FLUID-GRANULE MIXED FLOW DOWNSTREAM OF SCOUR HOLE AT OUTLET OF HYDRAULIC STRUCTURE

Jin-Hong Kim[1], MyungPil Shim[2], and Kyung-Sub Kim[3]

[1] Department of Civil Engineering, Chung-Ang University, Kyonggi-Do, Korea
[2] Department of Civil Engineering, Inha University, Inchon, Korea
[3] Dept. of Env. Engineering, Hankyong National University, Ansong, Korea

---

Abstract: This study presents the theoretical approach for volume concentration, velocity profile, and granular discharge on the fluid-granule mixed flow downstream of the scour hole at the outlet of the hydraulic structure. Concept of dilatant model was applied for the stress-strain relationships of fluid-granule mixed flow since the flow downstream of the scour hole corresponds to debris flow, where momentum transfers through particle collisions. Mathematical formulations were derived using momentum equation and stress-strain relation of the fluid-granule mixture. Velocity profile under the assumption of uniform concentration over flowing layer showed the downward convex type. Deposition angle of downstream hump was found to be a function of an upstream slope angle, a dynamic friction angle and a volume concentration irrespective of flow itself. Granular discharge and the overflow depth were obtained with given values of inflow rates. Experimental results showed relatively good agreements with theoretical ones.

Key Words: volume concentration, granular discharge, dilatant model, fluid-granule mixed flow

1. INTRODUCTION

Localized scour at the outlet of the hydraulic structures such as weirs, sluice gates and low dams can cause dangerous erosion problems. Jet flow, which occurs by the gate opening of such structures due to flood or for artificial sediment flushing, has a considerable potential for scour (Miwa, 1990). If the streambed is not hard or the bed protection is not sufficient, erosion occurs and a resulting scour hole is formed at the outlet of the hydraulic structure due to the high shear stress of the jet flow.

Once the scour hole is formed, a vortex occurs in it and this vortex causes additional erosion. Sediment particles eroded from the scour hole are circulated with the same direction of the vortex. They contribute the formation of a hump by deposition downstream of the scour hole. The volume concentration of the overflow in the deposition region is relatively high compared with that of the general bed load motion since the vortex has high erosion potential. The eroded sediment particles spread out to whole flowing layer uniformly due to dispersive pressure resulting from the exchange of momentum between the grains in neighboring layer. Thus the grain-inertia regime is formed and can be analyzed by the concept of the dilatant model initially proposed by Bagnold (1954). The dilatant model is based on the grain-inertia regime where the momentum transfer is made through collisions of sediment particles and the shear stress is essentially due to a grain interaction (Bagnold, 1954). He proposed the formulas on the dilatant model relating the dispersive pressure with the shear stress. Extensive studies on the dilatant model were carried out by several authors (Hanes and Inman, 1985; Kato et al., 1985; Shao et al., 1990). A comprehensive survey of the mechanics for the dilatant model was given by Savage (1984).

The main purpose of this study is to formulate the mathematical expressions for the volume concentration, velocity profile, and granular discharge downstream of the scour hole at the outlet of the hydraulic structure. Since the overflow corresponds to the fluid-granule mixed flow with sufficient water to disperse sediment grains uniformly throughout the whole depth, theoretical analysis were established using the concept of dilatant model.

2. THEORETICAL APPROACH ON FLUID-GRANULE MIXED FLOW DOWNSTREAM OF SCOUR HOLE

2.1 Governing Equations of Fluid-Granule Mixed Flow

Consider the two-dimensional shear flow of a constant depth h_1 along the downstream hump at an angle θ_1 to the horizontal in region Ⅲ where the volume concentration over depth y is almost constant as shown in Fig. 1.

During the deposition process, the hump height remains constant. This means that the deposition occurs in region Ⅱ. So, there occurs both the erosion and the deposition simultaneously in region Ⅱ, but the erosion rate is larger than the deposition rate. While in region Ⅲ, the dynamic equilibrium state is formed where the erosion and the deposition occurs simultane-

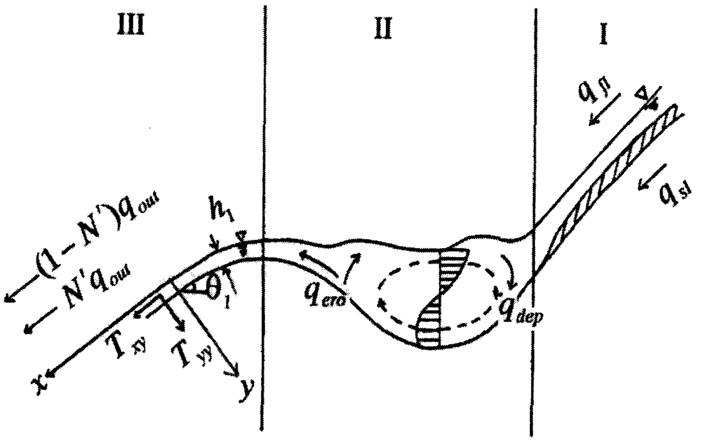

Fig. 1. Two-Dimensional Shear Flow

ously and their rates are almost same. The volume concentration of the overflow in region Ⅲ is relatively high since the eroded sediment particles spread out to whole flowing layer uniformly due to dispersive pressure. The granular jump and the line vortex occurred in the scour hole also act the accelerating effect on spreading out of sand particles (Kim and Choe, 2000). Thus, the grain-inertia regime is formed downstream of the scour hole and the flow can be analyzed by the concept of the dilatant model.

Continuity equation of fluid discharge is given as

$$q_{fl} = (1 - N')q_{out} \tag{1}$$

assuming that the storage effect in the scour hole of region II is small and the volume concentration over depth y is nearly the same. Here q_{fl} is the fluid discharge in region I, N' is the volume concentration and q_{out} is the disharge of fluid-granule mixed flow.

Continuity equation of granular discharge is given as

$$q_{sI} + q_{ero} - q_{dep} = N' q_{out} \tag{2}$$

where q_{sI} is the granular discharge in region I, q_{ero} is the vortex erosion in the scour hole and q_{dep} is the deposition which contributes to the formation of hump.

Two momentum equations of the fluid-granule mixed flow are given as

$$\begin{aligned} 0 &= \partial T_{xy} / \partial y + \rho_1 g \sin\theta_1 \\ 0 &= \partial T_{yy} / \partial y + (\rho_1 - \rho_f) g \cos\theta_1 \end{aligned} \tag{3}$$

where T_{xy} is a granular shear stress, T_{yy} is a granular normal stress, $\rho_1 = \rho_s + \rho_f(1 - N')$, ρ_s and ρ_f are granular and fluid density, respectively, and θ_1 is the slope of a downstream hump.

Stress-strain relations for the rapid shear

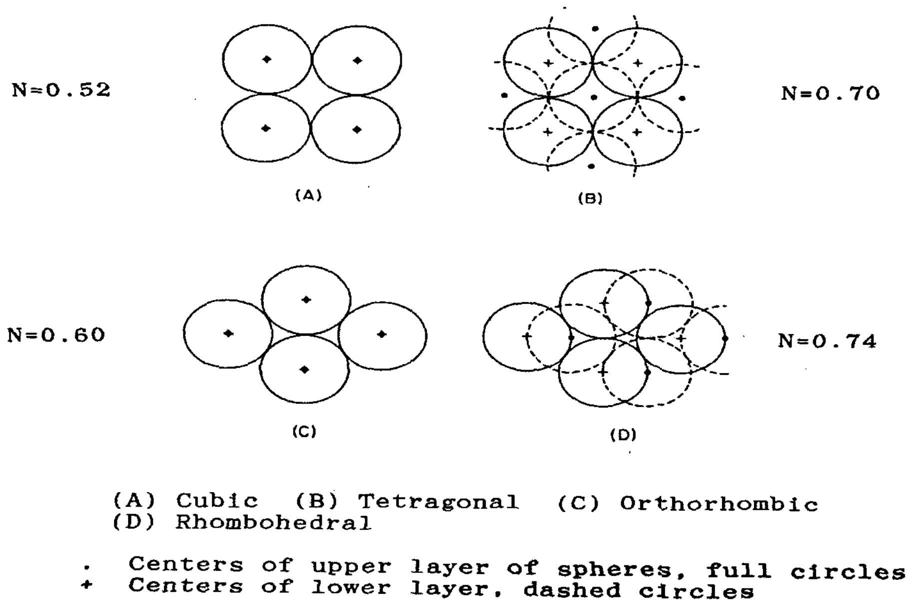

Fig. 2. Examples of Volume Fraction on Packing of Spherical Sediment Grains (Shibata and Mei, 1986)

flows of the grain-inertia regime are given as (Shibata and Mei, 1986)

$$T_{xy} = \mu_1 \left|\partial u / \partial y\right| \partial u / \partial y = -\mu_1 (\partial u / \partial y)^2$$
$$T_{yy} = -\alpha(b^{-1}+1)(\partial N' / \partial y)^2 - \mu_0 (\partial u / \partial y)^2 \quad (4)$$

where $\mu_0 = \beta_0 \{(N_\infty - N_0)/(N_\infty - N')\}^8$,
$\mu_1 = \beta_1 \{(N_\infty - N_0)/(N_\infty - N')\}^8$,

$b = k(N' - N_0)$, k, α, β_o and β_1 are constants. N_∞ and N_0 are volume fractions at the densest packing and at the top of mixture, respectively. Examples of the volume fractions on packing of the spherical sediment grains are shown in Fig. 2 (Shibata and Mei, 1986).

2.2 Volume Concentration of Fluid-Granule Mixed Flow

The volume concentration over depth is nearly the same with the effect of the line vortex and the granular jump. Thus from eq. (3), following equations for the granular shear stress and the granular normal stress can be given as

$$T_{xy} = -g\int_0^y \rho_1 \sin\theta_1 dy = -\rho_1 g y \sin\theta_1$$
$$T_{yy} = -g\int_0^y (\rho_1 - \rho_f)\cos\theta_1 dy = -(\rho_1 - \rho_f) g y \cos\theta_1 \quad (5)$$

Equations on the stress-strain relations of fluid-granule mixed flow can be simplified as follows with the assumption abovementioned.

$$T_{xy} = -\mu_1 (\partial u / \partial y)^2, \quad T_{yy} = -\mu_0 (\partial u / \partial y)^2 \quad (6)$$

Bagnold (1954) found that the viscosity ratio of the granular shear stress to the granular normal stress is virtually independent of the volume fraction and approximately equal to a dynamic friction angle ϕ_D. Thus,

$$T_{xy} / T_{yy} = \mu_1 / \mu_0 = \tan\phi_D \quad (7)$$

So, from eqs. (5) and (7), following equation for the volume concentration is obtained

$$\rho_1 /(\rho_1 - \rho_f) = \tan\phi_D / \tan\theta_1$$
$$N' = \{\rho_f - /(\rho_s - \rho_f)\} \cdot \{1/(\tan\phi_D / \tan\theta_1 - 1)\} \quad (8)$$

According to Egashira et. Al (1988), the equation for the deposition angle of the debris flow in open channel is given as

$$\theta_1 = \tan^{-1}[(\rho_s - \rho_f)N' \cos\theta \tan\phi_D$$
$$/\{(\rho_s - \rho_f)N' + \rho_f\}] \quad (9)$$

where θ is an upstream slope angle of the debris deposits. Eq. (9) means that deposition angle is a function of an upstream slope angle, a dynamic friction angle and the volume concentration irrespective of flow itself. Eq. (9) is the same as eq. (8) if the term $\cos\theta$ is omitted. In this study, $\cos\theta = 1.0$ and there is no difference between these equations. Thus, it can be said that the deposition angle of the downstream hump is related to sediment properties not to flow features in case that the upstream bed is flat.

2.3 Velocity Profile of Fluid-Granule Mixed Flow

To determine the vertical velocity profile over the depth y downstream of the scour hole, following equation is obtained from eqs. (5) and (6)

$$\partial u / \partial y = -(\rho_1 g \sin\theta_1 / \mu_1)^{1/2} y^{1/2} \quad (10)$$

Integration of the above equation gives

$$u = 2/3 \cdot (\rho_1 g \sin\theta_1 / \mu_1)^{1/2} {h_1}^{3/2} - y^{3/2} \quad (11)$$

using boundary condition $u = 0$ at $y = h_1$.

Here h_1 is the depth of flowing layer. The surface velocity u_0 at $y = 0$ is,

$$u_0 = 2/3 \cdot (\rho_1 g \sin\theta_1 / \mu_1)^{1/2} h_1^{3/2} \tag{12}$$

Thus, the non-dimensional velocity profile becomes

$$u/u_0 = 1 - (y/h_1)^{3/2} \tag{13}$$

2.4 Fluid and Granular Discharge

The outflow q_{out} of fluid-granule mixed flow in region III of Fig. 1 is given

$$\begin{aligned} q_{out} &= \int_0^{h_1} u\,dy \\ &= 2/5 \cdot (\rho_1 g \sin\theta_1 / \mu_1)^{1/2} h_1^{5/2} \end{aligned} \tag{14}$$

Thus, the fluid discharge q_f and the granular discharge q_s are given

$$\begin{aligned} q_f &= (1 - N')q_{out} \\ &= 2/5 \cdot \{1 - \rho_f/(\rho_s - \rho_f) \cdot 1/(\tan\phi_D / \tan\theta_1 - 1)\} \\ &\quad (\rho_1 g \sin\theta_1 / \mu_1)^{1/2} h_1^{5/2} \end{aligned} \tag{15}$$

$$\begin{aligned} q_s &= N' q_{out} \\ &= 2/5 \cdot \{\rho_f/(\rho_s - \rho_f) \cdot 1/(\tan\phi_D / \tan\theta_1 - 1)\} \\ &\quad (\rho_1 g \sin\theta_1 / \mu_1)^{1/2} h_1^{5/2} \end{aligned} \tag{16}$$

3. EXPERIMENTAL APPRATUS AND PROCEDURES

Experiments in a movable model were performed to investigate the hydraulic characteristics of fluid-granule mixed flow downstream of the scour hole. Fig. 3 shows an experimental arrangement.

A scour hole and a hump were made of sands inside the recirculating channel of 0.4m wide and 10m long. The median diameter of sands is about 0.5mm. The side wall of the channel was made of glass and a transparent scale was attached to the side wall to check the physical features of the fluid-granule mixed flow. The channel discharge controlled by the valve in a feed back loop could be measured with the manometer attached to a return pipe. Since the length between the upstream end of the channel and the location of a scour hole was relatively short, a thick screen was installed at the entrance of the channel. A video camera and a time-lapsed video tape recorder were used to measure the deposition angle and the overflow depth. The test section was illuminated by strong light for clean view. An electro-optical concentration meter was used to measure the sediment concentration in the state of the fluid-granule mixed flow. The analog signals obtained from the elec-

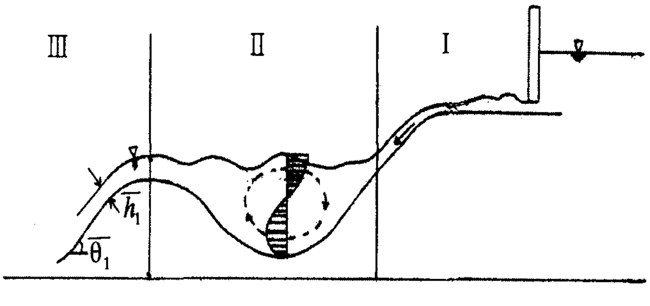

Fig. 3. Experimental Arrangement

tro-optical concentration meter were converted into digital signals, and then collected in the computer (Kim and Kim, 1999).

Measurements through a video recording were carried out every 10 seconds in order to estimate the time development of the scour hole and the hump.

4. EXPERIMENTAL RESULTS AND COMPARISONS

Flow over the hump downstream of a scour hole forms a fluid-granule mixed flow. The tractive stress is very large, resulting in high erosion potential. The hump becomes larger with time, but its height remains almost constant during the

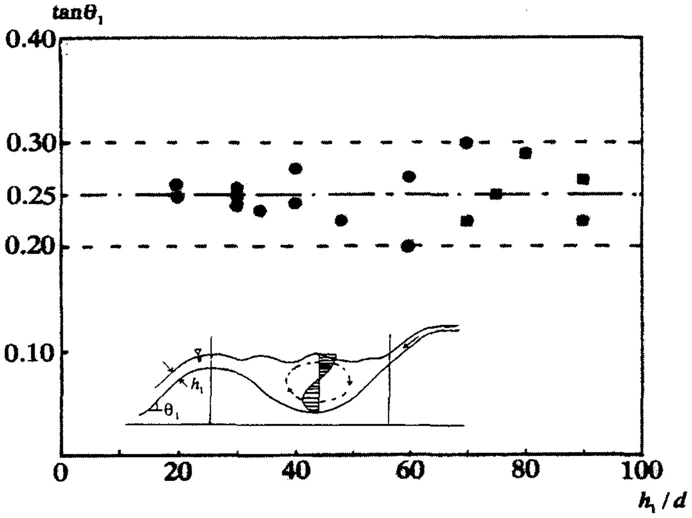

Fig. 4. Deposition Angle of Downstream Hump

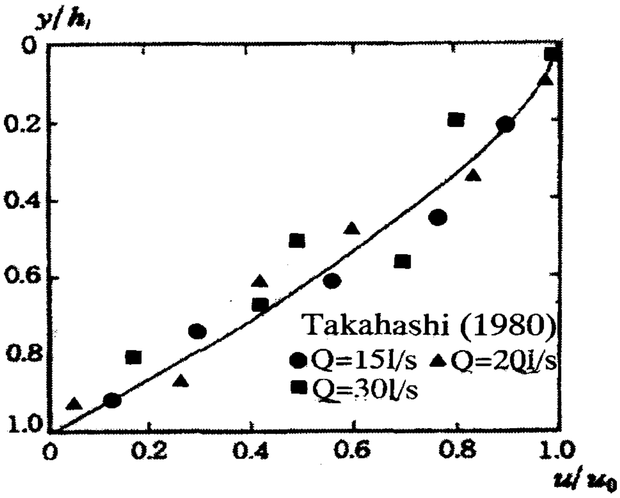

Fig. 5. Non-Dimensional Velocity Profile and Experimental Results

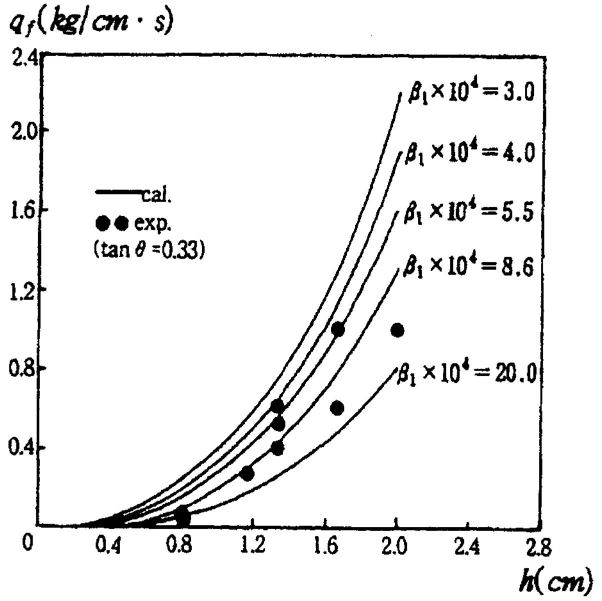

Fig. 6. Fluid Discharge Downstream of Scour Hole

erosion of the scour hole. This means that the deposition occurs only at the front side of the hump. The eroded sand particles from the scour hole spread out to a whole flowing layer with the effect of the granular jump and the line vortex. Momentum transfer is made through the collisions of sand particles and hence the grain-inertia regime on a fluid-granule mixed flow can be assumed.

Experimental results on the deposition angle of downstream hump are shown in Fig. 4. It can be said that the deposition angle lies between 0.20 and 0.30 irrespective of flow features, which shows the validity of eq. (9). In Fig. 4, h_1 is the overflow depth and d is the diameter of sediment particle.

Fig. 5 shows the non-dimensional velocity profile for eq. (13) and the Takahashi(1980)'s experimental results. It can be seen that the velocity profile is characterized by downward convex type over the whole layer. A fairly good agreement is seen between the theoretical curve and the experimental results although the variation of large discharge is somewhat large.

Fig. 6 shows the comparison between theoretical results of eq. (15) for some different values of β_1 and experimental results. Fairly good agreements can be seen in case $\beta_1=8.6\times10^{-4}$. From these results, it can be roughly said that the overflow depth and the granular discharge are estimated with given values of inflow rates.

5. CONCLUSIONS

Mathematical formulations of the volume concentration, the velocity profile and the granular discharge over the flowing layer of deposition hump downstream of the scour hole were derived using momentum equation and the stress-strain relationships of the fluid-granule mixed flow. Velocity profile under the assumption of uniform concentration over flowing layer showed the downward convex type and showed good agreement with experimental results. Deposition angle of downstream hump was found to be a function of an upstream slope angle, a dynamic friction angle and a volume concentration irrespective of flow itself. The granular discharge and the overflow depth were estimated with given values of inflow rates.

ACKNOWLEDGEMENT

This research was supported by a grant (code 2-5-1) from Sustainable Water Resources Research Center of 21st Century Frontier Research Program.

REFERENCES

Bagnold, R.A. (1954). "Experiments on a gravity free dispersion of large solid spheres in a Newtonian fluid under shear." *Proceedings of the Royal Society of London*, London, England, Vol. 225, pp. 49-63.

Egashira, S., Ashida, K., and Sasaki, H. (1988). "Mechanics of debris flow in open channel." *Proceedings 32th Hydraulic Conference of JSCE*, Tokyo, Japan, Vol. 1, pp. 485-490.

Hanes, D.M. and Inman, D.L. (1985). "Observations of rapidly flowing granular-fluid materials." *Journal of Fluid Mechanics*, Vol. 150, pp. 357-387.

Kato, Y., Hashimoto, H., and Fujita, K. (1985). "Experimental study on levee reinforcement against failure by overtopping." *Proceedings of 29th Hydraulic Conference of JSCE*, Osaka, Japan, pp. 627-632.

Kim, J.H. and Choe, J.W. (2000). "Vortex structure in the scour hole by gate opening of hydraulic structure." *Water Engineering Research,* KWRA, Vol. 1, No. 1, pp. 83-92.

Kim, J.H. and Kim, J.S. (1999). "Sheet erosion in embankment of noncohesive materials due to unsteady slow." *Journal of Civil Engineering,* KSCE, Vol. 3, No. 1, pp. 58-69.

Miwa, H. (1990). "Process of flushing sands accumulated under gates on a dam by sluice outlet." *Proceedings 34th Hydraulic Conference of JSCE*, Tokyo, Japan, Vol. 1, pp. 247-252.

Savage, S.B. (1984). "The mechanics of rapid granular flows." *Advances in Applied Mechanics*, Edited by Wu. T.Y. and Hutchinson. J., Vol. 24, Academic Press, pp. 289-366.

Shao, X., Tanaka, H., and Shuto, N. (1990). "Experimental study on scouring of a sand bar by overflow." *Proceedings of 34th Hydraulic Conference of JSCE*, pp. 373-378.

Shibata, M. and Mei, C.C. (1986). "Slow parallel flows of a water-granule mixture under gravity, part Ⅱ: Examples of free surface and channel flows." *Acta Mechanica*, Vol. 63, pp.195-216.

Takahashi, T. (1980). "Debris flow on prismatic open channel." *Journal of Hydraulic Engineering*, ASCE, Vol. 196, No. 4, pp. 381-396.

Jin-Hong Kim, Department of Civil Engineering, Chung-Ang University, Ansong, Kyonggi-Do, Korea.
(E-mail : jinhkim@cau.ac.kr)

MyungPil Shim, Department of Civil Engineering, Inha University, Inchon, Korea.
(E-mail : shim@inha.ac.kr)

Kyung-Sub Kim, Dept. of Envir. Engineering, Hankyong National University, Ansong, Korea.
(E-mail : kskim@hnu.hankyong.ac.kr)