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로버스트 변수모형의 비선형 목표계획법 접근방법 (Nonlinear Goal Programming Approach for Robust Parameter Experiments)

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Abstract

Instead of using signal-to-noise ratio, we attempt to optimize both the mean and variance responses using dual response optimization technique. The alternative experimental strategy analyzes a robust parameter design problem to obtain the best settings that give a target condition on the mean while minimizing its variance. The mean and variance are treated as the two responses of interest to be optimized. Unlike to the crossed array and combined array approaches, our experimental setup requires replicated runs for each control factor's treatment under noise sampling. When the postulated response models are true, they enable the coefficients to be estimated and the desired performance measure to be analyzed more efficiently. The procedure and illustrative example are given for the dual response optimization techniques of nonlinear goal programming.

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1. Introduction

Most engineering design problems involve a large number of decision variables. Repeated experiences with experiments and simulations are required to come up with the most desirable decisions related to these variables.

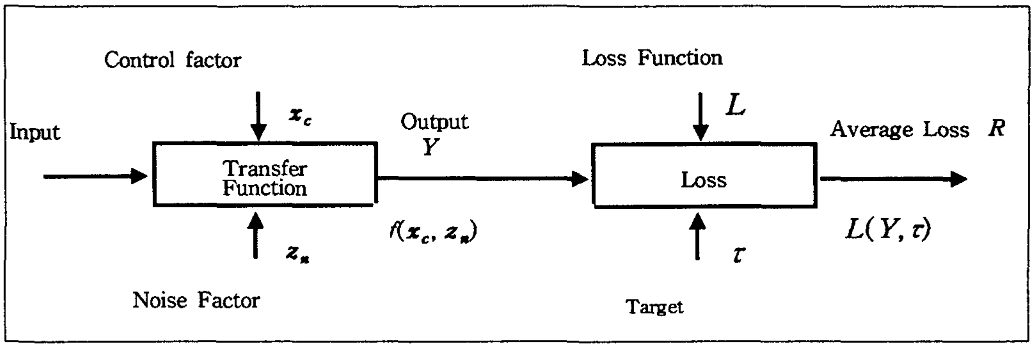
However, this process usually either takes long and/or is so expensive for completing a design that the design process is often terminated and the resulting product design is non-optimal. This results in a lower quality at an inflated cost. Taguchi [10,11,12] developed a robust parameter design method that uses mathematical tools to study a large number of decision variables with a small number of experiments to shorten the design time frame.

The main idea of robust parameter design is to reduce performance variation by reducing the sensitivity of an engineering design to the sources of variation rather than by controlling these sources. The objective of a robust parameter design is to choose settings of design factors to make a product(or process) performance insensitive to environmental variables, product deterioration, manufacturing imperfections and so on. The diagram in [Figure 1] shows elements and structure of a basic robust parameter design problem. The output Y is determined by two basic classes of experimental factors.

- *Control factors* : Design factors that can be easily controlled and adjusted at the designer's will.
- *Noise factors* : Factors that a designer cannot control for either physical or economical reasons.

Theses noise factors can be internal or external to the design. We denote that $\boldsymbol{x}_c$ and $\boldsymbol{z}_n$ are the vectors of p control factors $(x_1, x_2, \cdots, x_p)$ and q noise factors $(z_1, z_2, \cdots, z_q)$ respectively.

The most important aspect of Taguchi's approach is to decrease the system's sensitivity to noise variations on the product or process performance. The ultimate objective of robust parameter design is to find the best settings of control factors θ^* that minimize the variability of product performance attributed to noise factors.



[Figure 1] Block Diagram for Robust Parameter Design

By assuming that certain quality loss occurs through a loss function $L(Y, \tau)$ whenever the output response Y deviates from its target value τ, the robust parameter design problem can be formally stated as follow : Find the optimal value θ^* of design variables x_c that minimizes the average loss over the possible range of the noise variables z_n, that is, $R(\theta) = E_{z_n}L[Y, \tau]$. The simplest quadratic loss function is $L(Y, \tau) = k(Y-\tau)^2$, where k is a proportional constant.

Although the desired objective is to minimize the average loss $R(\theta)$, Taguchi uses a different performance measure, which is called signal-to-noise ratio Z. The signal-to-noise ratio is calculated for each design treatment combination from the response obtained from the experimental run at the noise treatment combination around each design point. Taguchi classifies the robust parameter design problems into different categories and defines over 60 signal-to-noise ratios corresponding to each class. The signal-to-noise ratio is an arbitrary combination of the mean and the standard deviation of the observations at each point of the control factors.

The three basic classes and their corresponding signal-to-noise ratio are as follows :

- On-target :

$$Z_{OT} = 10 \log_{10} [\overline{y}^2/s^2]$$

where the objective is closeness to specific target τ.

- Larger-the-better :

$$Z_{LTB} = -10 \log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{y_i^2}\right)\right]$$

where the goal is to make the response as large as possible.

- Smaller-the-better :

$$Z_{STB} = -10 \log_{10}\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right]$$

where the objective is to make the response as small as possible, where $y_1, y_2, \cdots, y_n$ represent response values of n observations, $\bar{y}$ is the sample mean, and s^2 is the sample variance. However, he does not provide any proofs or justifications of these performance measure classifications [7].

The objective of a robust parameter design is to determine the control factor levels which yield the largest value of signal-to-noise ratio. Taguchi depends on two experimental arrays which are saturated or nearly saturated orthogonal arrays; one for the control factors (control array) and another for the noise factors (noise array). Each point of the control array is replicated according to the noise array, which can result in a very large number of runs. Each noise array provides an estimate of the value of signal-to-noise ratio, which is the performance optimization criterion R. Then, the control factor levels that maximize the signal-to-ratio are selected by analyzing the main effects and some two-factor interactions, if they exist, based on the marginal means.

Although Taguchi's concept provides a powerful tool for improving product design, some of the methods have been the subject of both controversy and criticism [1,7]. In this paper, we propose an alternative approach, which is called a dual response optimization approach, for a robust parameter design problem to obtain the best settings yielding a target condition on the mean while minimizing its variance. The essential feature of our approach is treating both the mean and the variance as two responses of interest to be optimized[4,8].

We characterized the experimental setup and procedure, and formulate mathematical programming models for the robust parameter design problems as a multi-response optimization problem. The mathematical programming models include all the three basic areas: (1)on-target, (2) larger-the-better, and (3) smaller-the-better problems.

2. Dual response Optimization Approach

In this section, we propose an alternative experimental setup for robust parameter design. Instead of controlling the noise factors during the experiment, we run replicated trials for each combination of control factors and measure process variation with sample variance[4,8]. The replication scheme should be performed in a way that sufficient information is obtained about the noise factors. Our proposed procedure, called dual response optimization procedure, consists of five steps as follow:

- **Step 1**: Specify all the necessary elements in design, and define quality performance objective and constraints.
- **Step 2**: Plan a second-order response surface design replicating each point of control factor setting according to the appropriate randomized sampling scheme.
- **Step 3**: Conduct the experiment according to Step 2 from the observed data, characterize appropriate models relating to the mean and variance to control factors using the least square estimates.
- **Step 4**: Perform the multi-response optimization using two models from Step 3 to identify the improved control factor settings.
- **Step 5**: Conduct a confirmatory experiment to compare the performance of the new settings from Step 4 and the initial settings. If necessary, iterate the procedure.

2.1 Step 1

The necessary elements of a design in Step 1 include identifying all experimental variables by control factors and noise factors, and choosing those factor levels. Let $\mathbf{x}_c$ and $\mathbf{z}_n$ be the determined sets of control factors and noise factors respectively, where $\mathbf{x}_c = (x_1, x_2, \cdots, x_p)$ and $\mathbf{z}_n = (z_1, z_2, \cdots, z_q)$ if there are p control factors and q noise factors. Responses are the required product or process performance quality characteristics. The performance targets and constraints on the control factors are specified.

2.2 Step 2

Prior to Step 2, screening experiments are recommended to find an appropriate experimental region R as the preliminary phase of experimentation. As a framework for a basic sequential approach to the response surface design, the gradient of the linear response surface is repeated until it provides a sufficiently good fit.

A second-order model for the control factors will be used in Step 2 because it provides a good indication of general trends through the curvature of second degree polynomials and also because it captures the important interactions among the experimental variables.

The replication scheme at each combination of the control factor settings can be any proper randomization method in order that the variability of noise factors z_n could be fully investigated. It is recommended that the noise factor levels for each set of replicates are chosen randomly rather than at the extremes of their ranges so that an estimate of the variance is not inflated. This can be accomplished by using the fractional factorial designs as in the basic sampling, for example, two-level factorial designs for the two noise factors.

2.3 Step 3

Suppose that we have selected an n-point second-order response surface design and that each design point of this design is replicated a total of m times according to the randomization scheme in Step 2. Let y_{uv} be the vth response at the uth design point, where $u=1,\ 2,\ \cdots,\ n$ and $v=1,2,\cdots,m$. Then, the point estimators of the mean and variance at the uth design point are

$$\overline{y_u} = \frac{1}{m}\sum_{v=1}^{m} y_{uv} \qquad \text{and}$$

$$s_u^2 = \frac{1}{m-1}\sum_{v=1}^{m}(y_{uv} - \overline{y_u})^2$$

Each of these summary statistics of the mean and variance is then fitted as a separate quadratic function through the usual least square estimates. When the range of variance is large, the transformation of data is recommended. In general, the log of the variance is used as the dependent variable.

Let $f_\mu(x_c, z_n)$ and $f_\sigma(x_c, z_n)$ be the fitted response surface functions for the mean and variance (or standard deviation) of the performance characteristics of interest. Assume that the two response may be modelled by

$$f_\mu(x_c, z_n) = \beta_0 + \sum_{i=1}^{p} \beta_i x_i + \sum_{i=1}^{p} \beta_{ii} x_i^2 + \sum_{1 \le i < j}^{p} \beta_{ij} x_i x_j + \varepsilon$$

and

$$f_\sigma(x_c, z_n) = \gamma_0 + \sum_{i=1}^{p} \gamma_i x_i + \sum_{i=1}^{p} \gamma_{ii} x_i^2 + \sum_{1 \le i < j}^{p} \gamma_{ij} x_i x_j + \varepsilon$$

where ε are random errors.

2.4 Step 4, 5

Once the models are characterized, we can use dual response optimization techniques. In this paper, we use goal programming to optimize both responses according to the priority levels. The details of goal programming approach will be discussed in the following section. To check the adequacy of the solution to the dual response optimization problem, confirmatory experiments are conducted.

3. Framework of Goal Programming

Goal programming is a type of mathematical programming used to solve optimization problems that contain multiple conflicting objectives(goals). The goal programming approach can give extensive solutions by ranking the preemptive priorities for multiple goals according to the ultimate purpose of the experiment. The goal programming rationale then provides a solution to the problem that minimizes the absolute deviations from the specified objectives by the specified priority order of goal attainment.

Goal programming can be applied in application problems with a single objective and multiple subgoal as well as cases having multiple goals and subgoal. Within the framework of the goal programming model, goals may be achieved only at the expense of other goals. Although it may not be possible to optimize every goal, goal programming attempts to obtain satisfactory levels of goal attainment that represent the best possible combination of goal achievement. The only attempt to applying the goal programming approach to a quality type problem has been done by Sengupta [9]. He uses a linear goal programming model to satisfy multiple quality specification objectives for the paper production process.

The general goal programming model formulation for n variables with m goals to be summarized as follow :

$$Min \quad \bar{a} = \sum_{k=0}^{K}\sum_{i=1}^{m} P_k (w_{ik}^- d_i^- + w_{ik}^+ d_i^+)$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_{ij} + d_i^- - d_i^+ = b_i \qquad i = 1, 2, \cdots, m$$

$$\underline{x},\ \underline{d}^-,\ \underline{d}^+ \geq \underline{0}$$

where P_k is the preemptive priority weight ($P_k \gg P_{k+1}$) assigned to goal k, w_{ik}^- and w_{ik}^+ are numerical weights assigned to the deviational variables of goal i at a given priority level k, d_i^-, and d_i^+ represent the negative and positive deviations, a_{ij} is the technological coefficient of x_j in goal i, and b_i is the ith goal level.

We analyze each one of the m goals considered in the model in terms of whether over or under achievement of the goal is satisfactory. If over achievement is acceptable, d_i^+ can be eliminated from the objective function. On the other hand, if under achievement is satisfactory, d_i^- may not be included in the objective function. If the exact achievement of the goal is desired both d_i^- and d_i^+ must be represented in the objective function. The deviational variables d_i^- and d_i^+ must be ranked according to their preemptive priority weights from the most important and the least important in the achievement function, so that the low order goals are considered only after the higher order goals are achieved. In this paper, we will consider the three basic cases in the following sets of multiple goals, arranged in order of their importance :

[On-target]

- Keep μ at the specified target value μ_0
- Minimize σ^2

or

- Minimize σ^2
- Keep μ at the specified target value μ_0

[Larger-the-better]

- Maximize μ
- Control σ^2 at the minimum acceptable value σ_0^2

[Smaller-the-better]

- Minimize μ
- Control σ^2 at the minimum acceptable value σ_0^2

4. Goal Programming Formulation

4.1 On-target Problem

The decision on ranking the priorities between two goals depends upon the ultimate purpose of the experiment. For the on-target problem, we will consider both cases to observe the difference that caused by assignment of priority. For the remainder, we will find the best compromise solution by setting several possible goal levels for the variance.

In order to formulate these problems as a goal programming, we need a preliminary step. To satisfy the non-negativity constraints on variables x, we need to modify the specification limits of those variables. We transform both sided specification limits as $0 \leq x \leq \beta$ for the second-order model instead of $-\alpha \leq x \leq \alpha$. If $\alpha = 1$, for example, $\beta = 2$ is an appropriate value. Therefore, the performance response model of interest also needs to be modified accordingly.

Let us suppose $f_\mu(x)$ and $f_\sigma(x)$ are adjusted fitted models for the mean and standard deviation of the character of interest respectively, where $\underline{x}$ are modified as of non-negativity. The general robust parameter design problem can be formulated as a nonlinear goal programming as follow :

$$Min\ \bar{a} = \ P_1(d_1^- + d_1^+),\ P_2 d_2^+,\ P_3(\sum_{i=1}^{p} d_{i+2}^+)$$

Subject to

$$\begin{aligned} f_\mu(\underline{x}) + d_1^- - d_1^+ &= \mu_0 \\ f_\sigma(\underline{x}) + d_2^- - d_2^+ &= \sigma_0 \\ x_j + d_{j+2}^- - d_{j+2}^+ &= \beta \quad j=1,2,\cdots,p \\ \underline{x}\ ,\ \underline{d}^-\ ,\ \underline{d}^+ &\geq \underline{0} \end{aligned}$$

The first priority is to achieve the target value of mean μ_0 through minimization of d_1^- and d_1^+, and the second priority is not to exceed the pre-limited standard deviation σ_0 by minimizing d_2^+. The final priority is setting an appropriate experimental region R.

On the other hand, if we swap the priorities of two responses, the achievement function and the first two objective functions will be changed as follow :

$$Min\ \bar{a} = \ P_1 d_1^+,\ P_2(d_2^- + d_2^+),\ P_3(\sum_{i=1}^{p} d_{i+2}^+)$$

Subject to

$$\begin{aligned} f_\sigma(\underline{x}) + d_1^- - d_1^+ &= \sigma_0 \\ f_\mu(\underline{x}) + d_2^- - d_2^+ &= \mu_0 \\ x_j + d_{j+2}^- - d_{j+2}^+ &= \beta \quad j=1,2,\cdots,p \\ \underline{x}\ ,\ \underline{d}^-\ ,\ \underline{d}^+ &\geq \underline{0} \end{aligned}$$

That is, our first priority is to satisfy our objective of never exceeding the acceptable standard deviation σ_0 by minimizing d_1^+. The second priority is to achieve the target value of mean through the minimization of d_2^- and d_2^+.

4.2 Larger-the-better Problem

The larger-the-better problem can be formulated as follow by the same manner.

$$Min \quad \overline{a} = \quad P_1 d_1^-, \ P_2 d_2^+, \ P_3(\sum_{i=1}^{p} d_{i+2}^+)$$

Subject to

$$f_\mu(\underline{x}) + d_1^- - d_1^+ = \mu_0$$
$$f_\sigma(\underline{x}) + d_2^- - d_2^+ = \sigma_0$$
$$x_j + d_{j+2}^- - d_{j+2}^+ = \beta \quad j=1,2,\cdots,p$$
$$\underline{x} \ , \underline{d}^- \ , \underline{d}^+ \quad \geq \underline{0}$$

The first priority is assigned to the maximization of mean response and is achieved by minimizing d_1^-. The second priority is to control the standard deviation by minimizing d_2^+.

4.3 Smaller-the-better Problem

The smaller-the-better problem can be formulated as follow by the same manner.

$$Min \quad \overline{a} = \quad P_1 d_1^+, \ P_2 d_2^+, \ P_3(\sum_{i=1}^{p} d_{i+2}^+)$$

Subject to

$$f_\mu(\underline{x}) + d_1^- - d_1^+ = \mu_0$$
$$f_\sigma(\underline{x}) + d_2^- - d_2^+ = \sigma_0$$
$$x_j + d_{j+2}^- - d_{j+2}^+ = \beta \quad j=1,2,\cdots,p$$
$$\underline{x} \ , \underline{d}^- \ , \underline{d}^+ \quad \geq \underline{0}$$

The situation is similar to the larger-the-better problem except a minimization the mean response by minimizing d_1^+. Note that for one sided specification limits, only one of the positive or negative deviational variable will be optimized. But for the closeness type of constraint, both the negative and positive deviational variables have to be considered.

5. Example : Chemical Process

5.1 Problem Definition and Preliminary Phase

In this section, we show an example of nonlinear goal programming technique to the robust parameter designs. The following example comes from Lawson [6]. In the chemical process, side reactions creates tars that bring lower product quality. The product having high level of tars result in lower yield and require further blending that would increase the production cost.

In this experiment, reaction temperature at the initial mixing point (x_1), the catalyst concentration(x_2), and the excess of reagent B (x_3) are considered as the control factors (x_p, $p=3$), and purity of regent A (z_1) and purity of the solvent stream (z_2) are treated as important noise factors (z_q, $q=2$) by the priori knowledge of the chemical process. An interested quality response characteristic is the resulting level of tars in the chemical process, and the objective of this experiment is to find the best settings of control factors θ^* that gives the lowest level of tar without tightening the specification limits on purity of regent A or the purity of solvent.

The Box-Behnken design for three factors of 15 runs($n=15$) is selected as an appropriate second-order response surface design. To study the effect of the two noise factors, a 2^2 factorial design($m=4$) is replicated at
each combination of the three control factor settings. The total of $n \times m = 60$ runs of experiment are performed in random order. <Table 1> summarizes the data.

The fitted response surface for the mean of the characteristic of interest 'tar' is

$$\hat{y}_\mu = 14.9 - 8.19x_1 - 9.09x_2 - 0.13x_3 + 0.51x_1^2 + 5.01x_2^2 + 8.33x_1x_2 \qquad (1)$$

The analysis of variance for a full second-order model is in <Table 2>. Those terms x_3^2, x_1x_3 and x_2x_3 seem to be an insignificant. Therefore, a reduced equation resulting in (1).

We choose to model the log of variance rather than the standard deviation. By observing the wide range for the values of variance, the assumption of homogeneity for variance will be more suitable when the log of variance is used rather than the standard deviation itself

<Table 1> The Chemical Process Data

u	x_1	x_2	x_3	y_{u1} y_{u2} y_{u3} y_{u4}	$\overline{y}_u$	s_u
1	-1	-1	0	57.81 32.29 47.07 42.87	46.26	8.68
2	1	-1	0	24.89 4.35 14.69 8.23	13.04	8.98
3	-1	1	0	13.21 9.51 11.19 10.10	11.00	1.63
4	1	1	0	13.39 9.15 11.23 10.30	11.02	1.80
5	-1	0	-1	27.71 20.24 24.32 22.28	23.64	3.19
6	1	0	-1	11.40 4.48 8.23 5.44	7.39	3.11
7	-1	0	1	30.65 18.40 24.45 20.24	23.49	5.44
8	1	0	1	14.94 2.29 8.49 4.30	7.51	5.59
9	0	-1	-1	42.68 22.42 30.30 21.64	29.26	9.76
10	0	1	-1	13.56 10.08 11.38 9.85	11.22	1.70
11	0	-1	1	50.60 13.19 30.97 18.84	28.40	16.55
12	0	1	1	15.21 7.44 11.82 9.78	11.06	3.29
13	0	0	0	19.62 12.29 14.54 13.14	14.90	3.28
14	0	0	0	20.60 11.49 13.39 12.06	14.41	4.21
15	0	0	0	20.15 12.20 13.89 14.06	15.08	3.49

<Table 2> The ANOVA Table for the Mean Response

Source	df	SS	MS	F	R^2
Model	9	6255.5	695.1	15.65	0.7575
Lack of fit	5	3.7	0.7	0.02	
Pure	45	1998.7	44.4		
Variable		t-ratio		p	
Constant		7.690		0.001	
x_1		-6.937		0.001	
x_2		-7.709		0.001	
x_3		-0.117		0.429	
x_1^2		0.300		0.066	
x_2^2		2.892		0.001	
x_3^2		0.102		0.453	
x_1x_2		4.987		0.001	
x_1x_3		0.048		0.767	
x_2x_3		0.106		0.454	

<Table 3> gives the analysis of variance for the full second-order model. By observing that all interaction terms seems to be insignificant, the resulting fitted model for the log of the variance results in

$$\hat{y}_\sigma = 2.66 + 0.03x_1 - 1.54x_2 + 0.53x_3 - 0.19x_1^2 + 0.40x_2^2 + 0.45x_3^2 \quad (2)$$

<Table 3> The ANOVA Table for the Variance Response

Source	df	SS	MS	F	R^2
Model	9	22.8	2.5	91.4	0.994
Error	5	0.1	0.0		
Total	14	22.9	2.5		
Variable	t-ratio			p	
Constant	27.632			0.001	
x_1	0.477			0.605	
x_2	-26.221			0.001	
x_3	9.072			0.001	
x_1^2	-2.165			0.069	
x_2^2	4.594			0.013	
x_3^2	5.169			0.003	
x_1x_2	0.239			0.722	
x_1x_3	0.288			0.773	
x_2x_3	0.192			0.479	

The lack of fit for the reduced model is insignificant. Therefore we can use this fitted model in further developments.

Because that the goal of this experiment is identifying the set of three control factors settings to minimize the level of tars, the formation for an optimization technique is classified the smaller-the better situation.

5.2 Nonlinear Goal Programming Approach

In order to satisfy non-negativity, we transform the variables $-1 \le x \le 1$ to

$0 \le \underline{x} \le 2$. The fitted models in (1) and (2) are therefore modified as follow :

$$f_{\mu}(\underline{x}) = 11.68 + 1.21x_1 + 9.46x_2 + 0.50x_3 + 0.51x_1^2 + 5.01x_2^2 + 8.33x_1x_2 \quad (3)$$

and

$$f_{\sigma}(\underline{x}) = 2.78 - 0.31x_1 - 1.08x_2 + 1.47x - 0.19x_1^2 + 0.40x_2^2 + 0.45x_3^2 \quad (4)$$

Hence the goal programming formulation for the chemical process is as follow :

Find $\underline{x} = (x_1, x_2, x_3)$ so as to

$$Min\ \bar{a} = P_1 d_1^+ + P_2 d_2^+ + P_3(d_3^+ + d_4^+ + d_5^+) \quad (5)$$

Subject to

$$1.21x_1 + 9.46x_2 + 0.50x_3 + 0.51x_1^2 + 5.01x_2^2 + 8.33x_1x_2 + d_1^- - d_1^+ = \mu_0 - 11.68$$

$$-0.31x_1 - 1.08x_2 + 1.47x_3 - 0.19x_1^2 + 0.40x_2^2 + 0.45x_3^2 + d_2^- - d_2^+ = \sigma_0 - 2.78$$

$$x_1 + d_3^- - d_3^+ = 2.0$$

$$x_2 + d_4^- - d_4^+ = 2.0$$

$$x_3 + d_5^- - d_5^+ = 2.0$$

$$\underline{x},\ \underline{d}^-,\ \underline{d}^+ \ge \underline{0}$$

where μ_0 and σ_0 are selected values for the mean and log of the variance. Following the earlier assumption, we use 8.0 and 3.5 as the suggested values for μ_0 and σ_0. Then, the first two goal levels of constraints in (5) become -3.68 and 0.72, respectively.

The best solution is found at $x_1 = 1.998$, $x_2 = 0.998$, and $x_3 = 0.998$ which produces achievement function values of 40.1618, 0.7623, and 0.0 for each priority levels. The Hooke and Jeeves pattern search algorithm [3,5] was used for this optimization. After transforming back to the original variables, the resultant optimal settings θ^* are $x_1 = 0.998$, $x_2 = -0.002$, and $x_3 = -0.002$ which give predicted mean value 7.136 and 2.503 for the standard deviation.

5.3 Taguchi's Signal-to-Noise Ratio

In order to compare the results of the proposed method, we attempt to find the optimal settings θ^* that give a maximum value of the smaller-the-better performance measure $Z_{STB} = -10\log(\Sigma_{i=1}^{n} y_i^2/n)$ from Taguchi's signal-to-noise ratio. The Z_{STB} values for each set of replicated values are in <Table 4>.

Because the control array is not a type of a first-order orthogonal array, we cannot analyze the results by the marginal means. Thus we use a nonlinear programming method to find the maximum value Z_{STB} over the experimental region. The fitted second-order model of Z_{STB} is

$$\begin{aligned} f_Z &= -54.34 + 8.04x_1 + 9.42x_2 - 0.46x_3 + 1.57x_1^2 \\ &\quad -4.14x_2^2 - 0.21x_3^2 - 5.65x_1x_2 - 0.62x_1x_3 + 0.17x_2x_3 \end{aligned} \qquad (6)$$

The optimal settings θ^* determined by constrained nonlinear optimization is $x_1 = 1.0$, $x_2 = 0.435$ and $x_3 = -1.0$, which produce a Z_{STB} value of -42.807. Note that the location of the optimal values for x_1 and x_2 are not affected much by deviations in the values of x_3.

<Table 4> The Signal-to-Noise Performance Measure for Chemical Process Data

u	Z_{STB}	u	Z_{STB}
1	-76.94	9	-68.33
2	-54.40	10	-48.52
3	-48.12	11	-69.20
4	-48.19	12	-48.71
5	-63.39	13	-54.38
6	-41.25	14	-53.98
7	-63.48	15	-54.66
8	-43.80		

5.4 Comparison

Consistent with the insight we got from the contour plots in [Figure 2], the dual response optimization technique presented in this paper resulted in neighboring points of the optimal setting θ^* of the three control factors around $x_1 = 1.0$, $x_2 = 0.0$, and $x_3 = 0.0$. The resulting optimal settings are summarized in <Table 5>.

<Table 5> Comparison of Optimal Settings

Optimization Techniques	Optimal Settings(θ^*)			Predicted Value	
	x_1	x_2	x_3	Mean	Std. Dev.
Contour Plots	1.0	0.0	0.0	-	-
Nonlinear Goal Programming	0.998	-0.002	-0.002	7.136	2.503
Signal-to-Noise Ratio	1.0	0.435	-1.0	-	-

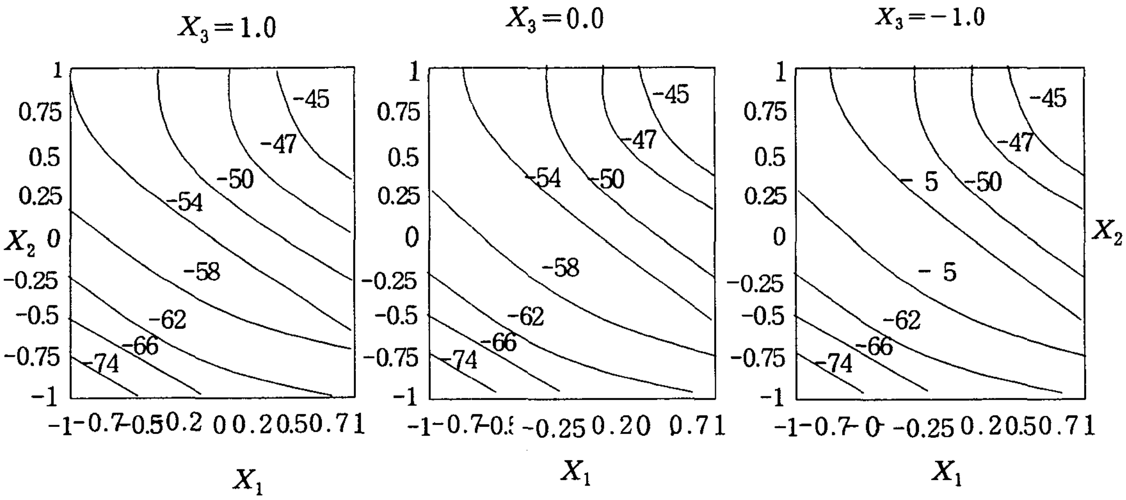
However, Taguchi's signal-to-noise ratio leads the optimal settings θ^* far from its projected optimum. By exploiting the fact that the location of optimal values of x_1 and x_2 are not sensitive to deviations from x_3 in the contour plot in [Figure 2], we explored different settings other than those derived from constrained nonlinear optimization models. For example, $x_1 = 1.0$, $x_2 = 0.472$ and $x_3 = 0.0$, which is found as an optimum by fixing $x_3 = 0.0$, produces the maximum Z_{STB} value of -42.872, and is a more desirable setting. Because it does not give direct point estimates of predicted values of the mean and variance of quality characteristic, we cannot directly compare them.

The potential difficulty for the nonlinear goal programming approach arises only from the limitation of the algorithm used to solve the goal programming problem. Based on our computational experience, the modified pattern search method by Ignizio [5] is very sensitive to the initial guess values. It is therefore suggested that the pattern search method should be attempted after getting the reasonable starting point estimate from other methods.

6. Discussion and Summary

The most obvious benefit of dual response optimization approach to robust parameter design is to allow the use of broad range of design, not restricted to a specific set of designs. The interaction effects, which have not been accounted for Taguchi's approach, are adequately dealt with by implementing response surface designs. Although Taguchi's signal-to-noise ratio has intuitive engineering interpretation, it has been criticized because the transformation of mean and variance information was postulated without proof or substantiation [1,7]. By avoiding the signal-to-noise ratio as the performance criteria, our methodology is a more natural approach to addressing the concurrent information about mean and variance measure.

Our approach can handle the more demanding multiple quality characteristics, which requires the simultaneous optimization of more than one quality response in the same products or processes. For example, in this chemical process, the catalyst activity which affects a production rate could be joined as an additional quality response to be optimized simultaneously. By using the dual response optimization approach, we can determine the optimal settings of control factors that result in low level of tars as well as high level of catalyst activity in a single experimental effort. In other words, our approach can be applied dual response models that provide the optimal settings for the control factors which optimize all response toward their targets.



[Figure 2] Contour Plots for Singal-to-Noise Ratio

It should be noted that the dual response optimization approach relies on a fitted model so that model selection, transformation, and model checking methods should be utilized prior to optimization to guard against false dependence on a specified model. In addition, although the proposed method allows the generalization of the optimization of the dual response, certain additional consideration needed to be explored because of the assumption utilized.

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