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An Elementary Teacher's Practical Knowledge of Using Mathematical Tasks for Promoting Students' Understanding and Discourse

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This study described an elementary teacher's practical knowledge of selecting and using mathematical tasks for promoting students' understanding and discourse. The informant of this ethnographic inquiry was a third grade teacher and has 10 years of teaching experience. According to the analysis of multiple data sources, this study showed that based on his beliefs about the development of understanding of mathematics and discourse, he continually employed two different types of tasks: open-ended tasks and tasks from students' mistakes and comments during discourse. Teachers' practical knowledge of teaching mathematics and the classroom norms for students' understanding and discourse are suggested to be given attention for further research on this area.

1. INTRODUCTION

How can mathematics teachers facilitate students' understanding of mathematics and discourse in mathematics classroom? According to the *Principles and Standards for School Mathematics* (NCTM 2000), students' understanding of mathematics, mathematical discourse, and their confidence in mathematics are all shaped by the teaching they encounter in mathematics classroom. Thus, teachers are responsible for creating such a classroom environment that promotes students' understanding of mathematics and discourse.

Worthwhile mathematical tasks are considered as one of the most important aspect for such a classroom environment because their quality determines the level of instructional activities in mathematics classroom. Teachers should choose and develop tasks that are likely to enhance the development of students' understanding and discourse (NCTM 1991). Good tasks can be completed in more than one legitimate way, draw students' curiosity, and invite students to reason about different strategies and outcomes so that the tasks facilite significant classroom discourse. Then, how and in what way do teachers implement the 'standard-oriented teaching' mathematics in everyday mathematics classroom?

In order to obtain answers to the question, this study is to describe how an elementary teacher teaches mathematics. The assumption of this study is that teachers' beliefs and teaching practices of mathematics would be better understood through studying the patterns of classroom interaction. The description of this paper is part of the data conducted by an ethnographic inquiry with a single informant. In particular, this paper focuses on describing how the teacher made use of mathematical tasks for promoting students' understanding of mathematics and discourse in mathematics classroom.

2. DESIGN OF THE STUDY

The rationale for pursuing the ethnographic approach was that this approach is to provide a vivid and think description of classroom activities by analyzing the teacher and students' behaviors. The selection of a single informant was based on Wolcott's (1992) recommendation that studies of multiple informants reduce the total attention that can be given to any one of them, weakening rather than to strengthening the study. He expressed a strong preference for studying just one informant in depth. Moreover, it would be a critical factor to establish a solid and dependable relationship with the teacher and students in the classroom in order to be accepted as a member of the classroom.

Teacher Lee, a third grade teacher and the informant of this study, was a 33-year-old male and has 10 years of teaching exeprience in elementary schools. In his classroom, 45 students (24 boys and 21 girls) came from predominantly working class families in the city of Inchon.

Three main data sources were used for the study. The first data source was from participant-observation, the second from formal and informal interviews, and the third from a variety of documents. The data for the study were collected over a three-month period from June through August 1999.

The analysis proceeded in two phases: analysis in the field and analysis after data collection (Bogdan & Biklen 1992). Data analysis was done by an on-going process that helped the researcher move back and forth between thinking about the existing data and generating strategies for collecting new, or better data (Miles & Huberman 1994). Once the fieldnotes from observation, interviews or documents were word-processed, they were coded by sentence or paragraph. The researcher continued to read the data to develop a holistic picture, writing summaries, and drawing diagrams. Once the data were coded, the researcher pursued confirming and disconfirming evidence from the multiple data sources e.g., fieldnotes, interview transcripts, documents (Erickson 1986). For triangulartion a matrix of findings was made by data sources and methods. The final analysis generated six major themes of Teacher Lee's beliefs about mathematics teaching and learning and classroom practices. The description of this paper is about one of these thems, 'Mathematical Tasks'.

3. Mathematical Tasks for Understanding and Discourse

Teacher Lee was responsible for the quality of the mathematics tasks in which his students engaged. He chose and developed the tasks that were likely to promote the development of his students' understanding of concepts and procedures, to foster their abilities to solve problems, and to reason and communicate mathematically. Based on his beliefs about the development of mathematical understanding and discourse, he usually employed two different types of tasks: open-ended tasks and tasks from students' mistakes and comments during discourse. Although he viewed the mathematics in the textbook as the final destination of his teaching mathematics, he considered the mathematics laid out in the textbook as a guideline, not an authoritative rule.

> My final goal of teaching mathematics is to teach the contents of the textbook because it shows the final conclusions. However, I am trying to teach mathematics by different ways from the textbook. At least, I want my students to perceive there are different ways of doing mathematics (Interview, August 7).

In the following sections, two types of mathematical tasks are discussed in more detail with classroom episodes and Teacher Lee's beliefs.

3.1. Open-Ended Tasks

Teacher Lee strove to foster the ability of his students' mathematical thinking. For this purpose, he consistently had his students solve tangram problems in the morning self-learning session before school started. Tangram activities were one way of forming his students' mental habits for mathematical thinking. On the other hand, in his teaching practice, he believed that his role was to show his students that many different ways of solving a mathematics problem existed. He emphasized the change of the students' perceptions about mathematics.

> I think there are not only many different ways of methods of solution in mathematics, but many different ways of representing an answer. So, I don't think it is good teaching of mathematics to lead students to the teacher's own way · · · Most of the students tend to think that only one answer exists in mathematics and the answer should be correct. I think this tendency of mathematical practice is the fault of the teachers, who did not show them different ways of solutions and answers in mathematics (Interview, August 12).

Because this belief was drawn from his primary belief, the students' own ways of understanding, it was considered to be a derivative belief (Green 1971). To implement the belief about his role and responsibility, he utilized the open-ended tasks or approaches in his mathematics classroom.

Teacher Lee's belief about different ways of doing mathematics was demonstrated in his teaching and assessing mathematics. In this problem, Teacher Lee made the problem open-ended by using a set of number cards.

By using a set of number cards as below, answer the problems.

1	2	3	4	5	6	7	8	9	0

Figure 1. A set of number card

1. Using the number cards, make a division problem of (two digits) ÷ (one digit) having zero quotient and solve it.
2. Using the number cards, make a division problem of (two digits) ÷ (one digit) having no zero quotient and solve it (From Performance Assessment Test, the first semester of 1999).

In fact, Teacher Lee frequently had the students generate mathematical problems using number cards. In doing so, he wanted the students to understand that mathematics problems and answers were not given by the authoritative teacher and textbook. Rather, he wanted his students to take over the authority of doing mathematics.

Teacher Lee consistently emphasized open-ended tasks and approaches in his mathematics teaching. It should be noted that these open-ended tasks and approaches were greatly associated with his teaching practices of discourse-oriented and conceptual development. The following episode was observed in his mathematics teaching of the second grade. A female classroom teacher was sick and missed that day. He brought several stacks of number cards.

Teacher: I teach the third graders but I am going to teach math at this period for you.

Teacher Lee had them make a group of four or five and nine groups were formed. He wanted to know these second graders' knowledge of mathematics. So he wrote 1, 2, 3, 4, 5, 6, 7, 8, 9 on the board.

Teacher: Now, I am going to ask a question. What is the sum if you add all of them?

Within less than 20 seconds, several students called out "45!" Teacher Lee asked a student to present his idea.

Student 1:	I tried to make 10s first and then find the rest.
Teacher:	What about 56? [Wrote 56 on the board]
Student 1:	It's not 56. I made tens by adding two numbers and got four tens. There were 5 left. So I got 45.

Teacher Lee showed a little bit of surprise at these second graders. So he decided to put more challenging problems on the board. He wrote 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 on the board.

Teacher:	What is the sum if we add all of them?

Of course, there were hands up again within 20 seconds. Teacher Lee called on the students who raised their hands.

Student 2:	There are nine tens so it's 90. Like what we did just before, the sum of 1 to 9 was 45. So I got 135.
Student 3:	First I separated tens and 1s. So I got nine tens and then I added the rest, it is 45. So I got 135.

After the students' presentation, Teacher Lee handed a set of number cards containing 1 to 9.

Teacher:	Now, each group should have 1 to 9. Using these cards, you are going to make the number I write on the board. If your group makes it, bring the cards to the front quickly. It's a game. Okay, the first number is 21 (Lesson transcript, June 22).

The first group made 21 with 1, 2, 3, 7, 8 and Teacher Lee wrote the numbers. Another group made 21 with 1, 4, 7, 9. The next number he put on the board was 37. One group made it with 3, 4, 6, 7, 8, 9 and the students in this group said, "Yeah, we are first!" Another group made 37 with 1, 4, 5, 7, 8, 9 but the sum was 34. Teacher Lee put a check mark for any group that got an correct combination. The next number was 23. One group made it with 1, 3, 4, 6, 9 and another group with 1, 2, 3, 8, 9. Both groups made 23 with 5 cards but the third group made it with 3 cards, 6, 8, 9. Teacher Lee commented that there were many different ways of making a combination for a number.

The following classroom episode was another example of his open-ended tasks and approaches. When he decided to give these third graders Pascal's Triangle, it appeared to be a difficult task for them. His decision was based on his ongoing assessment through discourse. It was the Unit 9, Problem Solving chapter. The lesson was to find patterns from sequences. He put the following sequences on the board and had the students present their ideas, reasoning, and justifications. Although there were several problems related to finding patterns in the textbook, he made them up extemporaneously.

1) 1, 3, 9, 27, ___, ___
2) 1, 4, 7, 10, 13, ___, ___
3) 1, 1, 2, 3, 5, 8, 13, ___, ___
4) 1, 2, 5, 10, 17, ___, ___

The students had no difficulty identifying the patterns of each sequence. In particular, they identified the pattern of the third one, a Fibonacci sequence that appeared difficult for them. The students enjoyed these problems. After these problems, most of the students understood what a pattern meant. Thus, Teacher Lee put up Pascal's Triangle to encourage the students' intellectual challenge.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
○ ○ ○ ○ ○ ○

Figure 2. Pascal's Triangle (1)

Teacher:	Now, there are lots of patterns and find them. Write this figure in your notebook and find as many patterns as you can.
Yun-ha:	Uhh ··· I did not learn this thing in the learning center.
Teacher:	I know. The teacher in a learning center did not teach this, did he?
Students:	No. [Several students responded and became busy.]
Teacher:	I do not teach you the problems you learned in the learning center. I don't deal with the problems in the textbook because you already know the answers. [After five minutes he convened the class.] Does anybody want to present the patterns you found? Ji-un?
Ji-un:	[Rising from her seat] 1 plus 1 comes to 2, 1 plus 2 comes to 3, and 1 plus 3 comes 4. Because of 2, there are two threes. 3 plus 3 comes to 6.
Teacher:	Do the rest of you understand what Ji-un said?
Students:	No ··· [Several students responded.]
Teacher:	Okay. Ji-un, can you come up to the board and explain your method

again by pointing? [Ji-un explained her method again.] Let's give her a hand. Does everybody agree with her? Eun-ho?

Eun-ho: [Rising from his seat] I think 1 plus 2 comes to 3 and 2 plus 1 comes to another 3, but Ji-un said there are two threes because of 2.

Teacher: Eun-ho was listening carefully to Ji-un's presentation. I think we found one pattern. [He explained the pattern Ji-eun and Eun-ho found and drew the following figure.] Who else? Yun-ha?

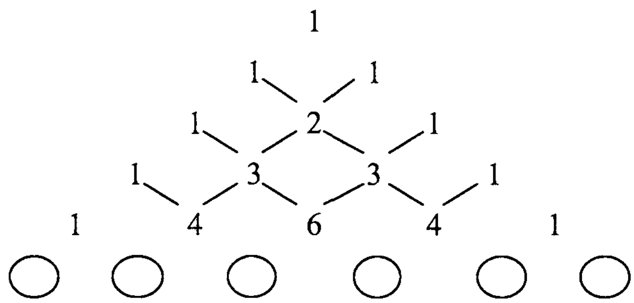

Figure 3. Pascal Triangle (2)

Yun-ha: [Came up to the board] 1 plus 1 is 2, 2 plus 1 is 3, 3 plus 1 is 4, 4 plus 1 is 5, 2 plus 6 comes to 8. Here 1, 2, 3, 4, and the next should be 5.

Teacher: Let's look at the patterns Yun-ha found. [He drew the following figure.] Does everybody agree with Yun-ha? Han-jin?

Han-jin: 1 plus 2 is 3, 3 plus 3 is 6, and 4 plus 6 is 10, not 8. [Teacher Lee wrote what Han-jin said.]

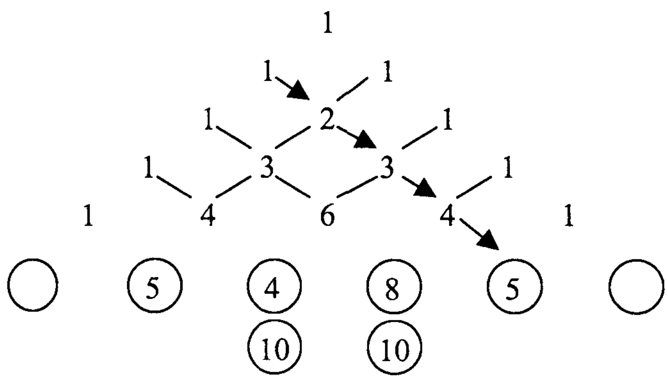

Figure 4. Pascal Triangle (3)

Teacher: Who else? Yung-jin?

Yung-jin: [Came up to the board] Here 1 plus 1 is 2, 1 plus 1 plus 1 is 3, 1 plus 1 plus 1 plus 1 is 4, and 1 plus 1 plus 1 plus 1 is 5. Adding 1, 2, 3 is 6.

Teacher: Everybody understand what Yung-jin said? [He drew the following figure and explained Yung-jin's method]

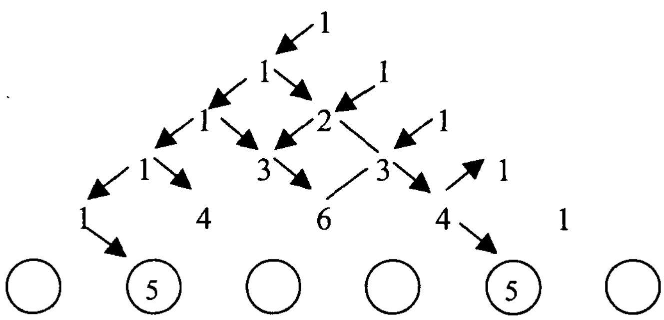

Figure 5. Pascal Triangle (4)

Teacher Lee had several more students present their methods of finding patterns (Lesson transcript, July 9).

The sequences Teacher Lee put on the board were not in the textbook. As he said in the episode, he knew that some of the students already had the answers to the textbook problems. So, if he used the textbook problems the students would not be interested in this lesson. Using this open-ended problem, Teacher Lee had the students perceive that doing mathematics was not obtaining a correct answer. Rather, the students actively participated in doing mathematics by making, communicating, and verifying mathematical assertions that were remarkably different activities in the traditional mathematics classroom.

To implement his belief about open-ended tasks, Teacher Lee consistently modified a problem or task in the textbook. Because the problems in the textbook narrowly defined mathematical situations, he indicated that such problems would not engage the students in discourse. Teacher Lee refused to simply introduce the problem in the textbook. He was aware that it was not a meaningful activity for his students. Thus, he modified the problem as an open-ended task so that the task could bring the students' active engagement in classroom discourse. It was apparent that open-ended tasks that Teacher Lee used fostered the students' participation and enhanced their mathematical discourse.

3.2. Using Students' Mistakes and Ideas for Tasks

Besides open-ended tasks and approaches, Teacher Lee frequently used the students' mistakes and ideas during discourse. This pattern demonstrated how he dealt with the students' mistakes and what he had to pursue in depth from among the ideas that the students brought up during a discussion.

Using the students' mistakes and comments for discussion provided the students with more a familiar context so that they could actively engage in a classroom discussion. The problems or tasks were produced from the students, not posed by the teacher. Another significant effect was that using the students' mistakes and comments conveyed Teacher Lee's interest in and care for their ideas and ways of thinking.

The following episode illustrates how Teacher Lee utilized a student's mistake for discussion. Teacher Lee's class worked on the issue of whether a number line between 0 and 1 could be divided by any number. Teacher Lee's intent was to have the students understand the need of fractions in representing parts between 0 and 1 on a number line. Now, in the middle of the lesson, he asked two students to try to divide the number line between 0 and 1. Tae-min was one of them. Jun-ho divided two parts but Tae-min did in a different way.

Teacher:	Let's listen to Tae-min's idea. Wow, Jun-ho divided it in two parts but Tae-min did in many parts.

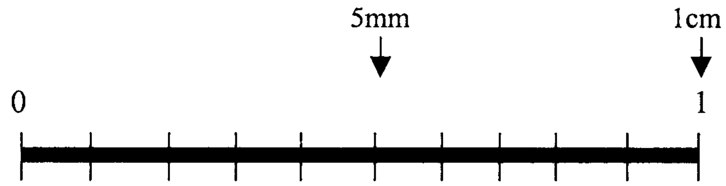

Figure 6. Ruler: Tae-min's method

Tae-min:	[Grinned] On the ruler this 1 means 1 centimeter. 1 centimeter has 10 millimeters, so the middle is 5 millimeters.
Teacher:	Let's give him a hand. [The students clapped.] This is a very unique way I never thought of it. Okay, let's study Tae-min's idea. How many parts did Tae-min divide?
Students:	Ten! [In unison]
Teacher:	Then what is the first skip?
Students:	One millimeter! [In unison]
Teacher:	Now, since we are studying fractions let's denote the skip-counts in the fraction. How many skips are there?
Students:	Ten! [In unison]
Teacher:	Then what is a fraction for the first skip-count?
Students:	One tenth! [In unison]
Teacher:	What about here? [Pointing the fourth skip-count]

Students:	Four tenths! [In unison]
Teacher:	What about here? [Pointing the seventh skip-count]
Students:	Seven tenths! [In unison]
Teacher:	So, do you think we can divide the number line between 0 and 1 in as many parts as we can?
Students:	Yes! [In unison]

Teacher Lee and the class examined a couple of more number lines between 0 and 1 to make sure their conjecture (Lesson Transcript, July 1).

In this episode, Tae-min thought about the number line between 0 and 1 as the length of 1 centimeter. Since the class had studied about measuring length prior to fractions, his idea appeared to be related to his previous knowledge. He was a good student in mathematics and told the researcher that he did not go to a learning center. Perhaps not being exposed to a learning center made him think in such a unique way. Teacher Lee accepted his idea as a valuable way of thinking and decided to pursue in depth his idea so that some students were able to connect the concepts between fractions and measuring units.

Teacher Lee made use of the students' ideas during discussion time. Sometimes he probed the students' ideas even though they went beyond what many of the students were trying to do at the point. He could have just mention that the idea was beyond the curriculum of the third grade. However, he never ignored the ideas that the students brought up during a discussion. Rather, he took the opportunity to stimulate discourse and to expand the students' interests. The following episode illustrates how Teacher Lee made an idea beyond their ability turn into a useful discourse task. Teacher Lee and the class had discussed whether a number line between 0 and 1 could be divided by using any numbers they knew.

Teacher:	So far, we divided number lines with 5, 8, 10, and so on as ending points. I wonder if this number line could be divided like what we did just before. Do you think it is possible?
Students:	No! Or No way! [In unison]
Teacher:	We don't · · · [Several students raised their hands to present their ideas.] Ho-rae? What do you think?
Ho-rae:	[Rising from his seat] I think it is divided by 0.5.
Teacher:	You can divide it by 0.5? Would you come up to the board to show us what you mean?

Ho-rae drew a number line like below.

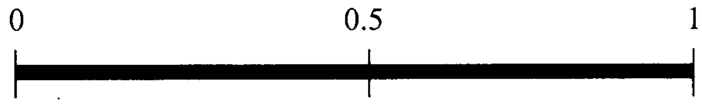

Figure 7. Number line

Teacher: Now, let's listen to Ho-rae's explanation. You know you should listen carefully to it in order to ask a good question.

Ho-rae: Since the starting point is 0 and the ending point is 1, this middle point is 0.5.

Teacher: Does anybody want to ask a question of him? Han-jin?

Han-jin: [Rising from his seat] How did you get the number 0.5?

Ho-rae: Because 0.5 means half way between 0 and 1.

Teacher: Who else? [No hands up.] Let's give him a hand. Now, what is the name of the number 0.5? [The class murmured.] I will write similar numbers to 0.5. [He wrote 0.5, 0.3, 0.9.] Do you know what these are?

Min-jung: [Called out] Prime numbers.

Teacher: Prime numbers. Where did you get that name? [Grinned to Min-jung and she did not respond] I am surprised that Ho-rae knows such a difficult concept in mathematics. These kind of numbers are called 'decimals.' [Wrote the term, decimals, on the board] Now, since we have not studied about decimals, let's think about it with what we already knew (Lesson transcript, July 1).

In this episode, most of the students did not connect the concept of fractions and number lines between 0 and 1. They easily recognized fractions in parts of bars, circles, or triangles but had difficulty imagining the fact that fractions are the numbers represented between 0 and 1. Since Ho-rae's idea of decimal was interesting and such an idea might be a fruitful task to expand the students' understanding of fractions, Teacher Lee decided to have him present his idea to the class. Han-jin's request for clarification was interesting because he used the same kind of questions Teacher Lee asked when the students were at the board to solve mathematics problems. It was observed that some of Teacher Lee's students asked this type of questions to the presenter for clarification, justification, or verification.

The tasks he chose, consequently, facilitated significant classroom discourse and the students' understanding because they required that the students reasoning about different strategies and outcomes, and weighed the pros and cons of alternatives.

4. DISCUSSION

This paper described the teaching practices of mathematics by an elementary classroom teacher who believes importance of the development of mathematical understanding and discourse. In order to do this, the teacher usually selected tasks for from two different sources: open-ended tasks and students' mistakes and comments in discourse.

Although his teaching of mathematics with understanding was consistent with the current reform movements that require the development of students' understanding and discourse, some of the teaching patterns observed were personal insights, or habits from his teaching experiences (Feinman-Nemser & Floden 1986) and were practical knowledge. Leinhardt (1990) called this type of knowledge "craft knowledge", the knowledge that very skilled teachers have about their own teaching practice. The term "craft knowledge" has been used to refer within their own classroom practice, the knowledge that enables them to employ the strategies, tactics, and routines that they do (Calderhead 1996). Thus, for the current reform in mathematics classroom to be successfully implemented this type of practical knowledge should be investigated as the legitimate knowledge of teachers.

Another aspect necessary for further research is classroom norms that focus on promoting students' understanding and discourse. The norms in a particular class determines the way mathematical tasks are used for learning, and they govern the nature of teaching and learning activities (Carpenter & Lehrer 1999). Although the selection of appropriate tasks can facilitate the development of understanding, the norms of a class determine whetehr will be used for that purpose. Therefore, the two aspects discussed here should be considered when designing mathematics instruction for students' understanding and discourse.

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