

# Maximum Entropy Spectral Analysis for Nonstationary Random Response of Vehicle

최대 엔트로피 스펙트럼 방법을 이용한 차량의 과도 응답 특성 해석

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## ABSTRACT

In this paper the nonstationary response of accelerating vehicle is firstly obtained by using nonstationary road roughness model in time domain. To get the result of nonstationary response in frequency domain, the maximum entropy method is used for processing nonstationary response of vehicle in frequency domain. The three-dimensional transient maximum entropy spectrum(MES) of response is given.

## 요 약

주행중인 차량의 동적 거동에 대한 응답을 해석하기 위하여 시간영역에서 뿐만 아니라 주파수 영역에서의 해석이 필요하다. 주파수 영역에서의 해석을 위하여 시간영역에서의 값을 FFT를 이용하여 주파수 영역으로 변화하는 방법이 일반적으로 사용되어 왔다. 본 연구에서는 최대 엔트로피 방법을 이용하여 기존의 FFT 방법 보다 차량의 과도응답특성을 시간영역 및 주파수 영역에서 편리하게 해석할 수 있는 방법을 제시하고 있다.

## 1. Introduction

It is well known that most researches of random vibration of vehicle caused by the excitation of rough road in frequency domain were based on two approaches. One approach is a direct method in which frequency characteristics can be obtained

directly by using transfer function, according to the theory of random vibration.<sup>(1~3)</sup> The other is an indirect method in which the response of vehicle in time domain is firstly obtained, and then the response of vehicle in frequency domain can be obtained by using FFT method which is widely used for processing stationary signals.<sup>(4,5)</sup> While a car is traveling at variable speed such as starting, accelerating as well as braking, the function of road roughness is a nonstationary random process in time domain despite it is stationary random process in spatial domain. Therefore the response of vehicle in time domain should be nonstationary random process response of vehicle to the excitation of rough road. It is not proper to use

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FFT to process input and response of accelerating vehicle, because FFT method is just suitable for processing stationary signals. As yet, there has no a satisfied method can be adopted to solve this problem in vehicle engineering, and to find a new way to replace FFT becomes very necessary. In this paper maximum entropy method<sup>(6)</sup> which contains more information with less data is used to obtain transient frequency characteristics. It should be point out that the nonstationary response of vehicle in time domain should be given with a brief statement first. The purpose of this paper is to propose a new method of obtaining the nonstationary response of vehicle in both time domain and frequency domain, which is suitable for processing nonstationary data.

## 2. Nonstationary Response of Vehicle

Generally, the function of a road roughness is regarded as a stationary random process in space domain. While a car is moving with constant velocity, it is also a stationary random process in time domain. In this case, some traditional methods such as Monte-carlo for simulating the road roughness and vehicle reponses had been developed in the past decades.<sup>(1~5, 13)</sup> However, the road roughness is a nonstationary random process in time domain while a vehicle is traveling at variable speed. Accordingly, the vibration caused by a rough road surface should also be considered as a nonstationary random process. Therefore, a state-space approach is adopted to analyze the response of a vehicle traveling on homogeneous rough road.<sup>(7)</sup> In that work, the dynamics are modeled by linear ordinary differential equations in time domain while the nonstationary excitation process is replaced by a equivalent stationary process, in which the equal covariances of the two processes is taken into account, the variance of response is simulated by the equivalent model in spatial domain. Based on this work, an improved method with more computational efficiency was

proposed by using complex modal analysis.<sup>(8)</sup> Another method for solving this problem is to adopt differential equations. To solve differential equations, the variable coefficients are established firstly in space domain, and then, the time changing covariance is computed.<sup>(9)</sup> However, the amount of computation of this method is too large. This section mainly investigates a new method in time domain, in which a nonstationary road excitation model is put forward and is used directly to simulate the nontationary response of vehicles.

### 2.1 Modeling

In this section, a new method of solving nonstationary vibration of vehicles is mainly investigated, which can also be used in various kinds of vehicle models. A an example, a vehicle model with 5 degrees of freedom is established, shown in Fig. 1.

Where  $Z_s$ ,  $Z_b$  represent the vertical displacements of seat and vehicle body at center of gravity,  $Z_p$  represents the pitch angular displacement of the vehicle body,  $Z_f$ ,  $Z_r$  represent the bouncing displacements of the unsprung mass of front and rear suspension,  $M_s$  denotes the mass of human body and seat,  $M_b$ ,  $M_p$  denote the mass of vehicle body and its moment of inertia with respect to y axis,  $M_f$ ,  $M_r$  denote the unsprung masses of front and rear suspensions,  $K_s$ ,  $K_f$  and  $K_r$  are the stiffness coefficients of the seat, the front and the rear suspensions, respectively,  $K_{if}$ ,  $K_{ir}$  are

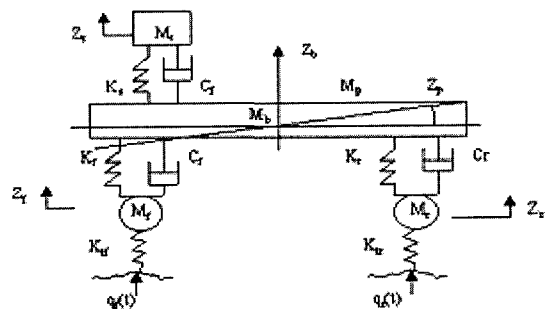


Fig. 1 Vehicle model with 5 degrees of freedom

the stiffness coefficients of the front and the rear tires,  $C_s$ ,  $C_f$  and  $C_r$  are the damping coefficients of the seat, the front and the rear suspensions, respectively,  $K_{yf}$ ,  $K_{yr}$  are the stiffness coefficients of the front and the rear tires,  $q_f$ ,  $q_r$  are the displacements of road input at the front and the rear tires, respectively,  $l_1$ ,  $l_2$  and  $l_3$  represent the distances from the centroid of the vehicle body to the seat, the front and the rear tires, respectively,  $L$  is the wheelbase.

In this model, damping of tires is neglected since it is so small.<sup>(8)</sup> The nonlinear damping and stiffness of suspension can also be considered if necessary.

The differential equations of the vehicle model can be described as follows,

$$[M]\ddot{Z} + [C]\dot{Z} + [K]Z = [F]Q \quad (1)$$

where,

$Z$  is the output vector,

$$Z = [z_s \quad z_b \quad z_p \quad z_f \quad z_r]^T$$

$[M]$  is the mass matrix,

$$[M] = \begin{bmatrix} M_s & 0 & 0 & 0 & 0 \\ 0 & M_b & 0 & 0 & 0 \\ 0 & 0 & M_p & 0 & 0 \\ 0 & 0 & 0 & M_f & 0 \\ 0 & 0 & 0 & 0 & M_r \end{bmatrix}$$

$[C]$  is the damping matrix,

$$[C] = \begin{bmatrix} C_s & -C_s & C_s l_1 & 0 & 0 \\ -C_s & C_s + C_f + C_r & -C_s l_1 - C_f l_2 + C_r l_3 & -C_f & -C_r \\ C_s l_1 & -C_s l_1 - C_f l_2 + C_r l_3 & C_s l_1^2 + C_f l_2^2 + C_r l_3^2 & C_f l_2 & -C_r l_3 \\ 0 & -C_f & C_f l_2 & C_f & 0 \\ 0 & -C_r & -C_r l_3 & 0 & C_r \end{bmatrix}$$

$[K]$  is the stiffness matrix,

$$[K] = \begin{bmatrix} K_s & -K_s & l_1 K_s & -K_f & 0 \\ -K_s & K_s + K_f + K_r & -K_s l_1 - K_f l_2 + K_r l_3 & K_f l_2 & -K_r \\ l_1 K_s & -K_s l_1 - K_f l_2 + K_r l_3 & K_s l_1^2 + K_f l_2^2 + K_r l_3^2 & K_f + K_l & -K_r l_3 \\ 0 & -K_f & K_f l_2 & K_f + K_{yf} & 0 \\ 0 & K_r & -K_r l_3 & 0 & K_r + K_{yr} \end{bmatrix}$$

$[F]$  is the excitation force matrix,

$$[F] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_{yf} & 0 \\ 0 & K_{yr} \end{bmatrix}$$

$[Q]$  is the excitation vector,

$$[Q] = [q_f \quad q_r]^T$$

When a car is running at variable speeds, the functions of  $q_f(t)$  and  $q_r(t)$  are nonstationary in time domain, but it is noted that they are stationary in spatial domain.

## 2.2 State Equations

To obtain the results of Eq. (1) in time domain, state vectors are utilized as follows,

$$[X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5]^T = [Z_s \quad Z_b \quad Z_p \quad Z_f \quad Z_r]^T \quad (1-a)$$

$$[X_6 \quad X_7 \quad X_8 \quad X_9 \quad X_{10}]^T = [\dot{Z}_s \quad \dot{Z}_b \quad \dot{Z}_p \quad \dot{Z}_f \quad \dot{Z}_r]^T \quad (1-b)$$

Hence,

$$[\dot{X}_1 \quad \dot{X}_2 \quad \dot{X}_3 \quad \dot{X}_4 \quad \dot{X}_5]^T = [X_6 \quad X_7 \quad X_8 \quad X_9 \quad X_{10}]^T \quad (1-c)$$

$$[\dot{X}_6 \quad \dot{X}_7 \quad \dot{X}_8 \quad \dot{X}_9 \quad \dot{X}_{10}]^T = [\ddot{Z}_s \quad \ddot{Z}_b \quad \ddot{Z}_p \quad \ddot{Z}_f \quad \ddot{Z}_r]^T \quad (1-d)$$

Substituting these state vectors into Eq. (1), and the following state equation can be obtained,

$$\dot{X} = [A]X + [B]U \quad (2)$$

where  $X$  is the state variables,

$$X = [X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \quad X_6 \quad X_7 \quad X_8 \quad X_9 \quad X_{10}]^T$$

$[A]$  is the state coefficient matrix,

$$[A] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -K_s/M_s & K_s/M_s & -K_s l_1/M_s & 0 & 0 \\ K_s/M_b & a_{72} & a_{73} & K_f/M_b & K_r/M_b \\ -K_s l_1/M_p & a_{82} & a_{83} & -K_f l_2/M_p & K_r l_3/M_p \\ 0 & K_f/M_f & -K_f l_2/M_f & a_{94} & 0 \\ 0 & K_r/M_r & l_3 K_r/M_r & 0 & a_{105} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -C_s/M_s & C_s/M_s & -C_s l_1/M_s & 0 & 0 \\ -C_s/M_b & a_{77} & a_{78} & C_f/M_b & C_r/M_b \\ -l_1 C_s/M_p & a_{87} & a_{88} & -l_2 C_f/M_p & l_3 C_r/M_p \\ 0 & C_f/M_f & -C_f l_2/M_f & -C_f/M_f & 0 \\ 0 & C_r/M_r & C_r l_3/M_r & 0 & C_r/M_r \end{bmatrix}$$

where,

$$\begin{aligned} a_{72} &= (K_s + K_f + K_r)/M_b \\ a_{73} &= (K_s l_1 + K_f l_2 - K_r l_3)/M_b \\ a_{73} &= (C_s l_1 + C_f l_2 - C_r l_3)/M_b \\ a_{94} &= -(K_f + K_r)/M_f \\ a_{105} &= -(K_r + K_r)/M_r \end{aligned}$$

[B] is the input coefficient matrix,

$$[B] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{qf}/M_f & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{tr}/M_r \end{bmatrix}^T$$

U is the input vector,

$$U = \begin{bmatrix} q_f \\ q_r \end{bmatrix}$$

### 2.3 Simulation of Road Roughness

In general case, the power spectral density (PSD) of rough road in frequency domain can be expressed as,

$$S_q(\omega) = S_q(\Omega_0) v / (\omega^2 + \omega_0^2) \quad (3)$$

where,  $S_q(\Omega_0)$  is the coefficient of road roughness. According to the classification of road, the values of  $S_q(\Omega_0)$  can be obtained from some references.  $\Omega_0$  is the reference spatial angular frequency,  $\Omega_0 = 1(\text{rad}/m)$ .  $v$  is the traveling speed of vehicle.  $\omega_0$  is the lowest cut-off angular frequency.<sup>(2)</sup> Eq. (3) can be considered as a response of a first order linear system to white noise excitation.

Based on the theory of random vibration,<sup>(2)</sup> following relationship is obtained,

$$S_q(\omega) = |H(\omega)|^2 S_w \quad (4)$$

where,  $H(\omega)$  is the transfer function, and  $S_w$  is the PSD of white noise where normally  $S_w = 1$ . From Eqs. (3) and (4),  $H(\omega)$  is written by,

$$H(\omega) = \frac{\sqrt{S_q(\Omega_0)} v}{\omega_0 + j\omega} \quad (5)$$

From Eq. (5), the differential equation about road roughness is expressed as,

$$\dot{q}(t) + \omega_0 q(t) = \sqrt{S_q(\Omega_0)} v \omega(t) \quad (6)$$

while a car is running with variable speed, despite road roughness is stationary random process in space domain, it is nonstationary random process in time domain.

Note that,

$$\omega = 2\pi n v(t) \quad (7)$$

where,  $n$  is the space frequency. Therefore, Eq. (6) becomes,

$$\dot{q}(t) + 2\pi n_0 v(t) q(t) = 2\pi \sqrt{S_q(n_0) n_0^2} v(t) \omega(t) \quad (8)$$

where  $v(t) = v_0 + at$ ,  $a$  is the acceleration of the car moving,  $t$  is the accelerating time.

According to Eq. (8), a numerical simulation can be carried out. Fig. 2 and Fig. 3 represent the simulation consequence of the nonstationary rough road and its PSD, respectively. From Fig. 2, it is concluded that with increasing of the velocity, the amplitude of road roughness increases in time domain.

### 2.4 Simulation of Nonstationary Responses

Inputting the values of road roughness above into Eq. (2), the nonstationary responses of a vehicle system to the excitation of road roughness can be calculated. The model parameters including the resonance frequencies of the 5 degrees of freedom are shown in Table 1.

Note that the resonance frequencies are obtained by taking  $1/2\pi$  times the square root of the eigenvalues of a matrix defined as  $[S] = [M]^{-1} [K]$ .

Fig. 4 represents the responses of vehicle system to nonstationary excitation, i.e., the responses of vehicle running with acceleration  $a=3\text{ m/s}^2$ . It has been seen that the components of the accelerations in low frequency range of seat, vehicle body, relative displacement and pitch acceleration have a rule of one periodicity per second. The acceleration of axle and dynamic load of the tire does not show an evident periodicity with the increasing of the velocity due to their higher inherent frequencies. Further analysis indicates that the

response periodicity comes from the road excitation in low frequency scope. The simulation method had already been verified by road tests of a light passenger car.<sup>(13)</sup> It is, therefore, reasonable to deem the results carried out above are reliable.

### 3. Maximum Entropy Spectral Analysis

Maximum entropy spectral analysis proposed by Burg in 1967 has developed in the past twenty years. It is also called modern spectrum compared with the classical FFT method. Although there were much more successful applications of FFT to process the stationary signals, the frequency resolution  $\Delta f$  is conflict with sampling time interval  $\Delta t$  in using FFT method. Furthermore,

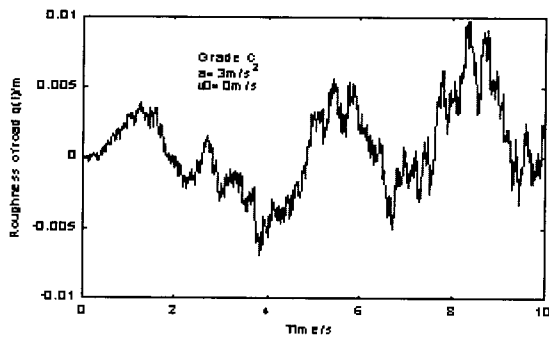


Fig. 2 Non-stationary road roughness

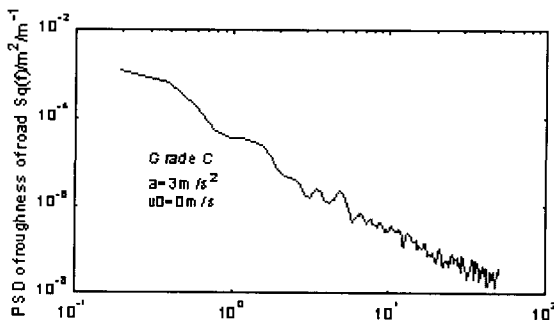


Fig. 3 PSD of nonstationary road roughness

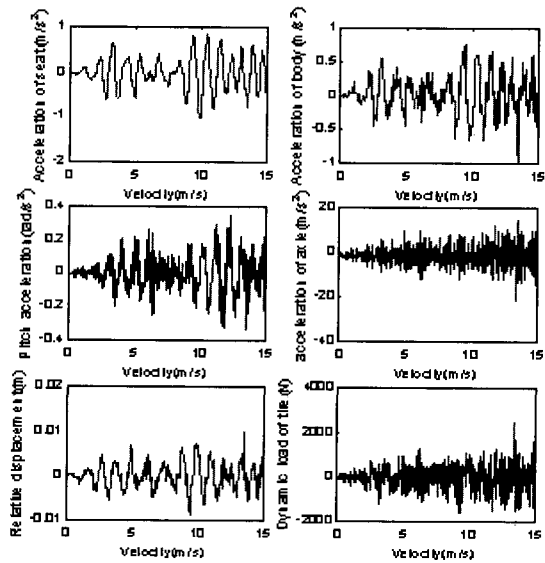


Fig. 4 Nonstationary response of vehicle ( $a=3\text{ m/s}^2$ )

Table 1 Parameters of vehicle model with 5 degrees of freedom

Symbol	$M_s$ (kg)	$M_b$ (kg)	$M_p$ (kgm <sup>2</sup> )	$M_f$ (kg)	$M$ (kg)
Value	70	2100	3500	140	210
Symbol	$K_s$ (N/m)	$K_f$ (N/m)	$K_r$ (N/m)	$K_{tr}$ (N/m)	$K_{tr}$ (N/m)
Value	12200	74000	120000	520000	520000
Symbol	$C_s$ (Ns/m)	$C_f$ (Ns/m)	$C_r$ (Ns/m)		
Value	550	1800	1200		
Resonance frequency	$F_s$ (Hz)	$F_b$ (Hz)	$F_p$ (Hz)	$F_f$ (Hz)	$F_r$ (Hz)
Value	2.15	1.51	1.34	8.83	10.38

the assumption refusing any data beyond the sampling time length  $T$  results in reduction of frequency resolution  $\Delta f$ . Therefore it is not suitable to use FFT method to process short time data. Maximum entropy spectral analysis is a parameter estimation method, in which a data model is first established and then the estimation of PSD is carried out with the model parameters. Therefore the frequency resolution is not limited by length of data, and higher frequency distinguishing ability can be obtained.

### 3.1 Data Modeling and Parameters Estimation

Given sampled discrete series  $x(n)$ , where  $n$  is the sampling number,  $n=1,2,3, \dots N$ . Assuming that  $x(n)$  is the response of a linear model subjected to an excitation of the white noise, it can be expressed as,

$$x(n) = -\sum_{k=1}^p a_k x(n-k) + w(n) \quad (9)$$

where,  $w(n)$  is the white noise with zero mean and  $\sigma_w^2$  variance,  $p$  is the number of model orders, and  $a_k$  is model parameters,  $k=1, 2, 3, \dots p$ . According to the definition of the autocorrelation,

$$R_x(m) = E[x(n)x(n+m)] \quad (10)$$

and considering Eq. (9), the following equation is obtained,

$$R_x(m) = E \left[ x(n) \left\{ -\sum_{k=1}^p a_k x(n-k+m) + w(n+m) \right\} \right] \quad (11)$$

Simplifying Eq. (11), it can be expressed as,

$$R_x(m) = -\sum_{k=1}^p a_k R_x(m-k) + E[x(n)w(n+m)] \quad (12)$$

when  $m \geq 1$ ,  $x(n)$ , is irrelevant to  $w(n+m)$ . Hence,

$$E[x(n)w(n+m)] = \begin{cases} 0 & (m > 0) \\ \sigma_w^2 & (m = 0) \end{cases} \quad (13)$$

Substituting Eq. (13) into Eq. (12), the following equations are obtained,

$$\begin{cases} R_x(m) + \sum_{k=1}^p a_k R_x(m-k) = 0 & (m > 0) \\ R_x(0) + \sum_{k=1}^p a_k R_x(-k) = \sigma_w^2 & (m = 0) \end{cases} \quad (14)$$

Eq. (14) can also be written in terms of matrix,

$$\begin{bmatrix} R_x(0) & R_x(-1) & \dots & R_x(-p) \\ R_x(1) & R_x(0) & \dots & R_x(-p+1) \\ \vdots & \vdots & \vdots & \vdots \\ R_x(p) & R_x(p-1) & \dots & R_x(0) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \sigma_w^2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (15)$$

If given the sampled discrete series  $x(n)$ , the autocorrelation  $R_x$  and the covariance  $\sigma_w^2$  can be carried out. Furthermore, the model parameters can be obtained easily by solving Eq. (15). So far there are many algorithms for computing model parameters among which Burg algorithm is a fast algorithm similar to FFT. While more data need to be calculated, this method is very effective.

### 3.2. Burg Fast Algorithm

Burg fast algorithm can obtain the model parameters directly with sampled data without calculating the autocorrelation. It is noted that the values obtained from Eq. (9) are evaluated values of sampled data. Define  $\tilde{x}(n)$  as evaluated values, the forward error of data model with  $p$  orders is expressed as,

$$e_{fn}^{(p)} = x(n) - \tilde{x}(n) = x(n) + \sum_{k=1}^p a_k^{(p)} x(n-k) \quad (16)$$

Similarly, the backward error of data model with  $p$  orders is written by

$$e_{bn}^{(p)} = x(n) - \tilde{x}(n-p) = x(n) + \sum_{k=1}^p a_k^{(p)} x(n+k-p) \quad (17)$$

The relationship between the high order error and the low order error can be expressed as,

$$e_{fn}^{(p)} = e_{fn}^{(p-1)} + \rho_p e_{bn-1}^{(p-1)} \quad (18)$$

$$e_{bn}^{(p)} = e_{bn}^{(p-1)} + \rho_p e_{bn}^{(p-1)} \quad (19)$$

Hence,

$$ES = \sum_{n=p}^{N-1} \left\{ \left[ e_{fn}^{(p)} \right]^2 + \left[ e_{bn}^{(p)} \right]^2 \right\} \quad (20)$$

$$\text{Let } \left( \frac{\partial E}{\partial \rho_p} \right)^p = 0 \quad (21)$$

Thus,

$$\rho_p = - \frac{2E \left[ e_{fn}^{(p-1)} e_{bn-1}^{(n-1)} \right]}{E \left[ e_{fn}^{(p-1)} \right]^2 + E \left[ e_{bn-1}^{(p-1)} \right]^2} \quad (22)$$

The relationship between model parameters and reflecting coefficients can be expressed as,

$$a_p^{(p)} = \rho_p \quad (23)$$

$$a_k^{(p)} = a_k^{(p-1)} + \rho_p a_{p-k}^{(p-1)} \quad k = 1, 2, 3 \dots p \quad (24)$$

According to Eqs. (22) and (24), the recursive calculation can be accomplished.

### 3.3 Maximum Entropy Spectral Estimation

Taking  $Z$  transform to both sides of Eq. (9), the discrete transfer function is expressed as,

$$H(z) = \frac{X(z)}{W(z)} = \frac{1}{A(z)} = \frac{1}{1 + \sum_{k=1}^p a_k z^{-k}} \quad (25)$$

hence, the PSD  $P_x(\omega)$  of the signal  $x$  referred to as maximum entropy spectral (MES) can be obtained,

$$P_x(\omega) = |H(e^{-j\omega})|^2 \sigma_w^2 = \frac{\sigma_w^2}{\left| 1 + \sum_{k=1}^p a_k e^{-j\omega k} \right|^2} \quad (26)$$

where  $a_k$  can be carried out from Eqs. (23) and (24). Fig. 5 is a processing result of vehicle body

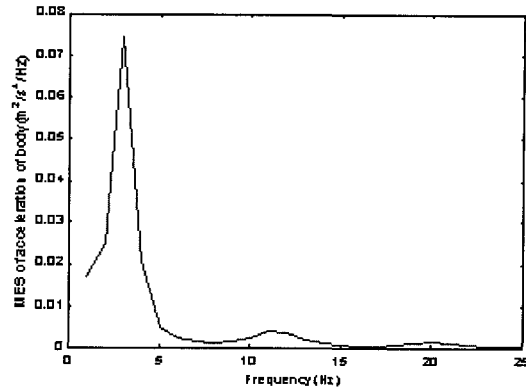


Fig. 5 MES of vehicle body acceleration

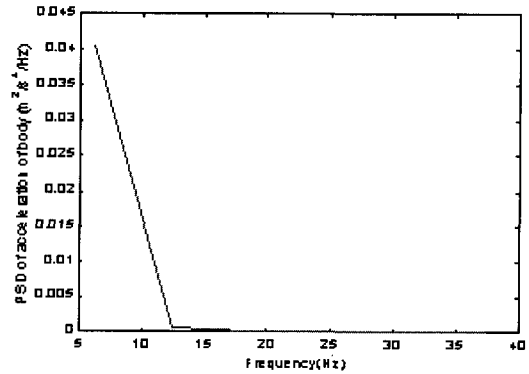


Fig. 6 PSD of vehicle body acceleration

acceleration by using the maximum entropy method with a data model of 32 orders, i.e.  $p = 32$ . For showing an exactly instantaneous PSD of the nonstationary random process, a short sampled time  $T$  should be considered, and the data points  $N=32$  is adopted in Fig. 5. Notice that, except for some small fluctuations along the curve, there has no difference of the trend in MES, if the data points a little more than 32 are chosen. It is seen that maximum entropy spectral estimation well reflects two main peaks with less sampled data. First peak represents resonant frequency of the vehicle body, second peak is close to resonant frequency of the unsprung mass. Moreover the frequency resolution is continuous. Fig. 6 is the PSD of the vehicle body acceleration by using FFT method with same sampled data. It illustrates that FFT method cannot reflect two main peaks,

and the frequency resolution is bigger. By the FFT method,

$$\Delta f = \frac{1}{T} = \frac{1}{N\Delta t} = \frac{1}{32 \times 0.005} = 6.25 \text{ Hz}$$

Therefore, the maximum entropy method has more advantage than FFT method in processing short time data. Fig. 7 and Fig. 8 represent MES and PSD of axle acceleration respectively. 2000 point data are used in both methods. It concludes that maximum entropy method can give the resonance frequency of axle exactly and FFT method cannot do. Fig. 9, Fig. 10 and Fig. 11 are three dimensional transient maximum entropy spectrums of vehicle body acceleration, axle acceleration and pitch angular acceleration, respectively. From Fig. 10 illustrates the amplitude of transient MES of axle acceleration increases with increasing of time.

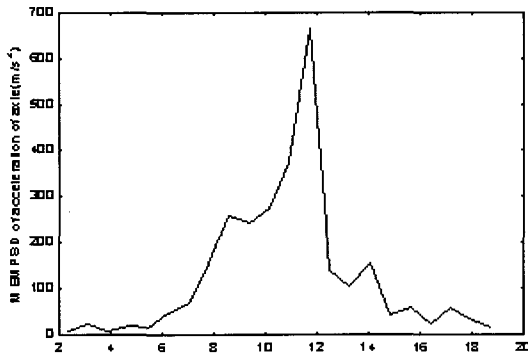


Fig. 7 MES of axle acceleration

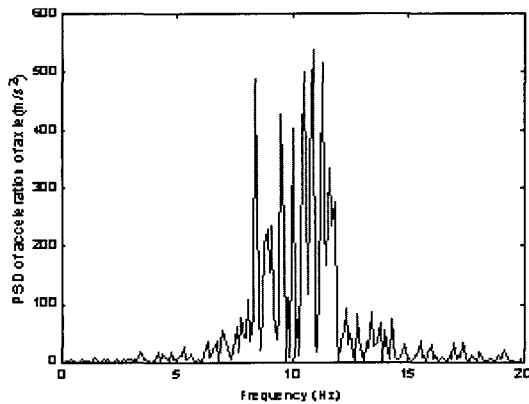


Fig. 8 PSD of axle acceleration

Fig.11 shows that transient MES of pitch acceleration has two main peaks in both low and high frequency. The peaks in low frequency increases with increasing of time, but does not always increase. The peak in high frequency increases with increasing of the time.

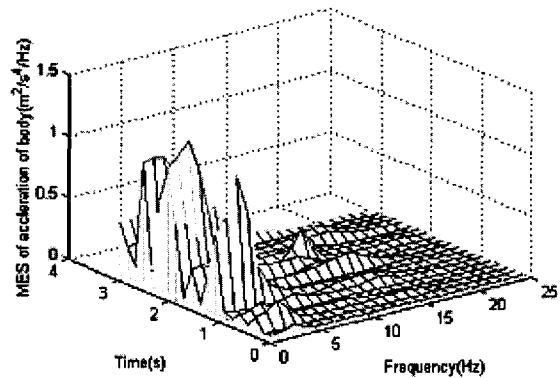


Fig. 9 Transient MES of vehicle body acceleration

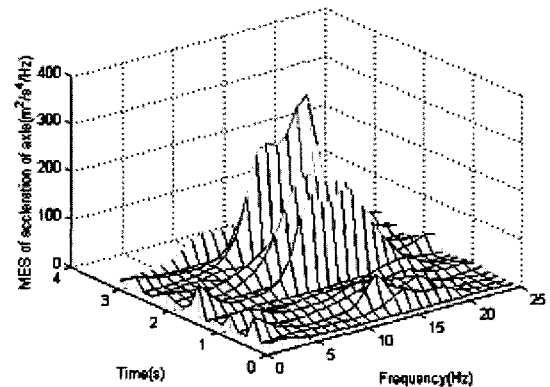


Fig. 10 Transient MES of axle acceleration

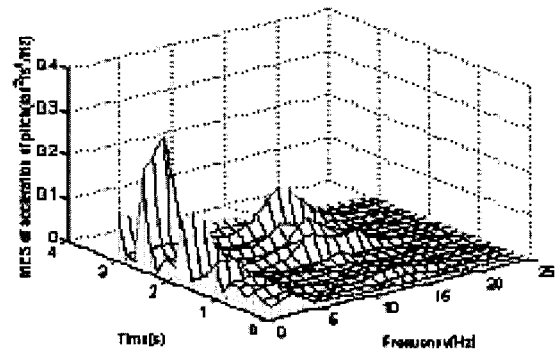


Fig. 11 Transient MES of pitch acceleration



#### 4. Conclusions

This paper firstly performs the numerical simulation of nonstationary response of a vehicle by using the nonstationary excitation model of the road roughness in time domain. To obtain frequency characteristics, the maximum entropy method is used to process nonstationary response in frequency domain. The results show that the maximum entropy method has more advantage than the traditional FFT method in processing with less data. The successive frequency resolution of MES, which shows higher frequency distinguishing rate than FFT, can give more frequency information in detail. The maximum entropy method is suitable for processing short time data, and is certainly a feasible method to solve nonstationary response problem of vehicles, especially to deal with the measured data in a vehicle experiment. Generally, it is possible to analyze any other signals of the nonstationary process. As an application, the instantaneous spectral characteristics of a vehicle response are firstly obtained in this paper.

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