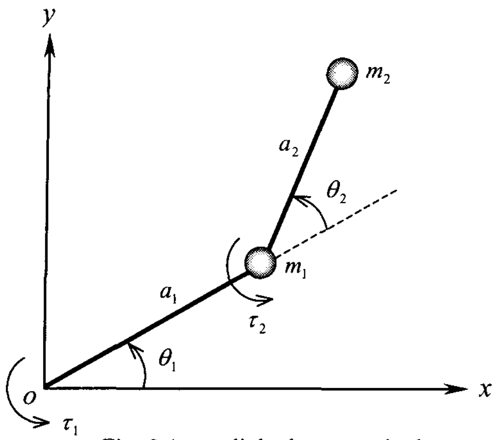


Fig. 5 A two-link planar manipulator

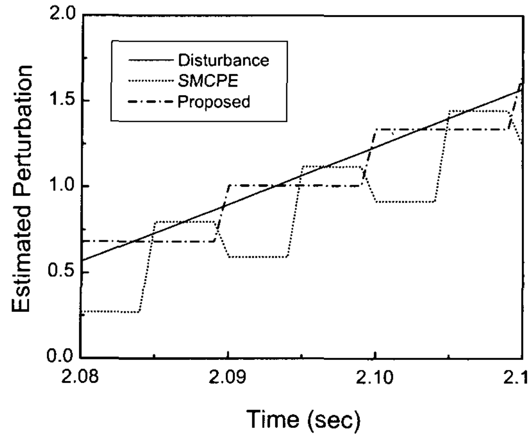
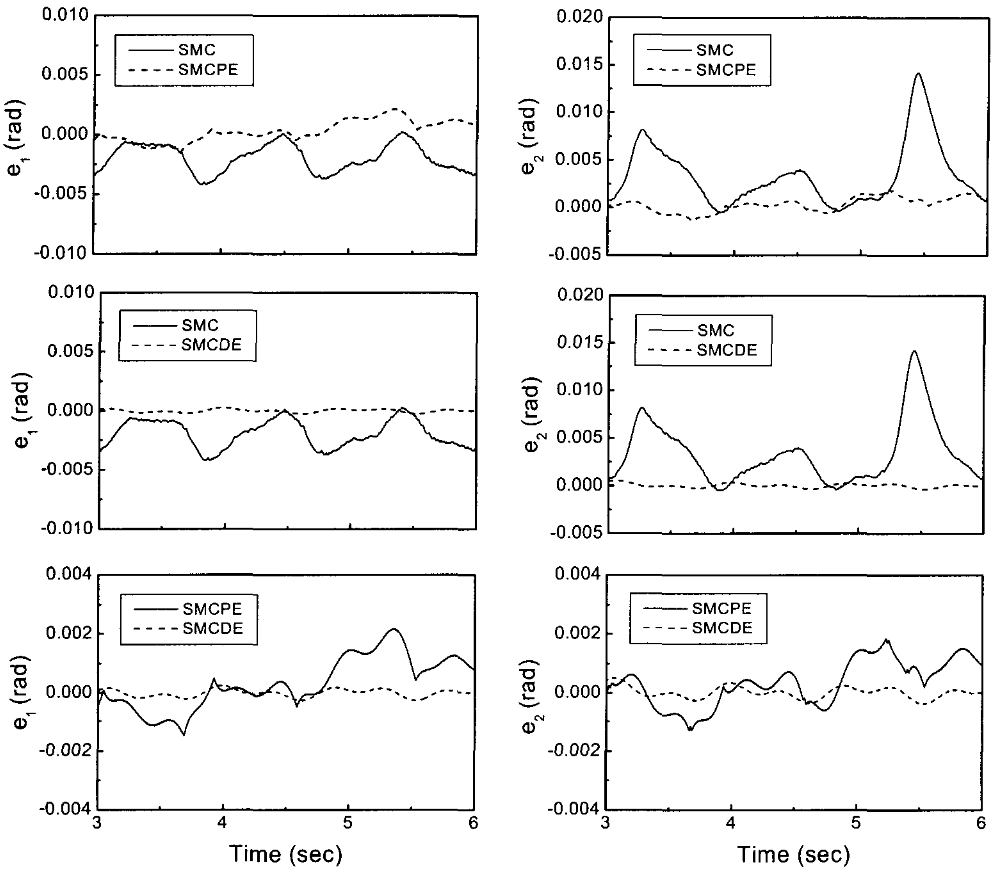


Fig. 6 Comparison of the estimation results



(a) Joint 1

(b) Joint 2

Fig. 7 Comparison of tracking errors

exhibits a relatively large variation in the steady state phase. In addition, it is observed that the proposed SMCDE offers faster and more accurate control responses than the SMCPE.

3.2 Two-Link Robotic Manipulator

As a second illustrative example, we adopt a planer two-link robotic manipulator shown in Fig. 5. The dynamic equation of the manipulator is obtained as follows [11] :

$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}=\begin{bmatrix}f_1\\ f_2\end{bmatrix}+\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}+\begin{bmatrix}d_1\\ d_2\end{bmatrix}=\mathbf{F}+\mathbf{B}\boldsymbol{\tau}+\mathbf{d}=\mathbf{F}+\mathbf{u}+\mathbf{d} \qquad (21)$$

$$\mathbf{F}=[f_1\ \ f_2]^T,\qquad \mathbf{d}=[d_1\ \ d_2]^T,\qquad \boldsymbol{\tau}=[\tau_1\ \ \tau_2]^T,$$

$$\mathbf{B}=\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix},\qquad \mathbf{u}=[u_1\ \ u_2]^T$$

$$f_1=\frac{m_2a_2(\dot{\theta}_1+\dot{\theta}_2)^2\sin\theta_2+m_2a_1\dot{\theta}_1^2\sin\theta_2\cos\theta_2}{(m_1+m_2+m_2\cos^2\theta_2)a_1}$$

$$f_2=\frac{-m_2a_2^2(\dot{\theta}_1+\dot{\theta}_2)^2\sin\theta_2-(m_1+m_2)a_1^2\dot{\theta}_1^2\sin\theta_2}{(m_1+m_2+m_2\cos^2\theta_2)a_1a_2}-\frac{m_2a_1a_2(2\dot{\theta}_1^2+2\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_2^2)\sin\theta_2\cos\theta_2}{(m_1+m_2+m_2\cos^2\theta_2)a_1a_2}$$

$$b_{11}=\frac{1}{(m_1+m_2+m_2\cos^2\theta_2)a_1^2}$$

$$b_{12}=\frac{-a_2.-a_1\cos\theta_2}{(m_1+m_2+m_2\cos^2\theta_2)a_1^2a_2}$$

$$b_{21}=\frac{-a_2-a_1\cos\theta_2}{(m_1+m_2+m_2\cos^2\theta_2)a_1^2a_2}$$

$$b_{22}=\frac{(m_1+m_2)a_1^2+m_2a_2^2+2m_1a_1a_2\cos\theta_2}{(m_1+m_2+m_2\cos^2\theta_2)m_2a_1^2a_2^2}$$

In the above equation u_i is the joint torque and d_i is the torque disturbance given by

$$d_1(t)=d_2(t)=2.5\sin(2\pi t)+1.5\cos(4.5\pi t) \qquad (22)$$

The control objective is to get $(\theta_1(t),\theta_2(t))$ to track a desired trajectory $(\theta_{d_1}(t),\theta_{d_2}(t))$. Thus, this is a tracking control problem. Since the controller formulations for the SMC, SMCPE and SMCDE are exactly the same as those for the previous example, we omit the details. For computer simulation, the following parameters are used : $m_1=m_2=a_1=a_2=1$, $k_1=k_2=5$, $c_1=c_2=4$, $\delta=0.05\,\text{sec}$ Fig. 6 compares the estimation results between the SMCPE and the proposed SMCDE. We clearly observe that the estimated disturbance obtained from the proposed method is closer to the imposed disturbance. This will directly affect the tracking control performance of the robotic system. Fig. 7 compares tracking control performances among SMC, SMCPE, and SMCDE. We see that favorable tracking performances have been achieved in all the three controllers. However, the tracking accuracies are different among them. We clearly see that the controller proposed in this work exhibits the best tracking performance in the sense of the tracking accuracy.

4. Conclusions

A new type of a sliding mode controller with disturbance estimator has been proposed and applied to two nonlinear systems: spring-mass-damper system and two-link planar robotic manipulator. The proposed estimator is featured by an integrated average value of the imposed disturbance over a certain sampling time. In addition, the proposed estimator does not include a time derivative of the highest-order state variable which may cause undesirable chattering. It has been demonstrated through computer simulations that the proposed estimation method can offer better estimation accuracy as well as better control performance than the conventional estimation technique.

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