

## 확률적 신경망 모델에서 느린 금지뉴런의 역할

박정주 · 신인선 · 박광석

서울대학교병원 의공학연구소, 한국교원대학교 수학교육과, 서울대학교병원 의공학과  
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### The Role of Slow Inhibitory Neurons in a Stochastic Neural Network Model with IF Neurons

C. J. Park\*, In Sun Shin\*\* and Kwang Suk Park\*\*\*

\*Institute of Medical and Biological engineering, Medical Research Center, Seoul National Univ.

\*\*Department of Mathematics Education, Korea National University of Education, Korea

\*\*\*Department of biomedical Engineering, Seoul National University.

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**요약**: 일반적으로 금지뉴런의 효과는 신경망을 안정시킨다고 알려져 있다. 본 연구에서는 확률적 평균 필드 이론에 근거한 신경망 모델에서 느린 금지뉴런의 역할을 살펴보았다. 흥분뉴런과 빠른 금지뉴런으로 구성된 신경망에 느린 금지뉴런을 더하면, 느린 금지 뉴런이 없는 모델에서보다 매우 낮은 역치에서 안정적인 동시적 활동이 유도된다는 것을 발견하였다. 이 역치는 대뇌 피질 신경의 생리학적 역치와 일치하며, 느린 금지 뉴런만이 신경망에 낮은 발화율과 낮은 역치를 유지시키는 네거티브 피드백을 줄 수 있다.

**Abstract**: We have investigated the role of slow inhibitory neurons in spontaneous activity using a model network controlled by stochastic mean field theory based on Integrated-and-Fire excitatory and fast inhibitory neurons. It is found that inputting slow inhibitory neurons to such network induces stable spontaneous activity at a much lower threshold than without slow inhibitory neurons in the network. This threshold range is low enough to be considered as biological threshold of cortical neurons. Only slow inhibitory neurons can give adjustable negative feedback in the network keeping lower rate and lower threshold.

## INTRODUCTION

In the brain there exist several kinds of neurons. The role of each of neurons has not been completely studied and is still under debate in many points. Especially several kinds of inhibitory neurons exist in cerebral cortex in brain. The inhibitory neurons are mediated by different kinds of neurotransmitters, GABAergic neurotransmitters. While fast

inhibitory neurons are mediated by GABA-A, slow inhibitory neurons are mediated by GABA-B. The integration time constants of each kinds neurons are diverse from 5~10 ms for fast inhibitory neurons to 100~1000 ms for slow inhibitory neurons. Many recent experiments have shown that the inhibitory neurons are involved in the stability of cortical network, long term adaptation, and high frequency bursting mode of brain rhythm in cerebral cortex or hippocampus [1,2,3,4,5,6].

Generally in neural network modeling many studies have described theoretically that the inhibitory neurons play a crucial role in giving stability to the network, and have proved that without inhibitory feedback a network can not converge to a stable state that is biologically

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통신저자: 박정주, (110-7444) 서울특별시 종로구 연건동 28번지  
서울대학교병원 의공학연구소

Tel. (02)760-3135, Fax. (02)745-7870

E-mail. punica@snuvh.snu.ac.kr

meaningful stable activity [1,2,7].

Neurophysiological experiments show two kinds of activities: spontaneous activity and selective delay activity in the cortex of monkey while performing cognitive tasks involving delayed response [8,9,10].

In this study we have investigated the effect of slow inhibitory neurons in the stochastic neural network model which has been proposed to show the coexistence of both types of activities by Amit [1]. This neural model had already been considered excitatory and inhibitory feedback, however the stable solution does not exist at the range of biological threshold, in which a neuron can fire after receiving 100~300 synaptic inputs[2]. As considering the slow inhibitory feedback in theoretical model network we conjecture that the role of slow inhibitory neuron is deeply related to the stability and the robustness of brain information processing.

In section 2, the model network will be described; here we consider only spontaneous activity prior to learning. The study of slow inhibitory neurons for selective delay activity will be handled in elsewhere. We chose the integration time constant as 100 ms. In section 3, the effect of inputting slow inhibitory neurons will be described.

## DESCRIPTION OF THE MODEL NETWORK

The model network is composed of a local module of about  $10^5$  neurons which consists of three kinds of neural populations: excitatory neurons, fast and slow inhibitory neurons. This local module has external noisy input from outside of module. Without considering slow inhibitory neurons, this kind of model network has already been formulated by Amit and Brunel [1]. It is known that each neuron in cerebral cortex receives  $C \sim 20000$  synaptic contacts from other cells [2].

In this model network, every neuron has synaptic input from excitatory and inhibitory cells. The number of synaptic contacts  $C_{iE}$  is about 20000, the numbers from fast inhibitory cells  $C_{iI}$  is 1000 and  $C_{iG}$  from slow inhibitory cells is also 1000. A fraction  $x$  of excitatory contacts comes from the local module, while remaining fraction  $(1-x)$  of excitatory connections comes from outside the local network, and are activated at the rate  $\nu_{ext}$ .

Each neuron in the local network is characterized by its depolarization at the soma  $V(t)$ , which obeys the integrator equation:

$$\tau \dot{V}(t) = -V(t) + I(t) \quad (1)$$

where  $\tau$  ( $\sim 100$  ms) is the integration time constant of the membrane depolarization at the soma, and  $I(t)$  is the synaptic current charges developed from the part of the soma. Both excitatory and inhibitory neurons are leaky integrated-and-fire neurons. Note that the current is expressed in the unit of potential. The absolute refractory period  $\tau_0$  ( $\sim 2$  ms) follows the emission of a spike when depolarization reaches the threshold  $\theta$ . The effective afferent current obeys the equation:

$$\tau' \dot{I}(t) = -I(t) + \sum_{i=1}^G J_i \tau \sum_k \delta(t_i^k - t) \quad (2)$$

where  $\tau'$  is the time constant of the conductance changes at the synaptic sites,  $i$  goes over the synaptic sites on the dendrites, the sum over  $k$  is over all spikes arriving at a given site, and  $t_i^k$  is the time of arrival of spike number  $k$  at synapse  $i$ .

If one assumes  $\tau' \ll \tau$ , Eq.(2) is reduced to:

$$\tau \dot{V}(t) = -V + \sum_{i=1}^G J_i \tau \sum_k \delta(t_i^k - t) \quad (3)$$

The depolarization  $V(t)$  is governed by Eq.(3), and a spike is emitted every time when the depolarization  $V(t)$  reaches the threshold. The output spike rate is given by:

$$\nu_{i,out} = \phi_i(\mu_i, \sigma_i) = (\tau_0 + \tau_i \int_{\frac{H_i - \mu_i}{\sigma_i}}^{\frac{\theta_i - \mu_i}{\sigma_i}} d\mu \psi(\mu))^{-1}, \quad i = E, I$$

with

$$\psi(\mu) = \sqrt{\pi} \exp(\mu^2) [1 + \text{erf}(\mu)], \quad (4)$$

where  $\nu_{i,out}$  is the output spike rate at synapse  $i$ ,  $\sigma_i$ , and  $\mu_i$  are mean afferent and its standard deviation in the absence of a threshold respectively,  $\theta_i$  is the threshold,  $H_i$  is the post-spike hyper polarization, and  $\tau_i$  is integration time constant characterized by the type of neurons, with  $i=E, I, G$  indicating whether the neuron is excitatory, fast or slow inhibitory neuron, respectively.  $\mu$  is the rate-dependent mean of the Gaussian distribution of afferents depolarizing the cell and  $\text{erf}(\mu)$  is the error function defined by the integral,  $\text{erf}(\mu) = \frac{2}{\sqrt{\pi}} \int_0^\mu \exp(-w^2) dw$ .

Prior to learning the network parameters are set for

the model:

1. Three types of neurons receive the equal number of excitatory synaptic input from inside the local module and from outside:  $x=0.5$  (the fraction of the excitatory contacts from local module)
2. The post-spike hyper polarizations  $H_i$  are set to zero for  $i$ 's.
3. The relative standard deviation of synaptic efficacies are equal to  $\delta=1$  for all synaptic types.
4. The ratio of average strengths of the fast inhibitory to the excitatory synapses are chosen as:  $\frac{J_{EI}}{J_{EE}} = \frac{J_{II}}{J_{IE}} = g_1$ , the slow inhibitory to the excitatory synapses are chosen as:  $\frac{J_{EG}}{J_{EE}} = \frac{J_{GG}}{J_{GE}} = g_2$ , and the fast inhibitory to the excitatory synapses are chosen as:  $\frac{J_{GI}}{J_{EE}} = \frac{J_{IG}}{J_{IE}} = g_3$ .

The mean field theory considered is based on the neural spike rate assuming that each neural spike is independent Poissonian train of spikes. If a neuron receives a large number of independent spike trains from synaptic contacts, then the depolarization  $I(t)$  has Gaussian distribution with mean afferent  $\mu$  and standard deviation  $\sigma$  [1,11,12].

In the mean field theory the output rate of a neuron is determined by mean  $\mu_i$  and standard deviation  $\sigma_i$  of its inputs in interval  $\tau_i$  for this model by introducing adjustable slow inhibitory feedback.

Those mean input and standard deviations of membrane potential of excitatory cells are determined from the three types of synaptic inputs:

$$\mu_E = C_{EE} J_{EE} [x\nu_E \tau_E + \nu_{ext} \tau_E] - C_{EI} J_{EI} \nu_I \tau_E - C_{EG} J_{EG} \nu_G \tau_E \quad (5)$$

$$\sigma_E^2 = (1 + \Delta^2)(C_{EE} J_{EE}^2 [x\nu_E \tau_E + \nu_{ext} \tau_E] + C_{EI} J_{EI}^2 \nu_I \tau_E + C_{EG} J_{EG}^2 \nu_G \tau_E) \quad (6)$$

, where  $C$  is the number of synapses per neuron,  $J$  is the mean of efficacies of the synapses, and  $\nu_{ext}$ ,  $\nu_I$ ,  $\nu_G$  are the rates of excitatory, fast inhibitory and slow inhibitory neurons in turn.

For a fast inhibitory neurons:

$$\mu_I = C_{IE} J_{IE} [x\nu_E \tau_I + \nu_{ext} \tau_I] - C_{II} J_{II} \nu_I \tau_I - C_{IG} J_{IG} \nu_G \tau_I \quad (7)$$

$$\sigma_I^2 = (1 + \Delta^2)(C_{IE} J_{IE}^2 [x\nu_E \tau_I + \nu_{ext} \tau_I] + C_{II} J_{II}^2 \nu_I \tau_I + C_{IG} J_{IG}^2 \nu_G \tau_I) \quad (8)$$

For a slow inhibitory neurons:

$$\mu_G = C_{GE} J_{GE} [x\nu_E \tau_G + \nu_{ext} \tau_G] - C_{GI} J_{GI} \nu_I \tau_G - C_{GG} J_{GG} \nu_G \tau_G \quad (9)$$

$$\sigma_G^2 = (1 + \Delta^2)(C_{GE} J_{GE}^2 [x\nu_E \tau_G + \nu_{ext} \tau_G] + C_{GI} J_{GI}^2 \nu_I \tau_G + C_{GG} J_{GG}^2 \nu_G \tau_G) \quad (10)$$

Self reproducing rates for neurons of the three populations are given by the simultaneous solution of the coupled equations:

$$\nu_E = \phi_E(\mu_E, \sigma_E) \quad (11)$$

$$\nu_I = \phi_I(\mu_I, \sigma_I) \quad (12)$$

$$\nu_G = \phi_G(\mu_G, \tau_G) \quad (13)$$

The dynamical equation, Eq.(1), implies the time dependence of  $\mu$  and  $\sigma$  of the depolarization in Ref. [2], i.e. in which  $\nu_E$ ,  $\nu_I$ , and  $\nu_G$  are given as function of  $\mu$  and  $\sigma$  via Eq.(4). The self-reproducing rates are the stable fixed points of the following dynamical equations:

$$\partial_t (\mu [V_E]^2) = -\mu [V_E] + C_{EE} J_{EE}^2 [x\nu_E \tau_E] - C_{EI} J_{EI}^2 \nu_I \tau_E - C_{EG} J_{EG}^2 \nu_G \tau_E \quad (14)$$

$$\partial_t (\sigma [V_E]^2) = -2\sigma [V_E]^2 + C_{EE} J_{EE}^2 [x\nu_E \tau_E + \nu_{ext} \tau_E] - C_{EI} J_{EI}^2 \nu_I \tau_E - C_{EG} J_{EG}^2 \nu_G \tau_E \quad (15)$$

The inhibitory neurons have similar dynamical equations.

Under numerical analysis with the help of computer program, programmed by one of authors, we can find the stable fixed point produced by the dynamical variation. Here, prior to learning we chose common parameter for the model:  $\tau_E = 10$  ms for excitatory neurons,  $\tau_I = 0.5 \tau_E$  for fast inhibitory neurons, and  $\tau_G = 100$  ms for slow inhibitory neurons.

## RESULTS AND DISCUSSION

The issue is whether the slow inhibitory feedback considered can bring to the biological range of the threshold that a neurons can fire after receiving 100~300 synaptic inputs. Previous models which does not consider

slow inhibitory feedback have very higher threshold beyond biological range even though the low spontaneous activity is stable [1].

We have found several parameter sets which can give low threshold of depolarization. We will use simple vector notation for describing parameters:  $T=(\tau_E, \tau_I, \tau_G)$  in unit ms is for the integration time constant,  $C=(C_{iE}, C_{iI}, C_{iG})$  is for the number of the synaptic contact,  $g=(g_1, g_2, g_3)$  is for the ratio of average strengths of the inhibitory to the excitatory synapse,  $\nu=(\nu_E, \nu_I, \nu_G)$  in Hz is for the mean rates of neurons of the three populations, and  $\theta=(\theta_E, \theta_I, \theta_G)$  in J is for the threshold.

For parameter set  $T=(10, 5, 100)$ ,  $C=(20000, 1000, 2000)$ , and  $g=(3, 4, 4)$  Low spontaneous rates for the three populations are  $\nu=(3, 6, 7.5)$  and their thresholds are  $\theta=(291.67, 181.43, 162.53)$ . All thresholds for three kinds of neurons stay at the range, 100~300 times of synaptic efficacy. The rate of slow inhibitory is higher in comparison with the rates of other neurons. As we can see in Eq.(5), due to the long integration time the input from slow inhibitory neurons can contribute to reduce mean potential with low spike rate. Simply inputting more fast inhibitory neurons to make mean potential lower causes high rates of fast inhibitory neurons due to short integration time. Only slow inhibitory neurons can give adjustable negative feedback without spoiling stability of network keeping lower rate and lower threshold.

In the view point of network, general inhibitory feedback controls the stability of network regardless of the magnitude of a threshold. In biological sense a high threshold means that it needs many synaptic inputs to generate spikes related to an information processing. It implies that neural system needs more energy and more time to perform a neural task in the brain.

We conjecture that the slow inhibitory neurons in the brain are also involved in minimizing energy and time to give much higher efficiency of information processing. After learning the slow inhibitory neuron may also play crucial role to lowering selective delay activity with the same mechanism shown here.

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