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Vibration Analysis of Rotating Composite Cantilever Plates

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A modeling method for the vibration analysis of rotating composite cantilever plates is presented in this paper. The coupling effects between inplane motions and the bending motion are considered and explicit mass and stiffness matrices are derived for the modal analysis. Numerical results are obtained and some of them are compared to those of a commercial program to confirm the accuracy of the present method. Numerical results show that the coupling effects become important only when laminates are stacked up unsymmetrically. Incidentally, natural frequencies loci veering, loci crossing, and associated mode shape variations are observed.

Key Words : Rotating Composite Cantilever Plate, Coupling Effect, Natural Frequency Loci Veering, Loci Crossing, Mode Shape Variation

1. Introduction

Composite materials, especially laminated composite plates, have been widely used in various kinds of engineering such as aeronautics, astronautics, and marine structures. In addition to the advantages of high strength (as well as high stiffness) and light weight, another advantage of laminated composite plate is the controllability of the structural properties through changing the fiber orientation angles and the number of plies and selecting proper composite materials, over a wide range.

As the importance of high strength and light weight rises, the study on vibration of composite structures has been actively progressed in the 1980s. Flexible structures having slender shapes are often idealized as beams since reliable and robust theories for beams, which can provide accurate numerical results in most cases, are available. Many structures, however, have plate-like shapes (rather than beam-like shapes). Solar panels of satellites, turbine blades, and aircraft rotary wings with small aspect ratios are such examples. These structures can be analyzed more accurately by modeling them as plates rather than beams. Recently, many research works for rotating cantilever plate have been made (Dokanish, 1971; Ramamurti, 1984). These works employed finite element techniques and strain energy expressions which were obtained from equilibrium conditions between the centrifugal inertia forces and the steady-state in-plane stress components. On the basis of this approach, the modal characteristics of rotating plates could be estimated by calculating explicit stiffness matrices. This approach, however, involves with unnecessary assumptions and complexities which result in two-step procedure to derive the equations of motion for rotating plates. Due to the complexities, this approach is extremely difficult to be applied to practical problems. Recently, a new modeling method, which employs a hybrid set of deformation variables, was introduced (Yoo, 2001). This modeling method is as efficient as the above

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method but much simpler than the previous method in deriving the equations of motion and performing the numerical analysis. This modeling method, however, was applied to only isotropic plates rather than composite plates so far.

The purpose of this paper is to investigate the variations of modal characteristics of the rotating composite plate. The equations of motion are derived and transformed into dimensionless forms. Dimensionless parameters are identified and the effects of angular speed and the fiber orientation angle on the modal characteristics are investigated. Especially, the importance of the coupling effects between inplane and bending motion is clarified. Incidentally, natural frequency loci veering, loci crossing, and associated mode shape variations are exhibited and discussed.

2. Formulation for Vibration Analysis

2.1 Stain energy of composite cantilever plates

Figure 1 shows that a rotating rectangular plate which is characterized by natural length a, width b and thickness h is attached to a rigid hub which rotates with a constant angular speed Ω. The laminated plate geometry and ply numbering system is shown in Fig. 2 and the coordinates and fiber direction of kth-layer cross-ply laminated composite plate are shown in Fig. 3. As shown in Fig. 2, the plies are numbered from bottom to top.

In the present work, two in-plane variables along with the lateral displacement variable are approximated to obtain the ordinary differential equations of motion. By using the Rayleigh-Ritz method, the approximations are given as follows:

$$s(x, y, t)=\sum_{i=1}^{\mu} \phi_{1i}(x, y)\, q_{1i}(t) \tag{1}$$

$$r(x, y, t)=\sum_{i=1}^{\mu} \phi_{2i}(x, y)\, q_{2i}(t) \tag{2}$$

$$w(x, y, t)=\sum_{i=1}^{\mu} \phi_{3i}(x, y)\, q_{3i}(t) \tag{3}$$

where ϕ_{1i}, ϕ_{2i}, and ϕ_{3i} are spatial mode functions. Any compact set of admissible functions, which satisfy the geometric boundary conditions of the plate can be used as the mode functions. q_i's are generalized coordinates and μ is the total number

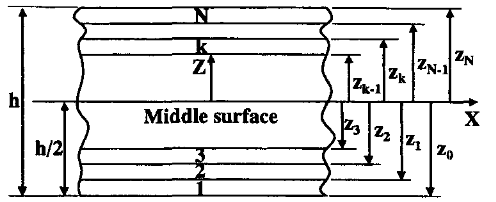


Fig. 1 Configuration of a rotating composite rectangular plate

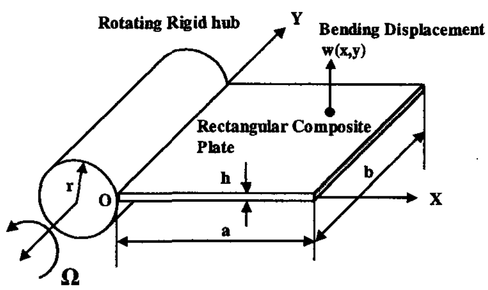


Fig. 2 Laminated plate geometry and ply numbering system

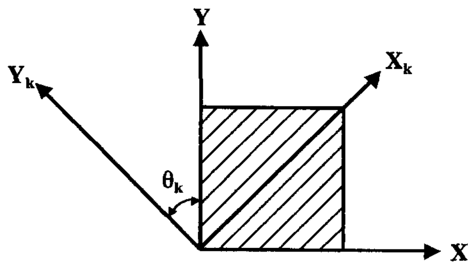


Fig. 3 Fiber direction of kth-layer cross-ply laminated composite plate

of the generalized co-ordinates. The transverse shear deformation is ignored to simplify the formulation employed in this work. The shear effect becomes important when the plate has a dimension with large thickness. Including the shear effect is not a major problem. Nonetheless, since it is not the major issue here, it is ignored in this work. Then, the elastic strain energy of a composite plate can be expressed as follows (Whitney, 1987):

$$U=\frac{1}{2}\int_0^b \int_0^a \Big[A_{11}\left(\frac{\partial s}{\partial x}\right)^2+2A_{12}\frac{\partial s}{\partial x}\frac{\partial r}{\partial y}$$
$$+A_{22}\left(\frac{\partial r}{\partial y}\right)^2+2\left(A_{16}\frac{\partial s}{\partial x}+A_{26}\frac{\partial r}{\partial y}\right)$$

$$\left(\frac{\partial s}{\partial y}+\frac{\partial r}{\partial x}\right)+A_{66}\left(\frac{\partial s}{\partial y}+\frac{\partial r}{\partial x}\right)^2$$
$$-2B_{11}\frac{\partial s}{\partial x}\frac{\partial^2 w}{\partial x^2}-2B_{12}$$
$$\left(\frac{\partial r}{\partial y}\frac{\partial^2 w}{\partial x^2}+\frac{\partial s}{\partial x}\frac{\partial^2 w}{\partial y^2}\right)-2B_{22}\frac{\partial r}{\partial y}\frac{\partial^2 w}{\partial y^2}$$
$$-2B_{16}\left[\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial s}{\partial y}+\frac{\partial r}{\partial x}\right)+2\frac{\partial s}{\partial x}\frac{\partial^2 w}{\partial x\partial y}\right] \quad (4)$$
$$-2B_{26}\left[\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial s}{\partial y}+\frac{\partial r}{\partial x}\right)+2\frac{\partial r}{\partial y}\frac{\partial^2 w}{\partial x\partial y}\right]$$
$$-4B_{66}\frac{\partial^2 w}{\partial x\partial y}\left(\frac{\partial s}{\partial y}+\frac{\partial r}{\partial x}\right)+D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2$$
$$+2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2$$
$$+4\left(D_{16}\frac{\partial^2 w}{\partial x^2}+D_{26}\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 w}{\partial x\partial y}$$
$$+4D_{66}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\Big]dxdy$$

where matrices A_{ij}, B_{ij}, and D_{ij} can be obtained by integrating material properties of each layer shown in Fig. 2 as follows:

$$A_{ij}=\int_{-h/2}^{h/2} Q_{ij}^{(k)}dz=\sum_{k=1}^{N} Q_{ij}^{(k)}(z_k-z_{k-1}) \quad (5)$$

$$B_{ij}=\int_{-h/2}^{h/2} Q_{ij}^{(k)}zdz=\frac{1}{2}\sum_{k=1}^{N} Q_{ij}^{(k)}(z_k^2-z_{k-1}^2) \quad (6)$$

$$D_{ij}=\int_{-h/2}^{h/2} Q_{ij}^{(k)}z^2dz=\frac{1}{3}\sum_{k=1}^{N} Q_{ij}^{(k)}(z_k^3-z_{k-1}^3) \quad (7)$$

where $Q_{ij}^{(k)}$ are the off-axis stiffnesses of kth layer (Whitney, 1987), z_k and z_{k-1} are the distance from the mid-plane to the top and bottom surface of the kth layer, and N is the total number of laminated layers.

2.2 Modal formulation

Using Eq. (4), equations of motion for the composite plate can be obtained. It is useful to rewrite these equations in a dimensionless form. For the purpose, the following dimensionless variables, parameter, and functions are introduced.

$$\tau\equiv\frac{t}{T},\ \xi\equiv\frac{x}{a},\ \eta\equiv\frac{y}{b},\ \vartheta_j\equiv\frac{q_j}{a}$$
$$\phi_i(x,y)\equiv\varphi_i(\xi,\eta),\ \omega\equiv\frac{\Omega}{\Omega_r},\ \sigma\equiv\frac{r}{a}$$
$$A_{ij}^1\equiv\frac{T^2}{\rho a^2}A_{ij},\ A_{ij}^2\equiv\frac{T^2}{\rho ab}A_{ij},\ A_{ij}^3\equiv\frac{T^2}{\rho b^2}A_{ij}$$
$$B_{ij}^1\equiv\frac{T^2}{\rho a^3}B_{ij},\ B_{ij}^2\equiv\frac{T^2}{\rho ab^2}B_{ij},\ B_{ij}^3\equiv\frac{T^2}{\rho a^2 b}B_{ij}$$
$$B_{ij}^4\equiv\frac{T^2}{\rho b^3}B_{ij},\ D_{ij}^1\equiv\frac{T^2}{\rho a^4}D_{ij},\ D_{ij}^2\equiv\frac{T^2}{\rho a^2b^2}D_{ij}$$
$$D_{ij}^3\equiv\frac{T^2}{\rho b^4}D_{ij},\ D_{ij}^4\equiv\frac{T^2}{\rho a^3 b}D_{ij},\ D_{ij}^5\equiv\frac{T^2}{\rho ab^3}D_{ij} \quad (8)$$

where ρ is the mass per unit area of the composite plate. Ω_r and T are given as

$$\Omega_r\equiv\sqrt{\frac{D_{11}}{\rho a^4}},\qquad T\equiv\frac{1}{\Omega_r} \quad (9)$$

Using these dimensionless variables and parameters, the following linear dimensionless equations of motion for composite plate can be derived eventually.

$$\sum_{j=1}^{\mu}\Big[M_{ij}^{11}\ddot{\vartheta}_{1j}+(-\omega^2M_{ij}^{11}+K_{ij}^{S1\,11,11}+K_{ij}^{S2\,11,12}$$
$$+K_{ij}^{S2\,11,21}+K_{ij}^{S3\,11,22})\vartheta_{1j}+(K_{ij}^{S2\,12,22}+K_{ij}^{S1\,12,12}$$
$$+K_{ij}^{S3\,12,11}+K_{ij}^{S2\,12,21})\vartheta_{2j}+2\omega M_{ij}^{13}\dot{\vartheta}_{3j} \quad (10)$$
$$+(-K_{ij}^{C1\,1,111}-K_{ij}^{C2\,1,122}-K_{ij}^{C3\,1,211}-2K_{ij}^{C3\,1,112}$$
$$-K_{ij}^{C4\,1,222}-2K_{ij}^{C2\,1,212})\vartheta_{3j}\Big]=0$$

$$\sum_{j=1}^{\mu}\Big[M_{ij}^{22}\ddot{\vartheta}_{2j}+(K_{ij}^{S3\,22,22}+K_{ij}^{S2\,22,21}+K_{ij}^{S2\,22,12}$$
$$+K_{ij}^{S1\,22,11})\vartheta_{2j}+(-K_{ij}^{C3\,2,211}-K_{ij}^{C4\,2,222}-K_{ij}^{C1\,2,111}$$
$$-K_{ij}^{C2\,2,122}-2K_{ij}^{C2\,2,212}-2K_{ij}^{C2\,2,112})\vartheta_{3j}+(K_{ij}^{S2\,21,21}$$
$$+K_{ij}^{S1\,21,11}+K_{ij}^{S3\,21,22}+K_{ij}^{S2\,21,12})\vartheta_{1j}\Big]=0 \quad (11)$$

$$\sum_{j=1}^{\mu}\Big[M_{ij}^{33}\ddot{\vartheta}_{3j}+(-\omega^2M_{ij}^{33}+K_{ij}^{B1\,11,11}+K_{ij}^{B2\,11,22}$$
$$+K_{ij}^{B2\,22,11}+K_{ij}^{B3\,22,22}+2K_{ij}^{B4\,11,12}+2K_{ij}^{B4\,12,11}$$
$$+2K_{ij}^{B5\,22,12}+2K_{ij}^{B5\,12,22}+4K_{ij}^{B2\,12,12}+\sigma\omega^2K_{ij}^{GX1}$$
$$+\omega^2K_{ij}^{GX2})\vartheta_{3j}+(-K_{ij}^{C1\,1,111}-K_{ij}^{C2\,1,122} \quad (12)$$
$$-K_{ij}^{C3\,1,211}-2K_{ij}^{C3\,1,121}-K_{ij}^{C4\,1,222}-2K_{ij}^{C2\,1,221})\vartheta_{1j}$$
$$+(-K_{ij}^{C3\,2,211}-K_{ij}^{C4\,2,222}-K_{ij}^{C1\,2,111}-K_{ij}^{C2\,2,122}$$
$$-2K_{ij}^{C2\,2,221}-2K_{ij}^{C3\,2,121})\vartheta_{2j}-2\omega M_{ij}^{31}\dot{\vartheta}_{1j}\Big]=0$$

where

$$M_{ij}^{kl}=\int_0^1\int_0^1 \varphi_{ki}\varphi_{lj}d\xi d\eta \quad (13)$$

$$K_{ij}^{S\alpha\,kl,mn}=A_{ij}^{\alpha}\int_0^1\int_0^1 \varphi_{ki,\Gamma_m}\varphi_{lj,\Gamma_n}d\xi d\eta \quad (14)$$

$$K_{ij}^{C\alpha\,k,lmn}=B_{ij}^{\alpha}\int_0^1\int_0^1 \varphi_{ki,\Gamma_l}\varphi_{3j,\Gamma_m\Gamma_n}d\xi d\eta \quad (15)$$

$$K_{ij}^{B\alpha\,kl,mn}=D_{ij}^{\alpha}\int_0^1\int_0^1 \varphi_{3i,\Gamma_k\Gamma_l}\varphi_{3j,\Gamma_m\Gamma_n}d\xi d\eta \quad (16)$$

$$K_{ij}^{GX1}=\int_0^1\int_0^1 (1-\xi)\,\varphi_{3i,\xi}\varphi_{3j,\xi}d\xi d\eta \quad (17)$$

$$K_{ij}^{GX2}=\int_0^1\int_0^1 \frac{1}{2}(1-\xi^2)\,\varphi_{3i,\xi}\varphi_{3j,\xi}d\xi d\eta \quad (18)$$

where a comma denotes partial differentiation with respect to subscripts that follow. If m is 1, Γ represents ξ and if m is 2, Γ represents η.

Using Eqs. (10), (11), and (12), the matrix form of the equations of motion can be derived as follows:

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{0} \quad (19)$$

where

$$\mathbf{M}=\begin{bmatrix} M_{ij}^{11} & 0 & 0 \\ 0 & M_{ij}^{22} & 0 \\ 0 & 0 & M_{ij}^{33} \end{bmatrix} \quad (20)$$

$$\mathbf{C}=\begin{bmatrix} 0 & 0 & 2\omega M_{ij}^{13} \\ 0 & 0 & 0 \\ -2\omega M_{ij}^{31} & 0 & 0 \end{bmatrix} \quad (21)$$

$$\mathbf{K}=\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \quad (22)$$

where **K** is the symmetric matrix whose respective element matrices K_{ij} are defined as

$$K_{11}=-\omega^2M^{11}+K^{S1\,11,11}+K^{S2\,11,12}+K^{S2\,11,21}+K^{S3\,11,22}$$

$$K_{12}=K_{21}=K^{S2\,12,22}+K^{S1\,12,12}+K^{S3\,12,11}+K^{S2\,12,21}$$

$$K_{13}=K_{31}=-K^{C1\,1,111}-K^{C2\,1,122}-K^{C3\,1,211}-2K^{C3\,1,112}-2K^{C4\,1,222}-2K^{C2\,1,212}$$

$$K_{22}=K^{S3\,22,22}+K^{S2\,22,21}+K^{S2\,22,12}+K^{S1\,22,11}$$

$$K_{23}=K_{32}=-K^{C3\,2,211}-K^{C4\,2,222}-K^{C1\,2,111}-K^{C2\,2,122}-2K^{C2\,2,212}-2K^{C2\,2,112}$$

$$K_{33}=-\omega^2M^{33}+K^{B1\,11,11}+K^{B2\,11,22}+K^{B2\,22,11}+K^{B3\,22,22}+2K^{B4\,11,12}+2K^{B4\,12,11}+2K^{B5\,22,12}+2K^{B5\,12,22}+4K^{B2\,12,12}+\sigma\omega^2K^{GX1}+\omega^2K^{GX2}$$

In order to use a complex modal analysis method, Eq. (19) is transformed into the following form.

$$\mathbf{M}^*\dot{\eta}+\mathbf{K}^*\eta=\mathbf{0} \quad (23)$$

where

$$\mathbf{M}^*=\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (24)$$

$$\mathbf{K}^*=\begin{bmatrix} \mathbf{C} & \mathbf{K} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \quad (25)$$

$$\eta=\begin{Bmatrix} \dot{\vartheta} \\ \vartheta \end{Bmatrix} \quad (26)$$

An eigenvalue problem can be derived by assuming that η is a harmonic matrix function of τ expressed as

$$\eta=e^{\lambda\tau}\Theta \quad (27)$$

where λ is the complex eigenvalue and Θ is the complex mode shape. Substituting Eq. (27) into Eq. (23) yields

$$\lambda\mathbf{M}^*\Theta+\mathbf{K}^*\Theta=0 \quad (28)$$

Table 1 Material properties of the composite plate

Material	E_1(Gpa)	E_2(Gpa)	G_{12}(Gpa)	ν_{12}
T300/5208	181	10.3	7.17	0.28

Table 2 Comparison of natural frequencies obtained by ANSYS and the present modeling

Mode	Present	ANSYS	Error(%)
1	1.0479	1.0422	0.55
2	1.9816	1.9567	1.27
3	4.6503	4.5453	2.31
4	6.6018	6.5249	1.18
5	8.0411	7.8710	2.16
6	10.0365	9.7834	2.58

3. Numerical Results

In this section, the numerical results are obtained by using the modal equations which are introduced in chapter 2. To solve the eigenvalue problem for the rotating composite plates, assumed mode functions are employed. In the present work, five cantilever beam functions and seven free-free beam functions which include two rigid body mode functions are employed to construct 35 plate mode functions. The number of mode functions are presumably sufficient to insure adequate convergence for the lowest six eigensolutions. First of all, the numerical results obtained by using the present modeling method are compared to those of ANSYS for a non-rotating composite plate. In the computation, the plate is made up of eight laminates with the fiber orientations[0, 45, −45, 90]s, and the composite material is T300/5208. The mechanical properties of the material are given in Table 1. It is shown in Table 2 that the lowest six natural frequencies obtained by the present modeling method agree well with those of ANSYS.

Figure 4 shows the variations of the lowest six dimensionless natural frequencies for rotating

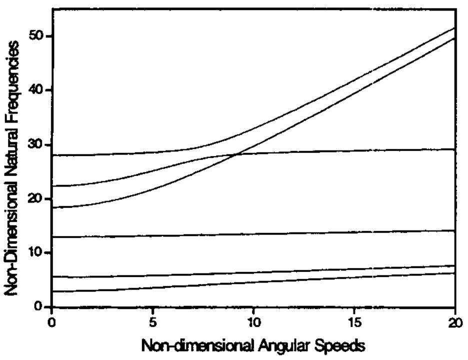
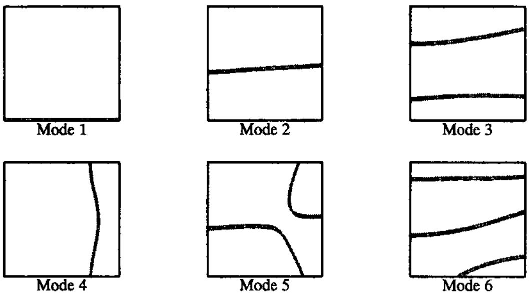
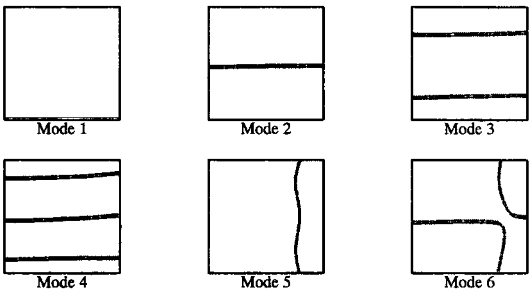


Fig. 4 Variation of the lowest six natural frequencies for rotating plate



(a) Lowest six mode shape without rotation(ω=0)

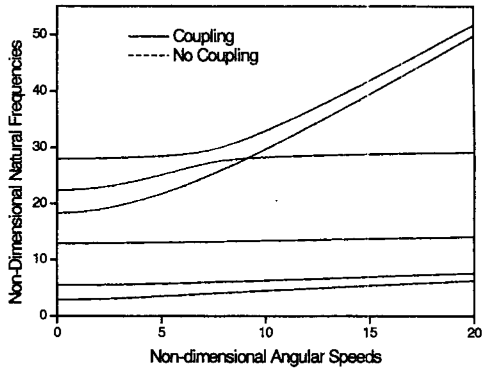


(b) Lowest six mode shape with rotation(ω=20)

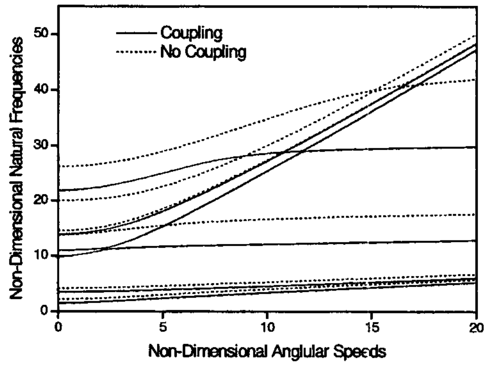
Fig. 5 Nodal line patterns of lowest six mode shapes with and without rotating motion

plate with fiber orientations[0, 45, −45, 90]s. The interesting phenomena observed from Fig. 4. The fifth and sixth eigenvalue loci veer around ω=7 and the fourth and fifth eigenvalue loci cross around ω=9.

Figures 5(a) and 5(b) show the nodal lines of the lowest six mode shapes when the dimensionless angular speed is 0 and 20. Comparing the mode shapes of rotating plate to those of the non-rotating plate, the fourth, the



(a) Fiber orientation[0, 45, −45, 90]s

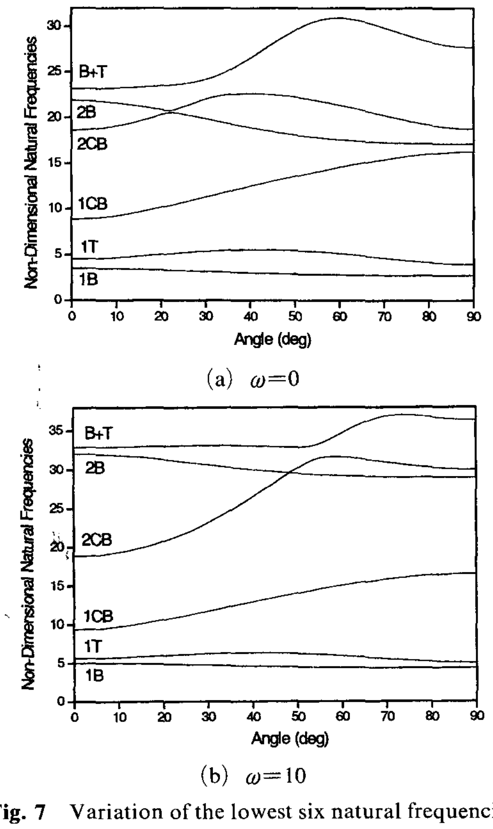


(b) Fiber orientation[10, 20, 30, 40, 50, 60, 70, 80]

Fig. 6 Variation of the lowest six natural frequencies for rotating plate with and without coupling effect

fifth, and the sixth modes seem to be switched one another. These mode shape variations result from eigenvalue loci veering and crossing as shown in Fig. 4. These phenomena are well explained by H. H. Yoo (1993). Although laminates are symmetrically stacked up with respect to neutral axis, modal line patterns are found to be unsymmetric. The reason for this comes from D_{16} and D_{26} terms which represent the flexural-torsional coupling effect in matrix D_{ij} of Eq. (7). This same phenomenon is also found by T. Maeda, V. Baburaj, Y. Ito, and T. Koga (1998). The flexural-torsional coupling effects, however seem to disappear as the increase of angular speed of the plate as shown in Fig. 5(b). This is because the motion-induced stiffness variation terms K^{GX1} and K^{GX2} become more dominant than the structural stiffness as the angular speed increases.

Figure 6(a) shows the variations of the lowest six dimensionless natural frequencies for rotating plate with fiber orientations[0, 45, −45, 90]s. The solid lines represent the results of considering the extensional-bending coupling effects and the dotted lines represent the results of ignoring the coupling effects. When laminates are symmetrically stacked up with respect to neutral axis as shown in Fig. 5(a), the results obtained by considering coupling effects are almost identical to those obtained by ignoring coupling effects. On the other hand, Fig. 6(b) shows the variations of the lowest six dimensionless natural frequencies for a rotating plate with unsymmetrical fiber orientations[10, 20, 30, 40, 50, 60, 70, 80]. It shows that the results obtained by considering the coupling effects are significantly different from those obtained by ignoring the coupling effects. Therefore, if laminates are unsymmetrically stacked up with respect to neutral axis, the coupling effects influence the modal characteristics considerably. In this case, B_{ij} which is the coupling stiffness matrix from Eq. (4) is not zero.



(b) $\omega=10$

Fig. 7 Variation of the lowest six natural frequencies due to fiber angle change

The variations of dimensionless natural frequencies of a square plate for the fiber orientations [0, θ, $-\theta$, 90]s are shown in Fig. 7. The effect of the fiber angle θ is shown in Fig. 7. Figure 7(a) shows the dimensionless natural frequency variations without rotation and Fig. 7(b) shows the dimensionless natural frequency variations with dimensionless angular speed $\omega=10$. These figures indicate that the natural frequencies of bending modes decrease as the fiber angle increases. Those of the chordwise bending modes, however, increase as the fiber angle increases.

4. Conclusions

In this paper, a modeling method for the modal analysis of rotating composite plates is presented. Using the modeling method, the effects of angular speed and the fiber angle orientation on the modal characteristics are obtained. As dimensionless angular speed and laminated angle are varied, natural frequencies loci veering, loci crossing, and associated mode shape variations are observed. Especially, when laminates are unsymmetrically stacked up with respect to neutral axis, it is found that the accurate results can be obtained by using equation of motions which consider the coupling effects between extensional motions and the bending motion. Finally, it is concluded that various mode shape variations can be observed by changing a laminate angles of layers. The presenting modeling method can be usefully employed for the design of rotating composite plate structures.

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Reference

Barton, M. V., 1951, "Vibration of Rectangular and Skew Cantilever Plates," *J. of Applied Mechanics*, Vol. 18, No. 1, pp. 129~134.

Chandrashekhara, K., Krishnamurthy, K. and Roy, S., 1990, "Free Vibration of Composite Beams Including Rotary Inertia and Shear Deformation," *Composite Structures*, Vol. 14, pp. 269~279.

Dokainish, M. and Rawtani, S., 1971, "Vibration Analysis of Rotating Cantilever Plates," *Int. J. for Numerical Methods in Engineering*, Vol. 3, pp. 233~248.

Ghosh, A. K., and Dey, S. S., 1992, "Simple Finite Element for the Analysis of Laminated Plates," *Computers & Structures*, Vol. 44, pp. 585~596.

Kapania, R. K. and Raciti, S., 1989, "Nonlinear Vibrations of Unsymmetrically Laminated Beams," *J. of AIAA*, Vol. 27, No. 2, pp. 201~210.

Kosmatka, J. B. and Friedmann, P. P., 1989, "Vibration Analysis of Composite Turbo-Propellers Using a Nonlinear Beam-Type Finite Element Approach," *J. of AIAA*, Vol. 27, No. 11, pp. 1606~1614.

Leissa, A. W., 1969, Vibration of Plates, NASA SP-160.

Maeda, T., Baburaj, V., Ito Y., and Koga, T., 1998, "Flexural-Torsional Coupling Effect on Vibration Characteristics of Angle-Ply Laminates," *J. of Sound and Vibration*, Vol. 210, No. 3, pp. 351~365.

Mohamed Nabi, S and Ganesan, N., 1994, "A Generalized Element for the Free Vibration Analysis of Composite Beams," *Computers & Structures*, Vol. 51, pp. 607~610.

Narita, Y. and Leissa, A. W., 1992, "Frequencies and Mode Shapes of Cantilevered Laminated Composite Plates," *J. of Sound and Vibration*, Vol. 154, pp. 161~172.

Ramamurti, V. and Kielb, R., 1984, "Natural Frequencies of Twisted Rotating Plates," *J. of Sound and Vibration*, Vol. 97, No. 3, pp. 429~449.

Rand, O., 1991, "Periodic Response of Thin-Walled Composite Helicopter Rotor Blades," *J. of the American Helicopter Society*, Vol. 36, No. 4, pp. 3~11.

Tsai, S. W. and Hahn, H. T., 1980, Introduction to Composite Materials, Technomic Publishing Co. Inc., Connecticut, pp. 217~274.

Yoo, H. H, and Chung, J., 2001, "Dynamics of Rectangular Plates Undergoing Prescribed Overall Motion," *J. of Sound and Vibration*, Vol. 239, No. 1, pp. 123~137.

Yoo, H. H., 1993, "Vibration Analysis of Rotating Cantilever Plates," *Trans. KSME*, Vol. 17, No. 3, pp. 652~657.

Young, D., 1950, "Vibration of Rectangular Plates by the Ritz Method," *J. of Applied Mechanics*, Vol. 17, pp. 448~453.

Whitney, J. M., 1987, *Structural Analysis Of Laminated Anisotropic Plates*, Technomic Publishing Co. INC. , Pennsylvania, pp. 41~44.