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제 28 권, 제 1 호, 2002. 6. 30.

계층적 분석기법을 활용한 그룹의사결정 지원 (Group Decision Support with Analytic Hierarchy Process)

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Abstract

The Analytic Hierarchy Process (AHP) is well suited to group decision making and offers numerous benefits as a synthesizing mechanism in group decisions. To date, the majority of AHP applications have been in group settings. One reason for this may be that groups often have an advantage over individual when there exists a significant difference between the importance of quality in the decision and the importance of time in which to obtain the decision. Another reason may be the best alternative is selected by comparing alternative solutions, testing against selected criteria, a task ideally suited for AHP.

In general, aggregation methods employed in group AHP can be largely classified into two methods: geometric mean method and (weighted) arithmetic mean method. In a situation where there do not exist clear guidelines for selection between them, two methods do not always guarantee the same group decision result. We propose a simulation approach for building group consensus without efforts to make point estimates from individual diverse preference judgments, displaying possible disagreements as is natural in group members' different viewpoints.

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1. Introduction and background

The increasing complexity of socio economic environments makes it less and less possible for a single decision maker to consider all relevant aspects of the problem. Therefore, many organizations employ groups in decision making problem and decisions are made collectively, regardless of whether the organization is public or private, national or international. Moving from a single decision maker's setting to multiple decision makers' setting introduces a great deal of complexity into the analysis. No longer is the problem concerned exclusively with the selection of a most preferred alternative from the nondominated set using the decision maker's preference structure. The analysis must be extended to account, somehow, for group decision makers, each one potentially exhibiting a unique preference structure, perceiving different consequences, and responding to a diverse array of aspirations [8]. Group decision making problem is usually understood to be the reduction of different individual preferences among objects in a given set to a single collective preference, or group preference.

There have been research efforts to represent a group preference as an additive individual value (utility) functions under some conditions. Harsanyi [9] presented the theory for an additive cardinal preference aggregation rule consistent with von Neumann and Morgenstern rationality axioms. The Bayesian rationality postulate (the group preferences satisfy the Bayesian rationality axiom) may not be appropriate for a group since they do not consider the equity of the outcomes. Keeney and Kirkwood [11] have specified sufficient conditions for a group cardinal social welfare function whose arguments are the individual utility functions of group members to have weighted additive form. The difficulties in the use of additive social welfare function lies in the assessment of group members' importance weights. The assessment involves addressing questions of trading-off utility to one individual against utility to another individual, so simultaneously trying to consider the inherent value of different measured utilities to each individual (interpersonal comparison of utility). Unfortunately, there is no entirely satisfactory procedure for making these tradeoffs.

Kemeny and Snell [12] have spearheaded a new thrust in the area of group consensus formation by proposing a distance measure between individual orderings. They have proposed a set of axioms to be satisfied by such a measure and then proved its existence and uniqueness. Further research on this distance measure has been carried out by many others [5, 6].

As group extension of a single decision maker's multiple criteria method for choosing among discrete alternatives [19, 20], Korhonen, et al. [13] suggested an interactive approach to multiple criteria optimization with multiple decision makers. They assume that group members are able to make pairwise choices among competing alternatives according to its (implicit) utility function in one of their procedural steps. The difficulty in interactive multiple objective methodology is how to determine most preferred alternative (s) which is consistent with the information (usually through the whollistic judgements between competing alternatives) given by decision makers interactively through adjusting attribute weights. Under multiple criteria, however, it is difficult for the group to build consensus between closely competing alternatives. If the group members are fairly in agreement, they can evaluate choices. Otherwise, the group will have to resolve a choice among pairs of solution. If not, a stalemate may result.

The Analytic Hierarchy Process (AHP), introduced by Saaty [15], has also been applied to group decision problems. Saaty [16] has discussed several practical and theoretical aspects of group decision making using AHP. There are at least two methods employed in AHP for aggregating group opinions. In the first, geometric mean method, as a most common group preference aggregation method in the AHP literature utilizes geometric mean of individual evaluations as elements in pairwise comparison matrices and then priorities are then computed [1, 3, 4, 17]. In the weighted arithmetic mean method, a simple arithmetic mean of individual priorities is used to arrive at the group consensus. In viewpoint of social choice axioms, the geometric mean method of combining individual opinions has been shown to violate at least one of the axioms of group preference aggregation, namely the Pareto optimality axiom. On the other hand, the weighted arithmetic mean method has been found to satisfy all the axioms except the independence of irrelevant alternatives and it has been shown that this does not limit the applicability of this method [14].

In this paper, we consider simulation approach as a group preference aggregation method rather than deriving group point estimates from individual pairwise judgments between criteria or between alternative with respect to criteria, which was adopted in many of group AHP applications. In applying a simulation approach, it is a prerequisite to have multitude of decision makers (at least the number of scales used in AHP) involved as is often case in public policy making for generating random observations from empirically observed frequency distribution which is determined from the frequency of responses. Using the simulation approach, which reflects diversification of group members' preferences as it is, analysis such as expected weights and expected ranks displays insights into group decision making context [2].

2. Simulation approach when multiple decision makers are involved

Let $A_1, A_2, \ldots A_n$ be a set of n alternatives compared in pairs according to a given criterion. We define a square matrix $A^k = (a_{ij}^k), \forall i, j \in [1, n], k \in K$ to be a reciprocal matrix with n alternatives where $a_{ij}^k = 1/a_{ji}^k$ and a_{ij}^k indicates that the ith alternative is a_{ij}^k times more dominant than the jth alternative on the criterion considered in kth group member's viewpoint. Similarly, let $C_1, C_2, \ldots C_m$ be a set of m common criteria which is shared among group decision makers. We define a square matrix $C^k = (c_{pq}^k)$, $\forall p, q \in [1, m], k \in K$ to be a reciprocal matrix with m criteria where $c_{pq}^k = 1/c_{qp}^k$ and c_{pq}^kindicates that the ith criterion is c_{pq}^k times more important than the jth criterion considered in kth group member's viewpoint.

Gathering K decision makers' pairwise judgments on criteria and between alternatives with respect to criteria considered, it can be thought that variable a_{ij} ranged from $a_{ij}^L = \min[a_{ij}^1, a_{ij}^2, \ldots, a_{ij}^K]$ to $a_{ij}^U = \max[a_{ij}^1, a_{ij}^2, \ldots, a_{ij}^K]$ and c_{pq} ranged from $c_{pq}^L = \min[c_{pq}^1, c_{pq}^2, \ldots, c_{pq}^K]$ to $c_{pq}^U = \max[c_{pq}^1, c_{pq}^2, \ldots, c_{pq}^K]$ can be regarded as variables bounded between 1/9 and 9 respectively. Let $f(a_{ij}), f(c_{pq})$ be the empirically observed relative frequency distribution and $F(a_{ij})$, $F(c_{pq})$ the cumulative frequency distribution on a_{ij} and c_{pq} respectively.

Let $a_{ij}^{(r)}$ and $c_{pq}^{(r)}$, $r = 1, 2, \ldots, R$, $i, j = 1, 2, \ldots, n$, $p, q = 1, 2, \ldots, m$, be pairwise comparisons of size r generated from the cumulative frequency distribution $F(a_{ij})$ and $F(c_{pq})$ respectively. Let $\Psi^{(r)}$ be matrix of which elements are eigenvectors calculated from generated pairwise comparison, $a_{ij}^{(r)}$ and $C^{(r)}$ be the eigenvectors associated with generated pairwise comparison, $c_{pq}^{(r)}$ in rth simulation run. Then final priorities of alternatives considered can be determined as displaying descending order of magnitude of $\Psi^{(r)} * C^{(r)'}$. The simulated final priorities are sometimes obtained in case simulated pairwise judgment matrices have high inconsistency ratio. To avoid this case, we consider generated pairwise comparison matrices with inconsistency ratio (IR) less than or equal to 0.1.

To illustrate the aforementioned simulation process, we consider an artificial example with three alternatives evaluated on three criteria and then the example is extended to illustrate more general case of four alternatives with five criteria. At first, let the hierarchy to be used in the example be as shown in Figure 1. It has three alternatives (A_1, A_2, A_3) to be compared using three criteria, (C_1, C_2, C_3). And the group members' pairwise judgment for this hierarchy is shown in Table 1.

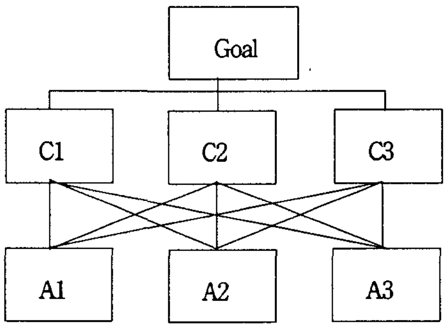

Figure 1.Typical AHP model

There is a continuum of decision making contexts ranging from (1) common objectives contexts where all parties have (basically) the same objectives, to (2) non-common objectives contexts in which parties (or groups of parties) have non-shared (and sometimes hidden) objectives, to (3) conflict contexts in which parties seek concessions from opposing parties. Further the common objectives context can be decomposed into following three situations consensus building, vote or compromise and separate models or players [7]. In our example, we assume that the participants involved in decision making process share common objectives for group consensus building. This approach is especially useful when judgments are elicited using (Web based) questionnaires as the group members will not have the chances to interact with each other so that the judgments are not influenced.

The preference judgments from group members (K=25) are shown in Table 1, where frequencies of preference judgments about pairs of criteria and alternatives with respect to criteria are denoted.

Before analyzing simulation results, let us scrutinize the preference frequency in Table 1. At first, we can infer group's preference tendency which implies criterion C_1 is most preferred, C_3 is next, and finally C_2, that is $C_1 > C_3 > C_2$. And we can infer $A_1 > A_3 > A_2$ with $F(C_{12})$; to criterion C_1, $A_3 > A_2 > A_1$ with respect to criterion C_2, and $A_1 > A_3 > A_2$ with respect to criterion C_3 from the preference frequency between alternatives on each criterion although there exist some extent of disagreements. On the other hand, roughly aggregated group preference, $A_3 > A_2 > A_1$ with respect to criterion C_2 does not have much influence on deciding the final priority because the weight of criterion C_2 is evaluated less important than the other two criteria. Consequently, we are strongly confident as a group opinion that A_1 is most preferred, A_3 is secondly, and A_2 is the least preferred, which is the result we want to show with simulation approach.

For each of the matrices of the example, discrete values for the judgments were generated from the empirically observed distribution. For example, let us consider the C_{12} column in Table 1. According to the relative frequency distribution, we can construct the cumulative distribution as shown in Figure 2.

Table 1. Preference frequency from group decision makers

Scale	Between criteria			Between alternatives with respect to criteria								
	C12	C13	C23	C1			C2			C3		
				A12	A13	A23	A12	A13	A23	A12	A13	A23
9				3						1		
7	1*	4		5	3			1	2	5	6	
5	6	5		7	4	1	1	1	3	3	7	1
3	7	7	1	3	5	3	5	4	3	6	3	3
1	5	3	4	2	8	5	7	6	2	4	4	2
1/3	4	2	3	4	3	7	7	3	6	3	4	5
1/5	2	3	10	1	2	7	5	5	4	2	1	7
1/7		1	7			2		4	5	1		5
1/9								1				2

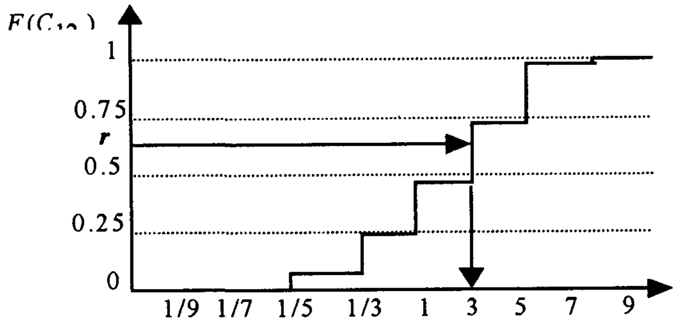

Figure 2. Cumulative distribution on C_{12}

Thus the random observation (i.e., pairwise ratio comparison) is generated from the equation, $r=F(C_{12})$, where $r\in[0,1]$ is a random number. In this manner, we can generate random observations for C_{13} and C_{23}, which are used for components of criteria matrix for calculating eigenvectors. And we can construct comparison matrices of three alternatives with respect to the three criteria. After all matrices (i.e., one criteria matrix and three comparison matrices of alternatives) were determined, the overall synthesized priorities were calculated and thus the final rank was recorded. This process was repeated 500 times. Of the 500 runs, 64% resulted in the first alternative with the first rank, 53% in the third alternative with the second rank, and 77% in the second alternative with the third rank.

Table 2. Composite results of 500 runs

Composite results						
Rank	A1	A2	A3	A1	A2	A3
1	320	25	155	64%	5%	31%
2	142	91	267	28%	18%	53%
3	38	384	78	8%	77%	16%

In the AHP with single decision maker, priorities with an IR greater than 0.10 are considered to have judgments which are too random-like [18]. In the group decision context, each preference judgment with small inconsistency is combined to build group consensus which can be ended with large inconsistency. Hence, pairwise comparison judgments with an IR less than with 0.10 for simulated criteria matrices are considered in Table 3. Although each alternative has seen each possible rank, it is clear that alternative one is inclined to be positioned in the first rank, 81% of the time. Likewise, alternative two is inclined to be positioned in the third rank, 89% of the time, and alternative three, 73% of the time. However, how much confidence can we have in first two rankings and others? In order to address this question, we consider the notion of *expected rank* and *expected weight* which are suggested by Hauser and Tadikamalla [10].

Table 3. Composite results of 500 runs with IR<0.1

Composite results						
Rank	A1	A2	A3	A1	A2	A3
1	405	15	90	81%	3%	18%
2	85	40	365	17%	8%	73%
3	10	445	45	2%	89%	9%

Expected score defined in (1) implies that we will sum together the product of the fraction of time each rank occurred and *n+1* minus the rank itself instead of the rank itself because the rank and the fraction of the time each rank occurred is inversely correlated.

$$ES_i = \sum_{k=1}^{n} (p_{i,k})(n+1-k) \qquad (1)$$

where is the expected score of the *i*th alternative and is the proportion of the trials that the *i*th alternative had rank *k*.

Next, let the expected weight to be the normalized expected scores. When alternatives are placed in descending order of the expected weights, the results reveal the expected rank of alternatives. Hence, we define

$$EW_i = \frac{ES_i}{\sum_{k=1}^{n} ES_k} \qquad (2)$$

The expected weights of (2) are determined from the frequency that each rank occurred for each alternative. Hence these weights are statistical weights indicating a composite frequency or a mean of feasible weights around which we expect the actual weight to be scattered (Table 4).

Table 4. Expected rank and weight

Alternative	Expected rank	Expected weight
A1	1	0.4637
A2	3	0.1883
A3	2	0.3479

Let us extend a simulation example to the hierarchy with four alternatives (A_1, A_2, A_3, A_4) to be compared using five criteria (C_1, C_2, C_3, C_4, C_5) such as in Figure 3. The preference judgments from group decision makers (K=25) are shown in Table 5 for frequencies of preference judgments about pairs of criteria and Table 6 for alternatives with respect to criteria

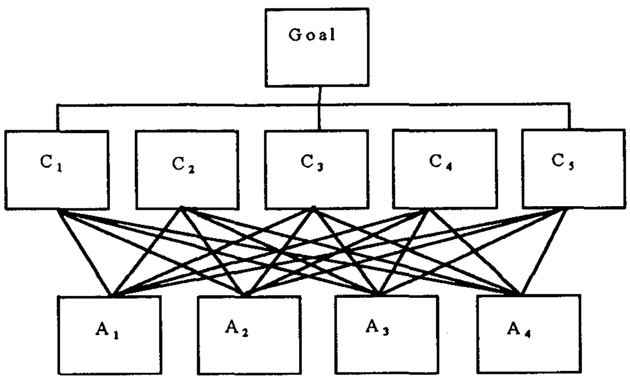

Figure 3. Typical AHP model

Table 5. Frequency of preference judgements about pairs of criteria

	Between criteria									
	C12	C13	C14	C15	C23	C24	C25	C34	C35	C45
9	1	1						1		1
7	3	1	2	1	1		1	2	3	2
5	6	4	4	3	1	2	1	3	3	4
3	7	5	3	4	3	4	5	3	6	5
1	4	7	10	12	6	9	10	9	7	6
1/3	2	6	5	5	6	4	3	5	5	6
1/5	2	1	1		7	5	5	2	1	1
1/7					1	1				
1/9										

Using these preference judgments, the results can be obtained as shown in Table 7. Although each alternative has seen each possible rank, it is clear that alternative one is inclined to be positioned in the first rank. Likewise, alternative two is inclined to be positioned in the third rank, alternative three in the second rank, and alternative four in the fourth rank. To concrete this conjecture, we use the concept of expected score, expected weight, and expected rank as was introduced in the above simulation example (See Table 8).

Table 6. Frequency of preference judgments between alternatives with respect to criteria

	Between alternatives on criteria																	
Scale	C1						C2						C3					
	A12	A13	A14	A23	A24	A34	A12	A13	A14	A23	A24	A34	A12	A13	A14	A23	A24	A34
9		1					1		1			1	1		1			
7	3	2	5		1	2	3	2	5		1	3	3	4	4		4	4
5	5	3	7	2	3	4	3	4	5	2	5	5	5	7	4	3	6	5
3	4	5	5	4	5	5	5	5	6	6	6	4	3	3	5	3	4	2
1	6	7	4	5	5	10	5	6	2	5	3	6	7	7	5	7	5	8
1/3	4	4	2	6	5	3	4	4	3	7	6	3	4	3	6	3	4	4
1/5	3	2	2	4	4	1	2	3	3	3	4	2	1	1		4	2	
1/7		1		4	2		2	1		1		1				5		
1/9										1								

Scale	Between alternatives on criteria											
	C4						C5					
	A12	A13	A14	A23	A24	A34	A12	A13	A14	A23	A24	A34
9		1						1			1	1
7	4	3	4		2	3	4	1	4		2	3
5	6	2	8	1	3	5	3	5	6	3	5	5
3	3	6	5	4	5	4	6	4	5	5	5	4
1	7	6	4	6	6	9	4	6	3	4	4	6
1/3	3	4	3	7	4	4	5	5	4	6	5	3
1/5	1	3	1	4	3		3	2	3	4	3	2
1/7	1			3	2			1		3		1
1/9												

Table 7. Composite of 500 runs with IR<0.1

Rank	Composite results							
	One	Two	Three	Four	One	Two	Three	Four
1	332	31	127	10	66%	6%	25%	2%
2	129	78	257	36	26%	16%	52%	7%
3	29	236	96	139	6%	47%	19%	28%
4	10	155	20	315	2%	31%	4%	63%

Table 8. Expected rank and weight

Alternative	Expected rank	Expected weight
One	1	0.3566
Two	3	0.1970
Three	2	0.2982
Four	4	0.1482

To distinguish this approach, we compare the simulation results with two group aggregation methods described in AHP literatures using the same input data (Table 5-6), as shown in Table 9.

In a situation where there do not exist clear guidelines for selection between geometric mean method and (weighted) arithmetic mean method, two approaches sometimes give conflicting results (See Table 9).

Although aggregated group judgments are so important and thus combining judgments for a group working together in a corporation is very important and generally can not be replaced by a statistical approach, simulation approach is useful as a supplementary tool which shows the possible disagreements among group members.

Table 9. Comparison with two aggregated methods

Alternative	Geometric mean	Arithmetic mean	Simulation
One	1	1	1
Two	2	3	3
Three	3	2	2
Four	4	4	4

3. Concluding remarks

To date, the majority of AHP applications have been in group settings. One reason for this may be that groups often have an advantage over individuals when there exists a significant difference between the importance of quality in the decision and the importance of time in which to obtain the decision. Another reason may be the best alternative is selected by comparing alternative solutions, testing against selected criteria, a task ideally suited for AHP.

In general, group decision making methods employed in the AHP can be largely classified into two ways: geometric mean method and arithmetic mean method. In the geometric mean method, as a most common group preference aggregation method in AHP literature, geometric mean of individual evaluations is used as elements in pairwise comparison matrices and then priority are computed. In the arithmetic mean method, a simple arithmetic mean of the individual priorities is used to arrive at the group consensus. Making group point estimates from individual judgments on each criterion is a solution alternative reflecting group members' diverse preferences. However, widely adopted aggregation methods adopted in AHP literatures do not guarantee the same group decision result and there do not exist clear guidelines for selection between two alternatives. In a situation where exact solutions are sometimes more important than probabilistic ones and thus combining judgments for a group working together is so important and can not be replaced by a statistical approach, aforementioned aggregation method is recommended to implement for deriving group judgments. Even in that case, a simulation approach which reflects diversification of group members' preference as it is, is useful as a complementary tool to get some analysis.

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