

A New Decomposition Method for Parallel Processing Multi-Level Optimization

Hyung-Wook Park

Graduate Research Assistant, Center of Innovative Design Optimization Technology, Hanyang University, 17 Haengdang-dong, Sungdong-gu, Seoul 133-791, Korea

Min-Soo Kim

Research Professor, Center of Innovative Design Optimization Technology, Hanyang University, 17 Haengdang-dong, Sungdong-gu, Seoul 133-791, Korea

Dong-Hoon Choi*

Director, Center of Innovative Design Optimization Technology, Hanyang University, 17 Haengdang-dong, Sungdong-gu, Seoul 133-791, Korea

In practical designs, most of the multidisciplinary problems have a large-size and complicate design system. Since multidisciplinary problems have hundreds of analyses and thousands of variables, the grouping of analyses and the order of the analyses in the group affect the speed of the total design cycle. Therefore, it is very important to reorder and regroup the original design processes in order to minimize the total computational cost by decomposing large multidisciplinary problems into several multidisciplinary analysis subsystems (MDASS) and by processing them in parallel. In this study, a new decomposition method is proposed for parallel processing of multidisciplinary design optimization, such as collaborative optimization (CO) and individual discipline feasible (IDF) method. Numerical results for two example problems are presented to show the feasibility of the proposed method.

Key Words : Parallel Decomposition, Multidisciplinary Design Optimization

1. Introduction

There are many design problems for which engineers should take into account multiple disciplines such as structural analysis, fluid dynamics and thermal influence, etc. In order to make a design more desirable, synthesis of various design conditions, automation of complex design procedures and optimization of multidisciplinary design are important.

When considering the efficiency and the cost of design, the multidisciplinary design optimization

(MDO) is known to have various advantages over conventional design (Kroo et. al, 1994; Sobieski, 1993; Stephen, 1999). However, MDO problems handle many variables and require complicated analysis procedures due to couplings of various disciplines. Therefore, the decomposition procedure is required for parallel computing, which is prerequisite to use MDO methodology such as collaborative optimization (CO) and concurrent subspace optimization (CSSO). The decomposition method for MDO should be developed not for conventional optimization methods which are composed of one optimizer and one analysis module defining design variables, objective and constraints, but for multidisciplinary optimization methods which are composed of several optimizers and several analyses module (Sobieski, 1982, 1988).

In decomposition method for MDO, couplings

* Corresponding Author,
E-mail : dhchoi@email.hanyang.ac.kr
TEL : +82-2-2290-0478; FAX : +82-24-2290-1639
Director, Center of Innovative Design Optimization Technology, Hanyang University, 17 Haengdang-dong, Sungdong-gu, Seoul 133-791, Korea. (Manuscript Received July 14, 2001; Revised February 4, 2002)

Table 1 Comparison of MDO methodologies

	All-at-once method	Individual Discipline Feasible Method	Collaborative Optimization	Bi-Level Integrated System Synthesis
No. of Optimizers	1	$1+n$	$1+n$	$1+n$
No. of MDASS	1	n	n	n

* n is the number of decomposed subsystems

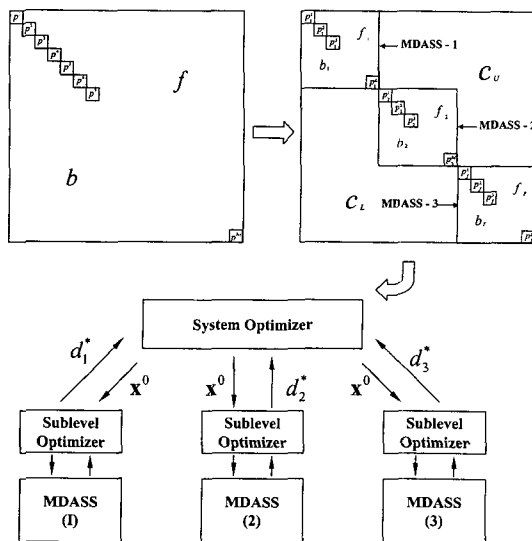


Fig. 1 CO procedure as an example of multi-level optimization problems

among subsystems and feedback couplings in the subsystems should be simultaneously reduced. The reason why couplings among subsystems should be reduced is that these couplings become consistency constraints, which are burdensome equality constraints, in applying MDO methodology. The reason why feedback couplings in the subsystems should be reduced is that these couplings bring about repetitive iterations in solving subsystem analyses.

In this study, a new decomposition method is proposed, which takes parallel computation into account. The method suggests qualitative value of parallel computation characteristics which is a function of the number of couplings. In order to reduce the number of feedback couplings in the subsystems, the established decomposition method is used. On the other hand, the objective function of genetic algorithms was changed

appropriately (to reduce the number of couplings among the subsystems).

In this work, methodology to solve multidisciplinary design problems and conventional decomposition method is explained. Parallel processing based decomposition method is proposed and numerical examples of the method are presented.

2. Review of Decomposition Method for Multidisciplinary Design Problem

2.1 Methodology for multidisciplinary design optimization

In order to solve a MDO problem, many analysis modules from various disciplines should be appropriately sequenced for optimization. For example, because the total analysis modules should operate as one analysis group, Rogers' decomposition method would be suitable for conventional optimizations, which use one optimizer such as all-at-once method, and this would reduce the number of iterations in the analysis groups.

There are many MDO methodologies which have several optimizers and several analyses groups. Table 1 compares the number of optimizers and analyses groups for each algorithm (Kroo, 1994; Tappeta, 1998; Sobieski, 1998). In these multi-level optimization methods, the total number of analysis modules should be decomposed into the appropriate number of groups. Therefore, a new decomposition method is required for parallel computation of multi-level optimization problems.

2.2 Conventional decomposition method

Considering the time consuming feedback

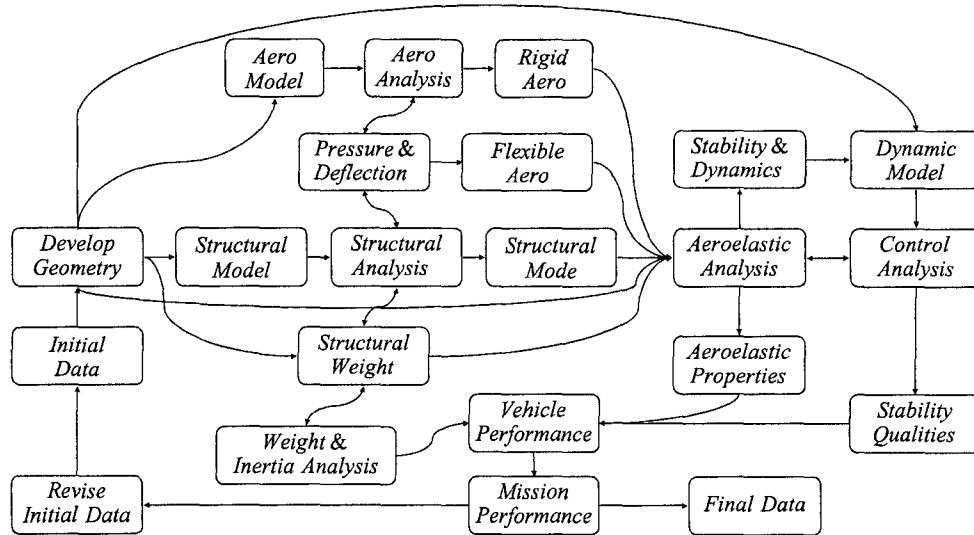


Fig. 2 Example of design process for MDO problem

coupling, Rogers (1992, 1994) and Altus (1995) proposed decomposition methods using genetic algorithms to reduce the number of feedback couplings. In their research, they used design structure matrix (DSM), which was devised by Steward (1981), to depict the coupling relations among disciplines and used genetic algorithm to reduce feedback couplings by reordering disciplines. Therefore, the objective function of genetic algorithm for their decomposition methods is formulated as:

$$\min_{\sigma \in \Pi} f(\sigma) = - \sum_{i=2}^{N_t} \sum_{j=1}^{i-1} \text{DSM}(i, j) (i-j) \quad (1)$$

where, Π denote a set of process sequences and the value of $\text{DSM}(i, j)$ is the number of state variables coupled between discipline i and discipline j in the design structure matrix in Fig. 5. If the i -th discipline and the j -th discipline do not have a coupling relationship, the value of $\text{DSM}(i, j)$ is set to 0. On the other hand, if the disciplines have n coupling variables, $\text{DSM}(i, j)$ is set to n . In improving this method, Rogers tried to minimize the number of feedback couplings of DSM and maximize parallel processing opportunities by decomposing the system into several levels (Rogers, 1996). However, his decomposition method of considering parallel computation is based on the experience of designer and does

not have quantitative criteria for the parallel computation characteristics of the DSM

Figures 2~4 are example used by Rogers to reorder processes for minimal feedback coupling. The processes and their couplings of aircraft design problems are displayed in the form of a process flow chart as shown in Fig. 2. Figure 3 depicts a DSM for the conceptual design project as shown in Fig. 3. In the DSM, the boxes on the diagonal indicate the processes in Fig. 2 and the numbers on the boxes represent which processes they are.

The output from each process is shown as a horizontal line that links to a numbered box, and the input is shown as a vertical line that enters a box. The off-diagonal dots that connect the horizontal and vertical lines represent couplings between two processes. Dots in the upper triangle of the DSM represent forward couplings and dots in the lower triangle of the DSM represent feedback couplings. Of the two, feedback couplings imply iterations in which coupled variables have to be converged and initial data estimates must be made. By using the Design Manager's Aid for Intelligent Decomposition (DeMAID), which is developed by Rogers, a DSM as shown in Fig. 4 can be obtained.

As explained before, Rogers proposed the decomposition method suitable or all-at-once

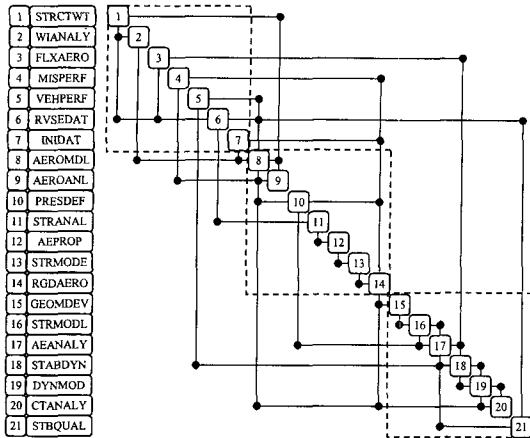


Fig. 3 Unsequenced DSM for sample MDO problem

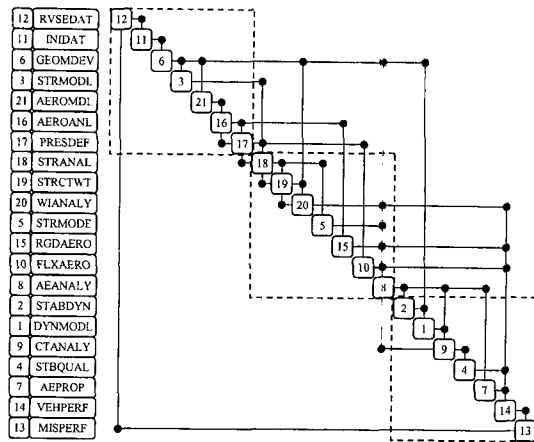


Fig. 4 Sequenced DSM for sample MDO problem

method in order to solve MDO problems. In his method, he tried to reduce iterative computations caused by feedback coupling in the design structure matrix through rescheduling of the order of modules using the genetic algorithm.

However, from the multi-level optimization viewpoint, designers can not expect parallel processing because the conventional decomposition methods solve the whole design problem all together at a time. Therefore, a new decomposition method, which divides the whole problem into several subproblems, is heeded for multi-level optimization.

3. Parallel Decomposition Method for Multi-Level Optimization

3.1 Decomposition method for parallel computing

In dealing with design problems where many disciplines are coupled each other, it is better to decompose a complex problem into several subproblems and apply a network based parallel processing to the decomposed problem.

As explained before, conventional decomposition method was performed to minimize the number of feedback couplings in the DSM using genetic algorithms. However, the objective function of genetic algorithm for the parallel decomposition should be formulated differently. Not only the feedback coupling factor but also the

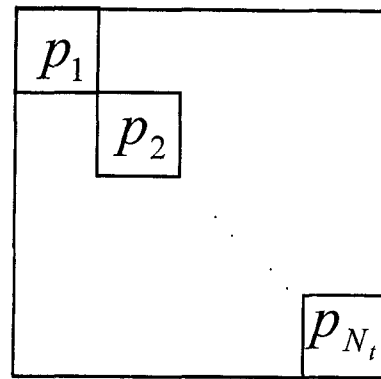


Fig. 5 DSM of system

coupling factor indicating the relationship among MDASS should be considered in the objective function of genetic algorithms.

Figure 6 shows the DSM when the total system composed of N_t disciplines is decomposed into N MDASS. Considering the k -th MDASS, symbols in the Fig. 6 are defined as:

$$f_k(\sigma) = \sum_{i=l}^{u-1} \sum_{j=l+1}^u \text{DSM}(i, j) \quad (2)$$

$$b_k(\sigma) = \sum_{i=l+1}^u \sum_{j=1}^{u-1} \text{DSM}(i, j) (i-j) \quad (3)$$

$$c_U(\sigma) = \sum_{i=1}^{N_t-1} \sum_{j=2}^{N_t} \text{DSM}(i, j) - \sum_{k=1}^N f_k \quad (4)$$

$$c_L(\sigma) = \sum_{i=2}^{N_t-1} \sum_{j=1}^{N_t} \text{DSM}(i, j) - \sum_{k=1}^N \left[\sum_{i=1}^u \sum_{j=1}^{u-1} \text{DSM}(i, j) \right] \quad (5)$$

$$l = \sum_{p=0}^{k-1} n_p + 1, \quad u = \sum_{p=1}^k n_p, \quad N_t = \sum_{p=0}^N n_p, \quad n_0 = 0 \quad (6)$$

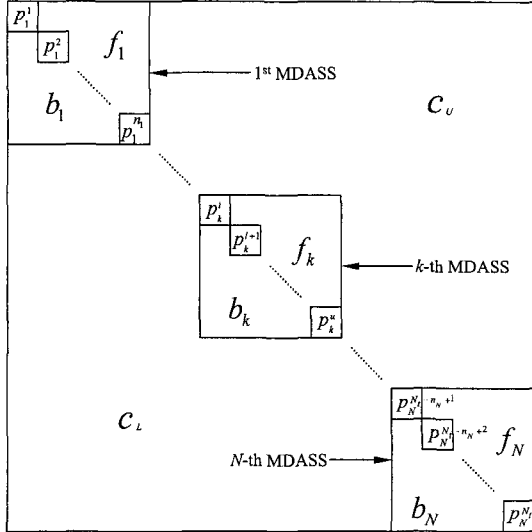


Fig. 6 DSM of decomposed system for parallel processing

Equation (2) is a forward coupling factor in each MDASS and this factor does not affect the computation time of subsystem analyses. Equation (3) is a feedback coupling factor in each MDASS and this factor increases the computation time of subsystem analyses. The distance factor ($i-j$) is multiplied in Eq. (3), since the analysis time of couplings, which are far away from each other, is longer than that of couplings, which are near each other. Equations (4) and (5) are upper and lower coupling factor of the system. In the Eq. (6), l and u denote the smallest and the biggest index of the disciplines in the k -th MDASS.

3.2 Objective function of genetic algorithm for parallel decomposition method

The genetic algorithm used in this paper is permutation based genetic algorithm (Mitsuo, 1997), in which a process sequence is regarded as a chromosome in the genetic algorithm and each discipline is supposed to be a gene which comprises a chromosome. By maximizing the objective function through the processes of cross-over and mutation in the genetic algorithm, a DSM as shown in Fig. 6 can be obtained. Using the Eqs. (2) ~ (6), the objective function of the genetic algorithm is proposed as:

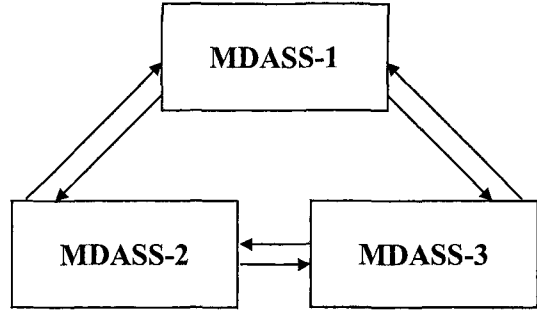


Fig. 7 Nonhierarchical structure of MDO problem

$$\underset{\sigma \in H}{\text{maximize}} \left[f(\sigma) = \frac{\min_{k=1 \dots N} f_k}{1 + \max \{ w_1 \cdot \max_{k=1 \dots N} b_k, w_2 \cdot \max(C_L, C_U) \}} \right] \quad (7)$$

k : MDASS No.

In the Eq. (7), the meaning of each term is as follow:

- $\min f_k$: the minimum forward coupling factor in the MDASS which should be maximized
- $\max b_k$: the maximum feedback coupling factor in the MDASS
- $\max(C_L, C_U)$: the larger value between the upper and the lower coupling factor in the DSM.
- $\left[1 + \max \{ w_1 \cdot \max_{k=1} b_k, w_2 \cdot \max(C_L, C_U) \} \right]^{-1}$: both $\max b_k$ and $\max(C_L, C_U)$ should be minimized. Since there is a tradeoff between them, the larger value between the weighted terms is selected as a coupling criterion in the objective function. However, the optimization problem of a genetic algorithm in Eq. (7) is not minimizing but maximizing the objective. The reciprocal is taken to make a maximization problem.

By maximizing the Eq. (7), which is a combination of the above terms, the optimal structure of DSM for multi-level optimization can be obtained. In this formulation, a min-max function, which is one of the preference functions in multi-objective optimization, is used to compose the objective function.

Figure 7 shows the result of the parallel

decomposition with three MDASS. Computation efficiency can be improved by applying an MDO methodology such as CO to decomposed problems.

4. Numerical Examples

Parallel decomposition of the aircraft design process and Colville's alkylation unit problem is demonstrated, and optimization of the decomposed problem is applied to the Colville's problem to validate parallel decomposition.

4.1 Parallel decomposition of aircraft design process

In this section, the parallel processing based decomposition method is applied to an aircraft design problem. Total analysis modules and the design structure matrix of the design are as shown in Figs. 2 and 3.

When the parallel processing based decomposi-

Table 2 Result of parallel decomposition for airplane design process

	Before parallel Decomposition	After Parallel Decomposition
Total Feedback Coupling	14	6
Total MDASS Coupling	15	12

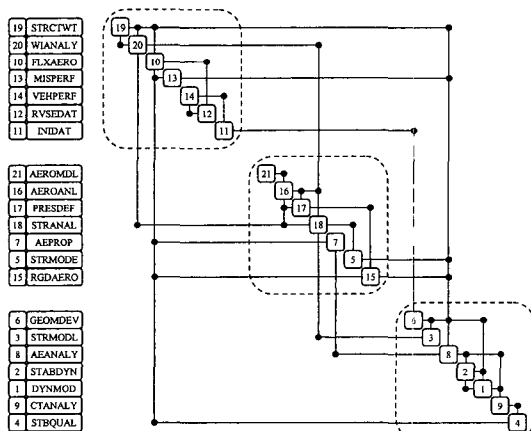


Fig. 8 Result of parallel decomposition for airplane design process

tion method is applied to the aircraft design problem, a DSM with three MDASS can be obtained (Fig. 8). As shown in Table 2, feedback couplings in the MDASS was reduced about 57% and couplings among the MDASS about 20%.

In performing the parallel decomposition above, it is important to decide the weighting factors of Eq. (3). The number of feedback couplings in the MDASS affects the analyses time of MDASS and the number of couplings among the MDASS affects the computation time of optimization. Because the iterations of optimization include the analyses of the MDASS, the weighting factor for the feedback coupling is regarded as more important compared to that of the MDASS couplings in this paper.

4.2 Parallel decomposition and collaborative optimization of colville's alkylation unit problem

A simplified process flow diagram of the alkylation process design problem used by Colville is given in Fig. 9. Design variables, state variables and optimization formulation are detailed in the Appendix. Figure 10 (a) is a DSM representation of the state variables and Fig. 10

Table 3 Result of parallel decomposition for alkylation unit process

	Before parallel Decomposition	After Parallel Decomposition
Total Feedback Coupling	5	2
Total MDASS Coupling	4	4

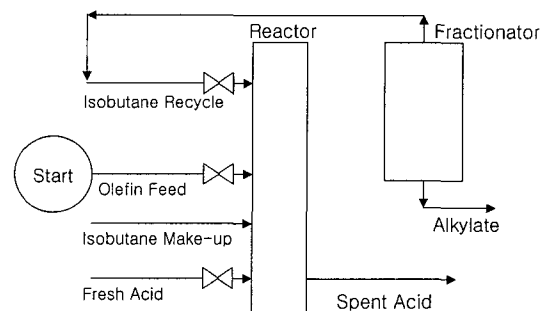


Fig. 9 Simplified flow diagram of an alkylation unit

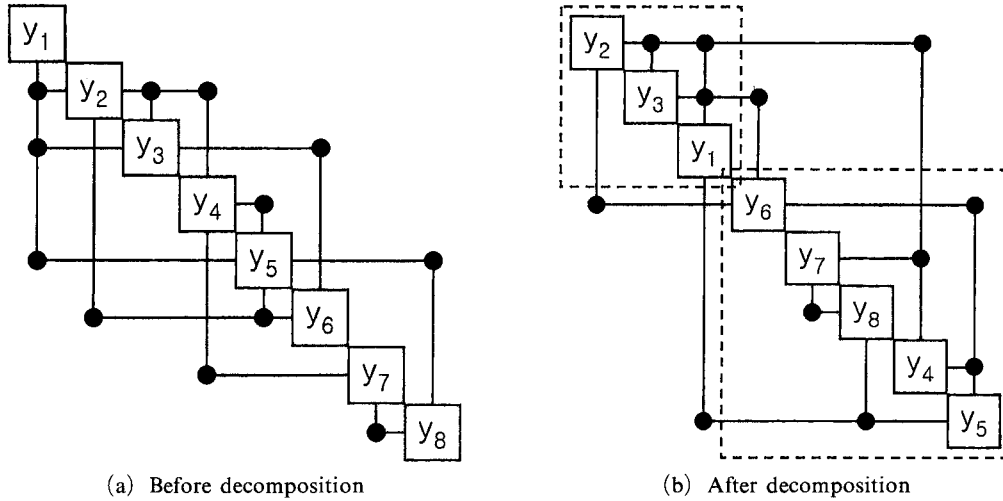


Fig. 10 Parallel decomposition of alkylation unit process

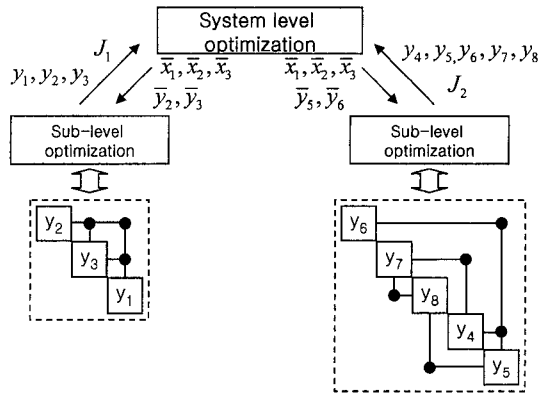


Fig. 11 Collaborative Optimization of alkylation unit process

(b) is the result of the parallel decomposition with two MDASS. From the Table 3, there is no change in couplings among MDASS, but the number of feedback couplings was reduced from 5 to 2. In order to validate the efficiency of the decomposed systems, CO of the Colville's problem is performed. The optimization problem of equation (A1) is reformulated as equations (A2) ~ (A4).

The structure for the CO of parallel decomposed Colville's problem is shown in Fig. 11. Both the conventional optimization of equation (A1) and CO of equations (A2) ~ (A4) are performed using DOT (Vanderplaats, 1995). In order to compare the pure decomposition effects,

no approximation methods were applied to the sample. The number of evaluation for the state variables $y_1 \sim y_8$ in the second method is much smaller (1503 evaluations) than that in the first one (4860 evaluations). The number of evaluation in the CO method is the summation of the bigger evaluation number between two sublevel optimizations because the state variables are evaluated in parallel under the two sublevel optimizations.

5. Concluding Remarks

The purpose of this work is to develop a decomposition method which is used for parallel processing based multidisciplinary design optimization problems. In applying the multi-level optimization methods to MDO problems, there is no proper method which can efficiently decompose the total system into several subsystems. A parallel decomposition method which is a precondition of multi-level optimization methods under the distributed computing environment.

When applying the distributed collaborative optimization to a MDO problem, the whole processes in the problem should be decomposed into several subsystems according to available network computing environment. Both the relations between processes in the subsystems and the effects of the relations on the analysis time should

also be considered. In a subsystem, backward relations, causing repetitive computation should be replaced by forward relations based on the theory of conventional decomposition methods. For the relations between the subsystems, the relation having the most state variables makes a major effect on the analysis time. Based on the results in this work, parallel decomposition method is more suitable to MDO problems than the conventional decomposition method.

The parallel decomposition method using the design structure matrix and genetic algorithm is proposed to apply collaborative optimization to MDO problems. The analysis time and the efficiency of collaborative optimization methods for MDO problems need to be conducted as a further study.

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Appendix

A simplified process flow diagram of the alkylation process optimized by Colville is given in Fig. 9. The problem includes three design variables and eight state variables. The process is constrained by fourteen inequality constraints. The design variables are controllable variables of the process, which can be controlled by the operator or automatic control system. The design and state variables for the alkylation process of Fig. 9 are detailed as below.

Definition of design variables

x_1 =Olefin Feed Rate, bpd
 x_2 =Isobutane Recycle Rate, bpd
 x_3 =Fresh Acid Addition Rate, Mbpd

Definition of state variables

y_1 =profit,
 $\$/\text{day}=0.063y_2y_5-5.04x_1-3.36y_3-0.035x_2-10.0x_3$
 y_2 =Alkylate Product Rate,
 bpd= $x_2(11+13.167x_2-0.6667y_6)/100$
 y_3 =Make-Up Isobutane Rate,
 bpd= $1.22y_2-x_1$
 y_4 =Spent Acid Strength,

$wt\%=98000x/(y_2y_7+1000x_3)$
 y_5 =Motor Octane Number
 $=86.35+1.098y_6-0.038y_6^2+0.25(y_4-89)$
 y_6 =External Isobutane to Olefin Ratio
 $=(x_2+x_3)/x_1$
 y_7 =Acid Dilution factor,
 $ADF=35.82-0.222y_8$
 y_8 =F-4 Performance No. @4.6cc Tel/gal.
 $=-133+3y_5$

The optimization problem of an alkylation process is given as :

$$\begin{aligned}
 & \text{Find} && x_1, x_2, x_3 \\
 & \text{Minimize} && y_1 \\
 & \text{Subject to} && 0 \leq y_2 \leq 5000 \\
 & && 0 \leq y_3 \leq 2000 \\
 & && 85 \leq y_4 \leq 93 \\
 & && 90 \leq y_5 \leq 95 \quad (\text{A1}) \\
 & && 3 \leq y_6 \leq 12 \\
 & && 0.01 \leq y_7 \leq 4 \\
 & && 145 \leq y_8 \leq 162 \\
 & && 0 \leq x_1 \leq 2000 \\
 & && 0 \leq x_2 \leq 16000 \\
 & && 0 \leq x_3 \leq 120
 \end{aligned}$$

The collaborative optimization (CO) formulation for an alkylation process is given in Eqs. (A2) ~ (A4). Equation (A2) is the system level optimization of the CO and Eqs. (A3) and (A4) are sublevel optimization of the CO.

$$\begin{aligned}
 & \text{Find} && \bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{y}_2, \bar{y}_3, \bar{y}_5, \bar{y}_6 \\
 & \text{Minimize} && y_1 \quad (\text{A2}) \\
 & \text{Subject to} && J_1=0 \\
 & && J_2=0
 \end{aligned}$$

$$\begin{aligned}
 & \text{Find} && x_1, x_2, x_3 \\
 & \text{Minimize} && J_1=(x_1-\bar{x}_1)^2+(x_2-\bar{x}_2)^2+(x_3-\bar{x}_3)^2 \\
 & && \quad + (y_2-\bar{y}_2)^2+(y_3-\bar{y}_3)^2 \\
 & \text{Subject to} && 0 \leq y_2 \leq 5000 \quad (\text{A3}) \\
 & && 0 \leq y_3 \leq 2000 \\
 & && 0 \leq x_1 \leq 2000 \\
 & && 0 \leq x_2 \leq 16000 \\
 & && 0 \leq x_3 \leq 120
 \end{aligned}$$

$$\begin{aligned}
 & \text{Find} && x_1, x_2, x_3 \\
 & \text{Minimize} && J_2=(x_1-\bar{x}_1)^2+(x_2-\bar{x}_2)^2+(x_3-\bar{x}_3)^2 \\
 & && \quad + (y_5-\bar{y}_5)^2+(y_6-\bar{y}_6)^2
 \end{aligned}$$

$$\begin{array}{l} \textit{Subject to} \\ 85 \leq y_4 \leq 93 \\ 90 \leq y_5 \leq 95 \\ 3 \leq y_6 \leq 12 \\ 0.01 \leq y_7 \leq 4 \\ 145 \leq y_8 \leq 162 \\ 0 \leq x_1 \leq 2000 \\ 0 \leq x_2 \leq 16000 \\ 0 \leq x_3 \leq 120 \end{array} \quad (\text{A4})$$