論 文
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# 광대역 다단 전력 분배기의 일반화된 설계 방법

# A General Design Method for the Broadband Multi-Section Power Divider

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**Abstract** - A novel multi-section power divider configuration is proposed to obtain wide-band frequency performance up to microwave frequency region. Design procedures for the proposed microwave broadband power divider are composed of a planar multi-section three-ports hybrid and a waveguide transformer design procedures. The multi-section power divider is based on design theory of the optimum quarter-wave transformer. Furthermore, in order to obtain the broadband isolation performance between the two adjacent output ports, the odd mode equivalent circuit should be matched by using the lossy element such as resistor. The derived design formula for calculating these odd mode-matching elements is based on the singly terminated filter design theory. The waveguide transformer section is designed to suppress the propagation of the higher order modes such as waveguide modes due to employing the metallic electric wall. Simulation and experiment show excellent performance of multi section power divider.

**Key Words** : Multi-section power divider, stepped transmission line quarter-wave transformer, general design formula, and isolation performance.

## I. Introduction

The multi-section three-port hybrid consider in this paper is useful both as a power divider and combiner applications with very wide operating frequency range. The hybrid T-junction is the basic element of the three-port hybrid. The lossless hybrid T-junction divider suffers from the problem of not being matched at all ports due to odd-mode operation. Thus, it does not have any isolation between output ports. It is well known that the odd-mode circuit of the hybrid T-junction is matched to guarantee the excellent isolation performance by using the lossy element. Design and analysis methods for single section lossy hybrids such as Wilkinson power divider have been well known through several papers and articles. [1][3] However, it has a limited frequency band characteristic because of the frequency limitation performance of the quarter-wave transformer, which is employed for the matching of even-mode circuit. Thus, several efforts for broadband three-port hybrid have been tried and reported. As one of great efforts, S. B. Cohn has reported and summarized the analysis and design method for a class of multi-section three port hybrids. [4] This previously reported design method has been utilized widely to solve limited frequency band problems in single section case. However, in the previous design methods one difficulty arises form calculating the resistance or conductance values for the implementation of wide band isolation performance. Little difficulty is experienced during the calculating the resistance or conductance values by employing the reported design formula for a given characteristic impedance level of the stepped transmission line quarter-wave transformer. The multi-section three-port hybrids for broadband frequency performance in this paper differ in that the conductance or resistance values for broadband isolation characteristic are simply determined. The design formula for determining the resistor can be easily derived based on a singly terminated filter theory; it would properly provide the odd-mode matching circuit elements for multi-section three-port hybrid. The main design algorithm for multi-section three-port hybrid, which is to implement the wide band even-mode matching circuit for three-port hybrid, is also based on optimum design of stepped transmission line transformer as the previous design methods. [4], [5] Compared to the earlier design method, there is no any performance improvement of a multi-section hybrid. However, design procedure in this paper is much easier and more convenient to use than that of the reported design method. Furthermore, the design formula for determining the resistance values is given by closed form for all cases. Simulations and measurements on several design results with multi-octave frequency band performance show the validity of this paper.

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Furthermore, in order to show the potential of the proposed design method to the microwave and millimeter-wave application, this theory is applied to the broadband power divider at the microwave and millimeter-wave frequency band.

## II. One-port resistive network based on singly terminated filters

Fig.1 shows the schematic of a conventional multi-section three-port hybrid and its even/odd-mode equivalent circuit representations. The multi-section three-port hybrid or power divider circuit is composed of a finite number of resistor and transmission lines with equal line length. The odd-mode circuit of the multi-section three-port hybrid is a one-port resistive circuit terminated in a 50ohm resistor. In order to synthesis this one-port resistive circuit, it is necessary to consider singly terminated prototype filter shown in Fig.2 (a), which is one-port reactive circuit terminated in a 1ohm resistor.

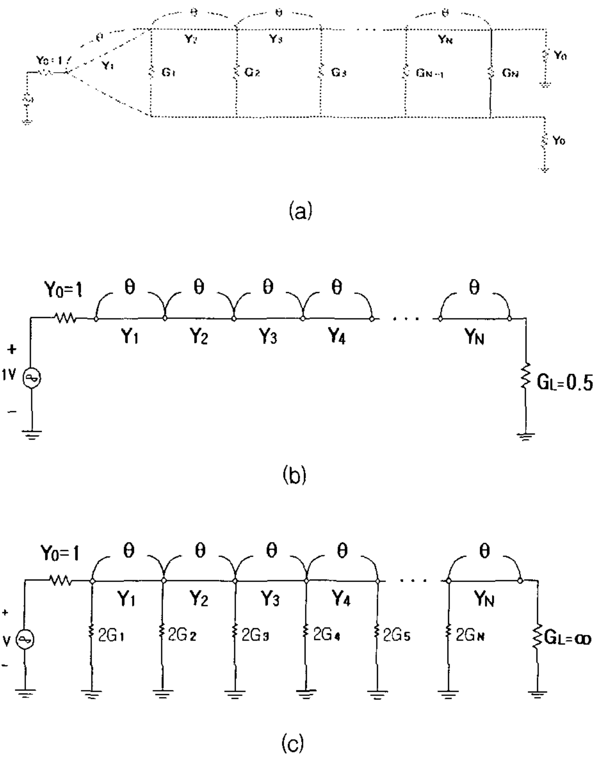


**Fig.10** A multi-section stepped transmission-line three-port hybrid (a) Schematic of multi-section power divider. (b) Even-mode equivalent representation. (c) Odd-mode equivalent representation

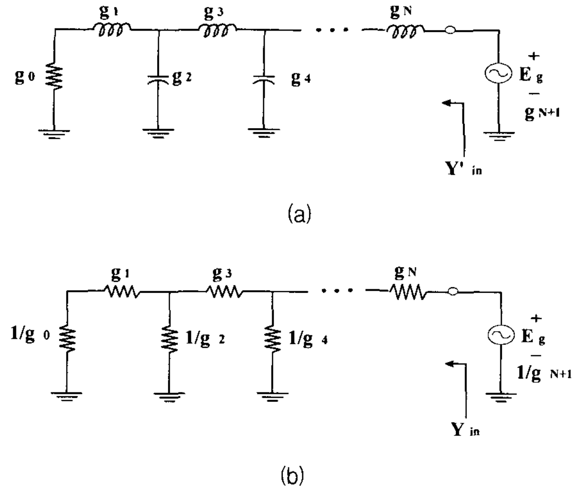


**Fig. 2** (a) Singly terminated prototype filter circuit. The terminated resistor g0 is 1ohm. (b) One-port resistive ladder circuit with an infinite internal-source conductance level.

The immittance function of the singly terminated prototype circuit may be expressed as the ration of two Hurwitz polynomials because that the immittance function is a LC positive real function, which is realizable condition of a one-port LCR circuit. Furthermore, owing to realizable condition of a one-port LC circuit terminated in a 1ohm resistor the roots of denominator are all located in imaginary axis. Since the one-port circuit is composed of reactive elements, the variable of the derived immittance function is pure imaginary shown in (1).

$$I(s)=\left.\frac{P(s)}{Q(s)}\right|_{s=j\omega}=\left.\frac{a_ns^n+a_{n-1}s^{n-1}+\cdots+a_1s^1+a_0}{b_ms^m+b_{m-1}s^{m-1}+\cdots+b_1s^1+b_0}\right|_{s=j\omega} \qquad (1)$$

This immittance function can be derived from the voltage or current attenuation function, which has an approximation characteristic function such as butterworth and chebyshev polynomials. If the immittance function with butterworth or chebyshev polynomials is known for a given specification, the element values of the singly terminated prototype circuit are determined.

Since all attenuation poles of the singly terminated prototype circuit with butterworth or chebyshev approximation polynomials are at infinity, the immittance function can be synthesized in the first Cauer form as shown in Fig.2 (a). For the variable of the derived immittance function is pure real, the rational function of two polynomials can be realizable with pure resistive elements shown in Fig.2 (b) because that the singly terminated prototype circuit is realized in the simple ladder circuit. It may be demonstrated by one to one mapping between the one-port reactive and resistive circuits.

$$I(s)=\left.\frac{P(s)}{Q(s)}\right|_{s=\sigma}=\left.\frac{a_n s^n+a_{n-1}s^{n-1}+\cdots+a_1 s^1+a_0}{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s^1+b_0}\right|_{s=\sigma} \tag{2}$$

Parallel elements in one-port resistive circuit have dimension of conductance on account of parallel capacitor in singly terminated prototype circuit. The immittance function of the one-port resistive circuit loses the frequency dependent characteristic. Thus, it provides the constant reflection characteristics for all frequency bands. This would be shown by simulations on the one-port resistive circuit. Fig.3 shows the simulated return loss characteristics of the one-port chebyshev resistive circuits at 1ohm-terminated junction plane with the shorted-voltage source. Performed simulations have 1.0dB, 0.5dB, 0.1dB, and 0.05dB ripple characteristics of the singly terminated prototype circuits, respectively. As shown in Fig.3, the reflection coefficient characteristics are improved in proportion to the ripple level of the singly terminated prototype circuit. As an example, the return loss characteristic of 0.1dB ripple case is about 10dB lower than that of 1.0dB case, which is 10 times greater than 0.1dB-ripple. Furthermore, instead of the chebyshev characteristic polynomials the butterworth polynomial can be used for the one-port resistive circuit.

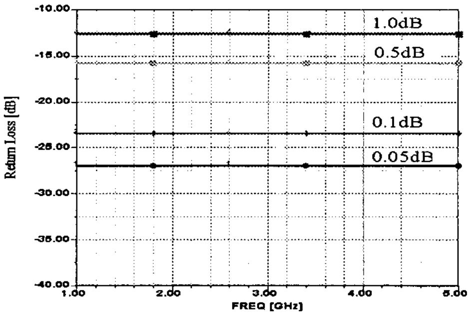


**Fig.3** Simulated return loss characteristics of the one-port chebyshev resistivecircuits at 1ohm terminated junction with the shorted-voltage source. Performed simulations have 1.0dB, 0.5dB, 0.1dB, and 0.05dB ripple characteristics, respectively.

## III. GENERAL DESIGN FORMULA for multi-section THREE-PORT hybrid

The schematic of a conventional multi-section three-port hybrid and its even/odd-mode equivalent circuit representations are shown in Fig.1. The multi-section power divider circuit for implementing the wide frequency band performance is composed of a finite number of resistors and stepped transmission lines with equal line length. The even-mode operation of a multi-section three-port hybrid is identical with a conventional multi-section quarter-wave transformer, which has 100ohm terminated impedance. Thus, each transmission line section of multi-section power divider for optimum performance can be easily determined by a optimum multi-section quarter-wave transformer design theory for a given specification.

However, the odd-mode operation differs from that of the even-mode. As we can Fig.1 (b), there are a number of parallel conductances for isolation performance of three-port hybrid. The power dividing can be pertinently achieved by determining the multi-section quarter wave transformer. Once the characteristic impedances or admittances of stepped transmission line section are determined, the remainder of the synthesis problem is to compute the parallel conductances. S. B. Cohn has summarized the synthesis method for determining these conductance values. An almost exact synthesis is possible for N=2 case in his research. Furthermore, for N3 a set of approximate design formula has been derived heuristically. However, as increase the number of section the synthesis procedure is increasingly difficult for N3.

In this paper, a general synthesis method for determining parallel conductances is newly proposed based on the singly terminated filter synthesis theory. Fig.4 shows the odd-mode equivalent circuit of a multi-section power divider and one-port resistive circuit corresponding to the singly terminated prototype filter. All admittance values in Fig.4 (a) are normalized by source admittance. Electrical lengths of transmission lines are equal to 90 degree. Then the input admittance to load, which has infinite admittance level, can be deduced by finite continued fraction as follow

$$Y_{in}=2\,G_1+\cfrac{Y_1^2}{2G_2+\cfrac{Y_2^2}{2G_3+\cfrac{Y_3^2}{2\,G_4+\cdots+\cfrac{Y_{N-1}^2}{2\,G_N}}}} \tag{3}$$

$g_0$, $g_1$, $g_2$, ..., $g_N$ in Fig.4 (b) is prototype element values of the singly terminated chebyshev prototype filter, where $g_0$ is also normalized source admittance to 1. These prototype element values are given by simple formulas.[6] Then, the input admittance of one-port resistive circuit toward the terminated load side is also given by the following finite continued fraction

$$Y'_{in}=\frac{1}{g_1}+\cfrac{1}{g_2+\cfrac{1}{g_3+\cfrac{1}{g_4\cdot\cdot+\cfrac{1}{g_{N-1}+\cfrac{1}{g_N}}}}} \tag{4}$$

In order to achieve an excellent isolation performance of the multi-section power divider, the input admittance of the oddmode equivalent circuit, which has an infinite terminated admittance level as shown in Fig.4 (a), should be matched to source. Fortunately, the one-port resistive circuit, which corresponds to the singly terminated filter, can provide the matched circuit for infinite admittance or impedance load conditions. Thus, for achieving the excellent isolation performance two circuits shown in Fig.4 should have same input admittance level. In order to simplify the comparison with (4), (2) may be modified as follow

$$Y_{in} = 2G_1 + \cfrac{1}{\cfrac{2G_2}{Y_1^2} + \cfrac{1}{\cfrac{2G_3Y_1^2}{Y_2^2} + \cfrac{1}{\cfrac{2G_4Y_2^2}{Y_1^2Y_3^2} + \cdots}}} \cdots + \frac{Y_{N-1}{}^2 Y_{N-3}{}^2 \cdot\cdot Y_2^2}{2G_N Y_{N-2}{}^2 Y_{N-4}{}^2 \cdot\cdot Y_1^2} \tag{5}$$

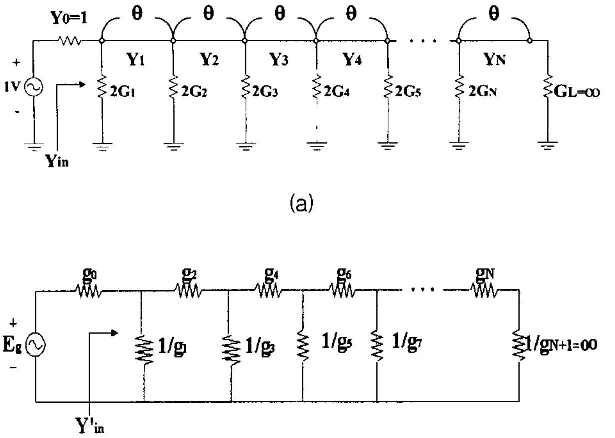
In order to have identical input admittance levels, the corresponding terms in (3) and (4) must be equal as follow With the use of (6),

$$\begin{aligned} &G_1 = \frac{1}{2g_1}, G_2 = \frac{Y_1^2 g_2}{2}, G_3 = \frac{Y_2{}^2 g_3}{2Y_1^2}, G_4 = \frac{Y_3{}^2 Y_1{}^2 g_4}{2Y_2{}^2}, \\ &\cdots, G_N = \frac{Y_{N-1}{}^2 Y_{N-3}{}^2 \cdots Y_3{}^2 Y_1{}^2 g_N}{2Y_{N-2}{}^2 Y_{N-4}{}^2 \cdots Y_2{}^2} \quad for\ N = even \\ &G_N = \frac{Y_{N-1}{}^2 Y_{N-3}{}^2 \cdots Y_4{}^2 Y_2{}^2 g_N}{2Y_{N-2}{}^2 Y_{N-4}{}^2 \cdots Y_1^2} \quad for\ N = odd \end{aligned} \tag{6}$$

the generalized conductance formula for multi-section power divider can be deduced by

$$G_1 = \frac{1}{2g_1} \ and\ \ G_i = \frac{Y_{i-1}{}^2 g_{i-1} g_i}{2G_{i-1}}, \text{where } i = 2,3,\cdots N \tag{7}$$

Since the transmission line sections of multi-section three-port hybrid have admittance values corresponding to given passband ripple level and cutoff frequency, it is recommended that the ripple level of singly terminated prototype filter is chosen to be equal to that of the optimum multi-section quarter-wave transformer. In addition, one thing we have to consider is that the input admittances of both circuits shown in Fig.4 have an opposite direction with the input admittances of the singly terminated prototype circuit and the corresponding resistive circuit shown in Fig.2. The prototype values of the singly terminated prototype circuit are calculated from the input admittances toward 1-ohm terminated resistor. However, the reactive two-port circuit between 1-ohm terminated resistor and the infinite source admittance is reciprocal. Thus, the matched condition at the junction plane of 1-ohm terminated resistor is maintained with the unknown ripple level. Further investigations will be necessary to clarify the exact ripple level at the junction plane of 1-ohm terminated resistor.



(a)

(b)

**Fig.4** (a) Odd-mode equivalent circuit of a multi-section power divider. (b) One-port resistive circuit corresponding to the singly terminated chebyshev prototype filter with the infinite source conductance.

## IV. DESIGNS AND EXPERIMENTS

### 1) RF Application Examples

To verify the derived design formula, we designed and simulated on a three-section power divider at the center frequency of 3GHz. The stepped transmission line quarter-wave transformer sections of the three-section power divider have been determined with 0.05dB ripple level. The conductance values were determined with 0.5dB, 0.1dB, and 0.05dB ripple levels, respectively. The design results are shown in Table 1. Fig.5 shows the comparison of the simulated isolation performance on the designed 3-section power dividers.

**Table 1** Designed results of3-section power dividers. The stepped transmission line quarter-wave transformer is designed with 0.05dB ripple level. The resistance values are determined with 0.5dB, 0.1dB, and 0.05dB ripple levels, respectively.

| Ripple Level | R1 | R2 | R3 |
|---|---|---|---|
| 0.05dB | 87.942 ohm | 203.55 ohm | 578.97 ohm |
| 0.1dB | 103.16 ohm | 184.09 ohm | 563.88 ohm |
| 1.0dB | 159.63 ohm | 153.83 ohm | 584.68 ohm |
| Characteristic impedance of transformer | | | |
| Ripple Level | Z1 | Z2 | Z3 |
| 0.05dB | 83.42 ohm | 70.77 ohm | 60.04 ohm |

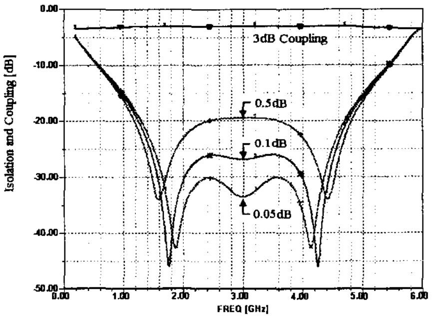
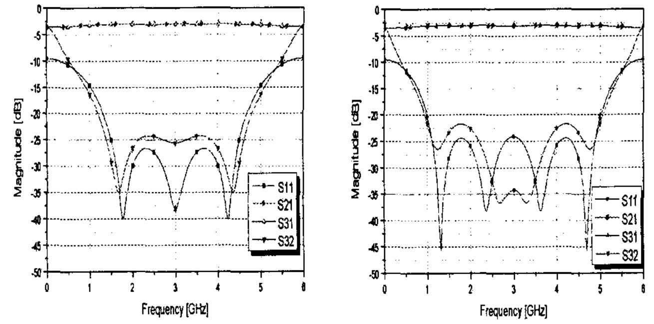


**Fig. 5** Comparison of the simulations on the designed 3-section power dividers. All the simulations are performed with ideal transmission lines. The stepped transmission line quarter-wave transformer is designed with 0.05dB ripple level. The conductance values are determined with 0.5dB, 0.1dB, and 0.05dB ripple levels, respectively.

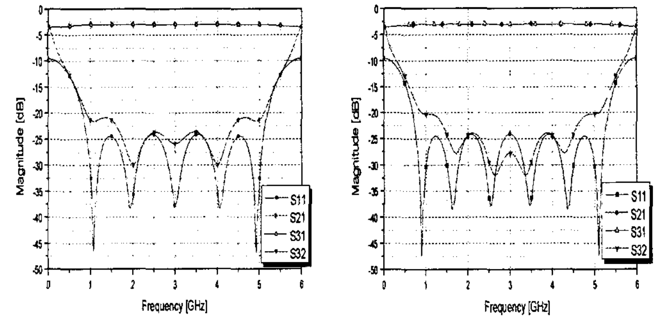
All the simulations have considered the junction capacitance at each stepped transformer junction. As the ripple level decreases the isolation performance is improved as demonstrated in Fig.5. The optimum isolation performance is achieved by maintaining the identical ripple levels of the stepped transmission line quarter-wave transformer section and the singly terminated prototype circuit as previously recommended.

Furthermore, we have designed a set of multi-section power dividers by using general design formula of this paper. Some of the examples designed in this paper are as follows; three sections, f2/f1=3, ripple level=0.01dB; four sections, f2/f1=4, ripple level=0.01dB; five sections, f2/f1=6, ripple level=0.01dB; six sections, f2/f1=8, ripple level= 0.01dB; and seven sections, f2/f1=10, ripple level=0.01dB. All the resistance values are determined with 0.1dB ripple-level prototype elements for the convenient practical implementation. In order to investigate the practical performance of higher order multi-section power divider, laboratory three-, five, and seven sections power dividers have been fabricated. The substrate for simulations and fabrications was RT/Duroid 5880 with 31-mil thick and dielectric constant r of 2.2. Fig.6 shows the simulated results for designed multi-section power dividers. Each simulation shows good agreements with design goals. However, as increase the number of section a degradation of the power divider performance is introduced in isolation characteristics. This might be due to the accumulated mismatched ripple level between the singly terminated prototype filter and the optimum multi-section quarter-wave transformer. It should be improved by matching the ripple level of singly terminated prototype filter with that of the optimum multi-section quarter-wave transformer. Fig.7 shows the measurements on fabricated multi-section power dividers. As we can see in these measurements, the excellent isolation performance supports the utility of the derived design formula in this paper. Since the relatively high junction capacitance level at each stepped transformer junction both the reflection and transmission performance is deviated from the simulated performance especially higher frequency region. This is the reason for requirements of the optimum design of stepped transmission line transformer. In order to improve the reflection performance, junction capacitance is accurately calculated for a gi



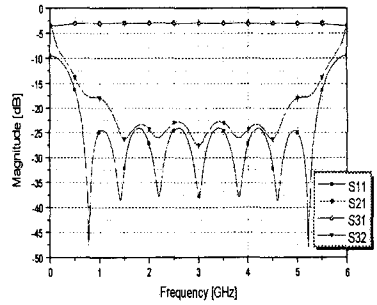
(a) Three-sections

(b) Four-sections



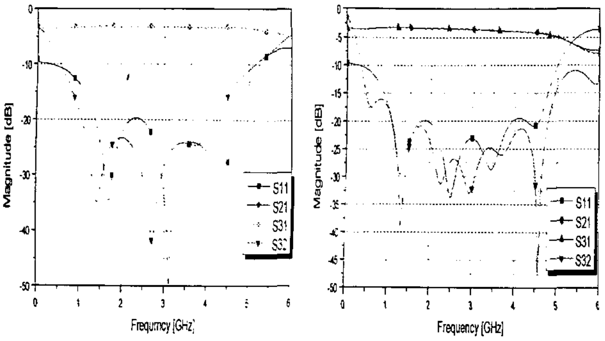
(c) Five-sections

(d) Six-sections

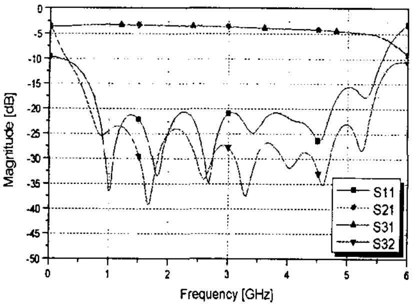


(e) Seven-sections

**Fig. 6** Simulations on designed multi-section three-ports hybrids. All the simulations have considered the junction capacitance at each stepped transformer junction. The stepped transmission line quarter-wave transformer is designed with 0.01dB ripple level. The conductance values are also determined with 0.1dB ripple levels. (a) Three-sections (b) Four-sections (c) Five-sections (d) Six-sections (e) Seven-sectio



(a) Three-sections (b) Five-sections



(c) Seven-sections

**Fig. 7** Measured results for designed multi-section three-ports hybrids.

### 2) Microwave and Millimeter Wave Application Examples

Furthermore, we designed a three-section power divider with a frequency range of 5GHz ~ 20GHz. The stepped transmission line quarter-wave transformer sections of the three-section power divider have been also determined with 0.05dB ripple level. Fig.9 shows the layout of the design power divider in that the spurious mode propagation such as spike was suppressed by employing waveguide transformer, which is composed of sections with evanescent mode propagation at the operating frequency range.

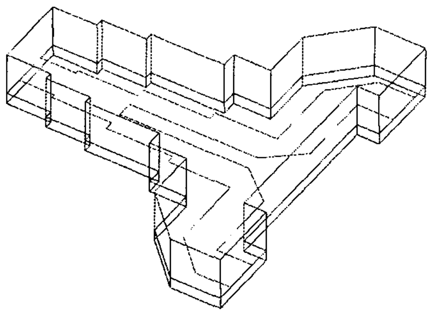


**Fig. 9** Layout of the designed three-section microwave power divider with the 0.05dB ripple level.

Fig.10 shows the simulated S-parameters for the designed broadband power divider shown in Fig.9. We fabricated the designed microwave band power divider. Substrate with dielectric constant of 2.2 was used to fabricate. The thickness of substrate was chosen to be 20mil thick. The measurement on the fabricated power divider is demonstrated in Fig.11. Basically, the designed power divider should have the parallel resistors to guarantee the isolation performance of the fabricated one. For the fabricated microwave power divider, the microwave chip resistors are required. However, the microwave chip resistors were not available in the fabrication. The measurements show the good agreements with the simulation except the isolation performance. But the isolation performance is expected to be excellent with the several RF application experimental examples.

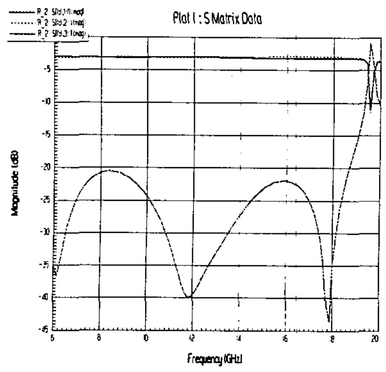


**Fig. 10** Simulated S-parameters on the designed three-section microwave power divider. The 3-D EM simulation is computed by HFSS.

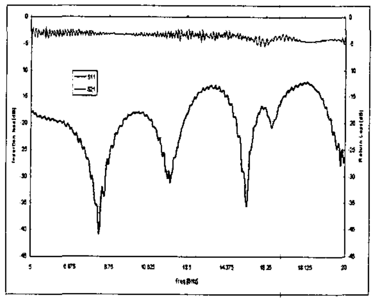


**Fig. 11** Measurement on the fabricated three-section microwave power divider.

## V. CONCLUSION

It is well known that the operating bandwidth of the multi-section hybrid power divider increases with its number of section. Furthermore, multi-section parallel resistors are required to guarantee the excellent multi-octave frequency band isolation performance. In this paper, the synthesis method for a multi-section hybrid power divider is based on the well-known optimum quarter-wave transformer theory. This paper has newly proposed a simple synthesis method based on singly terminated chebyshev prototype filter for determining the parallel conductance values for achieving excellent isolation performance of multi-section power divider. The derived general design formula in this paper is much easier and more convenient to use than that of the previous design method. The design formula for determining the resistance values is given by closed form for all cases. Several simulations and experiments on the designed multi-section power dividers including the microwave application show the validities of this theory.

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