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Prediction of Hydraulic Conductivity from Grain-size Distribution Parameters

입도분포를 이용한 투수계수의 예측

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요 지

투수계수는 지반공학의 문제를 해결하는 데 중요한 인자의 하나이다. 그렇지만 현장이나 실험실에서 투수계수를 구하려면 시간과 비용이 많이 든다. 이 논문에서는 입도분포를 반영하는 통계적인 계수를 이용하여 모래의 투수계수를 예측할 수 있는 식을 제안하였다. 이를 위해 36가지 입도분포의 시료를 통계적인 방법으로 조성하여 투수시험을 한 후 그 결과를 회귀분석하였다. 제안식은 변수로서 체분석시험에서 구한 모래 입경의 기하평균과 기하표준편차 또는 D_{10}, D_{50}, D_{60} 등과 같은 입도분포계수를 사용한다. 제안된 식의 성능을 검증하기 위해 국내 20개 지역에서 채취한 시료에 대한 투수계수의 예측치와 실측치를 비교한 결과 비교적 잘 맞는 것으로 판명되었다. 또한 제안식의 성능이 Hazen 등 다른 연구자들의 식과 비교되었다.

Abstract

Hydraulic conductivity k is one of the most important engineering properties of soil. However, both field and laboratory procedures for the determination of k are often tedious and expensive. This paper presents new models to predict k using statistical parameters from grain size distribution. A number of permeability tests for 36 types of sands mixed based on statistics were conducted to develop the regression-based models. Parameters used to estimate k are both the geometric mean and geometric standard deviation of the soil samples, or the particle-size distribution curve parameters such as D_{10}, D_{50}, D_{60}. Hydraulic conductivity predicted by this model is in good agreement with the laboratory measurements for the soil samples obtained at 20 locations within the Korean Peninsula. The performances of the proposed models were also compared with those of existing models including Hazen's.

Keywords : Geometric mean, Geometric standard deviation, Grain-size distribution, Hydraulic conductivity, Regression

1. Introduction

The hydraulic conductivity k is one of the most important engineering properties of soils in relation to some geotechnical problems, such as the determination of seepage, settlement computation, and stability analysis. Estimates of k in the field environment are limited by lack of precise knowledge of the aquifer geometry and hydraulic boundaries (Uma et al, 1989), whereas laboratory permeability tests take time in duplication of field void ratio, and in de-airing procedure.

In general, hydraulic conductivity represents the ability

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of a porous medium to transmit water through its interconnected voids (Alyamani and Sen, 1993). Consequently, hydraulic conductivity is strongly affected by characteristics of voids that depend on the type of grain packing and gradation. Since soil void can be considered as the mirror of the grain size distribution, a certain relationship would be expected to exist between k and grain-size distribution. Recent predictive methods developed by a number of researchers contain rather complicated expressions except for Hazen (1911).

The objective of this research is to establish more simple empirical relationship between the hydraulic conductivity of sands and particle-size distribution. Two parameters, namely geometric mean (GM) and geometric standard deviation (GSD), are used to depict the grain-size distribution. Numerous permeability tests for soils in a variety of grain-size distributions have been analyzed to develop the regression-based model between k and two parameters. Soil samples to meet the purpose could be prepared by mixing soils under the model derived from analogy between probability density function and grain-size distribution curve. In this case, field soil samples appear to be less appropriate, because they may not cover wide spectrum of grain-size distribution, which leads to the difficulty in developing models to predict k for a wide variety of soils.

Hydraulic conductivity predicted by proposed model is compared with the measurements for several field soil samples to check the reliability and accuracy of the model. Also, the performances of the proposed models are compared with those of existing models such as Hazen's.

2. Previous Works

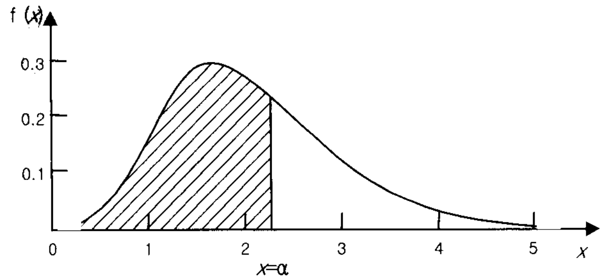
There have been two predictive methods to estimate the hydraulic conductivity, namely, theoretical relations and empirical relations. Kozeny-Carman equation was theoretically developed from Hagen-Poiseuille equation applied for laminar flow in the pipe. Empirical predictive methods have been developed by a number of researcher including Hazen (1911), Krumbein and Monk (1942), Amer and Awad (1974), Alyamani and Sen (1993), and Boadu (2000). Their developments summarized in Table 1 contain rather complicated expressions except for Hazen (1911).

3. Representation of Grain-size Distribution Data

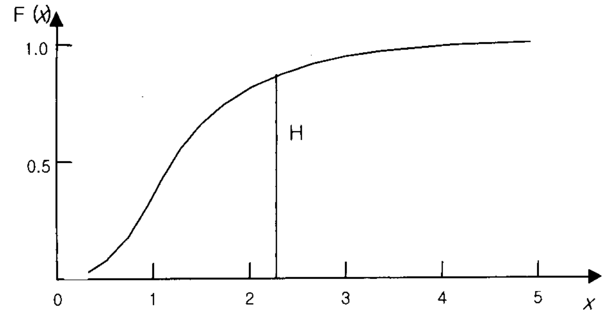
Referred to the literatures on statistics such as Milton and Arnold (1995), Montgomery and Runger (1994), the lognormal mean can be obtained from the following equation when weighing factors are given to variables.

Table 1. Empirical relations for coefficient of permeability of soil

Investigator	Relation	Notation	Remarks
Hazen (1911)	$k = CD_{10}^2$	C = Constant D_{10} = Effective size	effective for clean sand, Cu < 5
Krumbein and Monk (1942)	$k = (760d_w^2)e(-1.31\sigma_\psi)$	d_w = Geometric mean σ_ψ = Function of distribution	effective for unconsolidated sand, porosity < 40%
Kozeny (1927)-Carman (1956)	$k=\frac{1}{C_sS_s^2T^2}\frac{\gamma_w}{\mu}\frac{e^3}{1+e}$	C_s = Shape factor S_s = Hydraulic gradient T = Tortuous	effective for sand and silt
Amer and Awad (1974)	$k = C_1D_{10}^{2.32}C_u^{0.6}\frac{e^3}{1+e}$	C_u = Uniformity coefficient	
Alyamani and Sen (1993)	$k = 1300[I_0 + 0.025(D_{50} - D_{10})]^2$	I_0 = Intercept	
Boadu (2000)	$\ln k = 33.09+0.10P+0.18\Phi +0.33S-7.36D-11.09\rho$	P = Percent of fines Φ = Fractional porosity S = entropy D = Fractal dimension ρ = Bulk density	based on fractal-concept



(a) Lognormal distribution



(b) Cumulative probability

Fig. 1. Lognormal distribution curve

$$Log\mu = \frac{f_1 \log x_1 + f_2 \log x_2 + \ldots + f_n \log x_n}{\sum f} \qquad (1)$$

where f_1, f_2, ... f_n = weighing factors, x_1, x_2, ... x_n = variables, μ = GM

Likewise, the lognormal standard deviation can also be obtained from

$$Log\sigma = \sqrt{\frac{\sum f(\log x - \log \mu)^2}{\sum f}} \qquad (2)$$

where σ = GSD

The probability function for the lognormal distribution is

$$F(x) = \int_0^x f(x)$$
$$= \int_0^x \frac{1}{x\sqrt{2\pi}\log\sigma} e^\wedge\left(-\frac{1}{2}\left(\frac{\log x - \log\mu}{\log\sigma}\right)^2\right)dx \qquad (3)$$

where $f(x)$ = probability function, $F(x)$ = cumulative distribution function

Making computation easy to get a value of $F(x)$ in the probability table, x may as well be converted into

$$x' = \frac{\log x - \log\mu}{\log\sigma} \qquad (4)$$

An example for design of a soil mix by this model will be followed by next section.

A sieve and hydrometer analysis, whose results are plotted and sketched on a semi log paper to represent grain-size distribution, has been routinely performed in soil tests in order to classify soils. On the other hand, an attempt to represent grain-size distribution mathematically has also been made to identify soils. Gardner (1956) and Campell (1985) have tried to represent grain-size distribution data as a lognormal distribution that is expressed by geometric mean (GM) and geometric standard deviation (GSD). However, a limitation associated with using a lognormal type of equation is the assumption that the grain-size distribution is symmetric (Fredlund et al. 2000).

A physical appearance of lognormal distribution is shown in Fig. 1(a), and the shape of the curves, of course, can be altered with varying GM and GSD. If GM and GSD are known, the shape of lognormal distribution curve and cumulative probability curve (Fig. 1(b)) are determined accordingly, and vice versa.

Total area between this curve and the x axis in Fig. 1(a) is one square unit. Thus the height H indicated in Fig. 1(b) is numerically equal to the area shaded under the curve and thus gives the frequency with which observation less than or equal to any given values of $x=\alpha$ will occur. Preceding remarks imply the similitude between the value of cumulative probability and percent finer in grain size distribution curves. Therefore lognormal distribution would be considered to have ability to depict the grain-size distribution.

4. Experiment

4.1 Design of Soil Mixture

Principle for design of soil mix was based on geometrical distribution and its probability function as mentioned in section 3. Thirty-six types of soil mixtures having GM equal to 2.0, 1.18, 0.85, 0.60, 0.425 and 0.25mm were prepared, and every mean was respectively

Table 2. An example for design of soil mix (1800g) with geometric mean of 0.6 and geometric standard deviation of 3.0

Sieve number	Sieve size x (mm)	x'=(logx−log μ)/log σ	Cumulative Probability (Percent finer)	Retained %	Required Quantities, gr
4	4.75	1.88	97.0	3.0	54.2
8	2.36	1.25	89.4	7.6	135.9
10	2.00	1.10	86.4	3.0	54.2
16	1.18	0.62	73.2	13.2	237.4
20	0.85	0.32	62.6	10.7	192.4
30	0.6	0.00	50.0	12.6	225.9
40	0.425	−0.31	37.8	12.2	219.1
50	0.3	−0.63	26.4	11.4	205.2
60	0.25	−0.80	21.2	5.2	94.3
80	0.18	−1.10	13.6	7.6	137.2
100	0.15	−1.26	10.4	3.2	57.4
140	0.106	−1.58	5.7	4.7	84.1
170	0.088	−1.75	4.0	1.7	30.6
200	0.075	−1.89	2.9	1.1	19.3
pan				2.9	52.9

associated with GSD equal to 1.5, 2.0, 3.0, 4.0, 5.0 and 8.0.

Table 2 shows an example for design of a soil mix to get 1800g of soil mixture having GM equal to 0.6mm and GSD equal to 3.0. A sieve size is considered to be the value of x, however, in the probability function, normalized value x' by equation (4) is to be used in the probability table and get the cumulative probability that corresponds to percent finer than sieve size x in the grain size distribution curve. Once soil weight to be retained at corresponding sieve size is calculated by using percent finer, a soil mixture can be constituted by mixing corresponding volume of soil. Required soil volume of the particle size to meet the statistics could be prepared by sieving a large quantity of soil. All material retained on each sieve was gathered in a separate container labeled with a corresponding sieve size. Under the above systematic method, 36 types of sand mixture could be prepared. Masih (2000) has applied normal distribution in his work to representing grain-size distribution for the purpose of getting desired soil density.

5. Results and Discussions

5.1 Representation of Grain-size Distribution by Statistical Parameters

A Grain-size distribution curve sketched for the Kum-River sand was compared with the one duplicated by using two parameters, GM and GSD in the Fig. 2. It is observed that two-parameter model provides a close fit of the real grain-size distribution curve.

Among 36 types of samples, the grain-size distribution curves feature for samples with GSD for GM equal 0.6mm are shown in Fig. 3, in which a curve varies in shape with GSD, even though GM is constant through the samples, and the smaller the GSD, the stiffer the slope of the straight part in the middle of the curves. Fig. 4 illustrates the grain-size distribution curves features for samples with varying GM, for GSD equal to 3.0.

It is observed from Fig. 4 that the slopes of the straight part of the curves are almost the same, when GSD is constant through the samples even though GM varies. And the curves in the figure shift from left to right with an increase in GM.

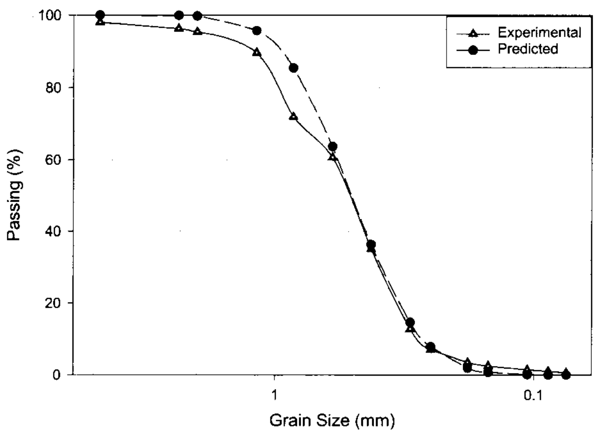


Fig. 2. Grain size distribution fit with GM and GSD

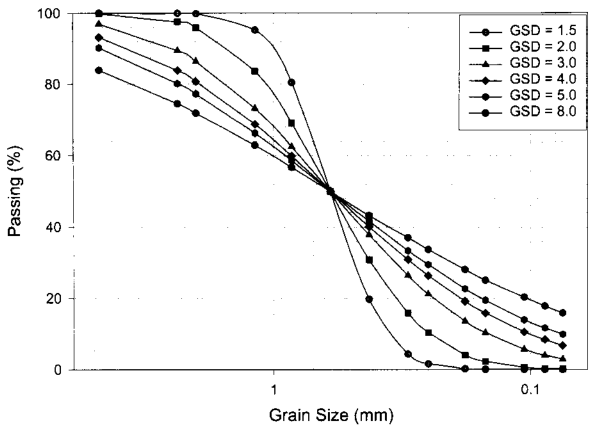


Fig. 3. Grain size distribution curves for GM = 0.6mm

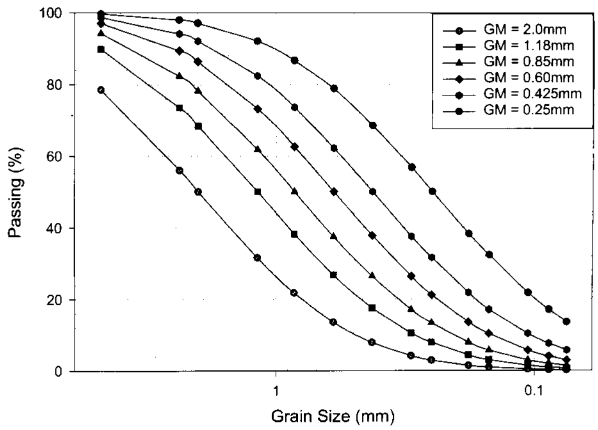


Fig. 4. Grain size distribution with variation of GM, GSD = 3.0

Thus two parameters could be used to represent grain-size distribution. However, there may be limitation associated with this application. Among three categories of grain-size distributions (well-graded soils, uniform soils, and gap-graded soils), unfortunately, it may be unsatisfactory to fit gap-graded soils by two parameters, because its grain-size distribution is often non symmetrical.

5.2 Statistical Parameters and Hydraulic Conductivity

In the compaction mold, each soil sample was compacted at OMC ranging 10-15%, and tested in constant head permeameter according to KS F2322. Thirty-six samples were tested 3 times respectively, consequently 108 permeameter tests were conducted so that reasonable regression-based model could be yielded. Hydraulic conductivity-geometric mean relationships for mixtures with constant GSD are presented in Fig. 5, which shows decrease of GM from 2.0mm to 0.25mm and results in about 10^2 times decrease of k from $2.60*10^{-1}$cm/sec to $2.80*10^{-3}$cm/sec for GSD = 1.5.

It is clear from the Fig. 5 that the lesser the geometric mean is, the lower the hydraulic conductivity is. On the other hand, Hydraulic conductivity-GSD relationship for the mixtures with constant GM could also be interpreted from Fig. 5. A change in GSD from 1.5 to 8 results in 1280 times decrease of k from $2.60*10^{-1}$cm/sec to $2.03*10^{-4}$cm/sec for GM=2.0mm. It is also clear that the higher the geometric standard deviation, the lower the hydraulic conductivity.

The preceding discussion leads to the conclusion that both geometric mean and geometric standard deviation affect the hydraulic conductivity of soil samples. However, a GSD seems to be more dominant factor affecting hydraulic conductivity than geometric mean. In addition, the value of k decreases more rapidly at the early portion of curves, and afterward it decreases slowly.

5.3 Prediction of Hydraulic Conductivity

Method 1

A number of hydraulic conductivity test data shown in Fig. 5 were analysed by DataFit® that is a software used for nonlinear regression. The relationship between the coefficient of permeability and GM, as well as GSD, is shown in Fig. 6 and can be written as

$$k = 0.3357 * \mu^{2.077} * \sigma^{-4.137} \fallingdotseq \frac{\mu^{2.1} * \sigma^{-4.1}}{3} \quad (5)$$

where μ = geometric mean, σ = geometric standard deviation. The model has the coefficient of correlation R^2 of 0.985 which means roughly 99% of the variance in hydraulic conductivity is explained by the model. Once both GM and GSD of the sample are calculated by equations (1) and (2) respectively, hydraulic conductivity could be estimated by equation (5). Alternatively, hydraulic conductivity of a sample can be read from Fig. 6.

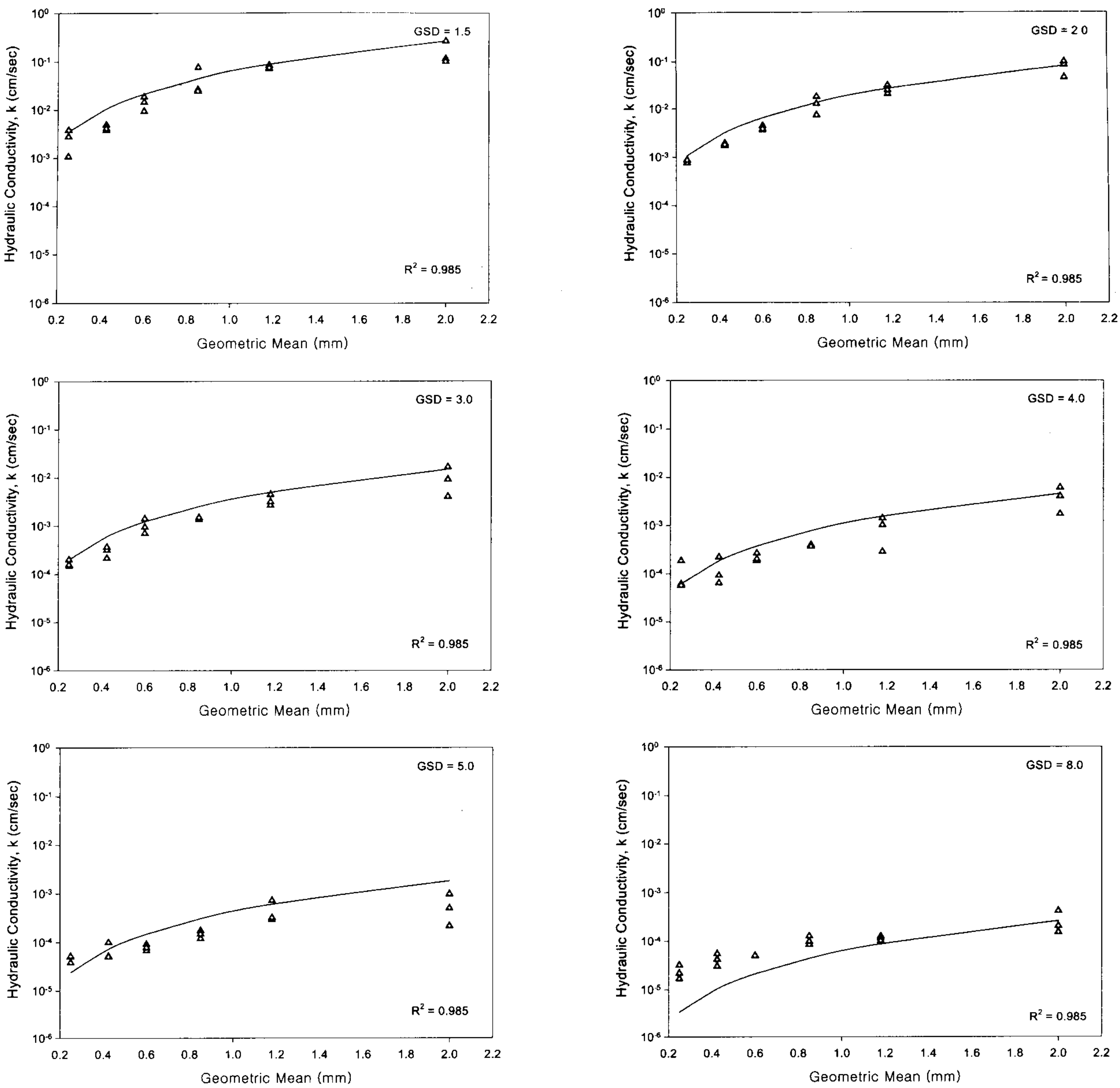


Fig. 5. Hydraulic conductivity with GM and GSD

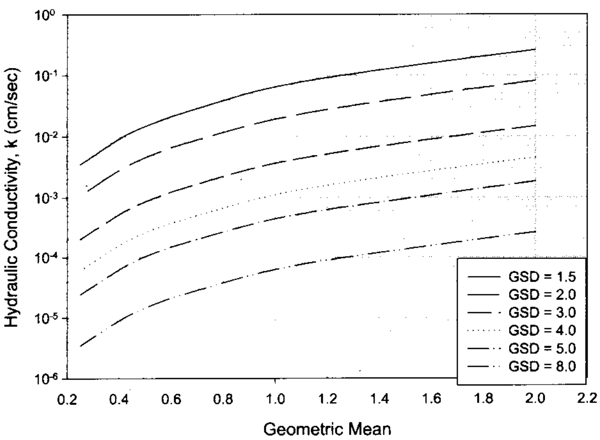


Fig. 6. Hydraulic conductivity with GM and GSD

Method 2

As shown in Fig. 3, the smaller the GSD, the stiffer the slope of the middle part of grain-size distribution curve. And the slopes seem to be closely related with the coefficient of uniformity commonly used in soil engineering. An attempt has been made to establish a relationship between Cu and GSD. Fig. 7 shows that GSD bears a relationship with the coefficient of uniformity Cu with R^2 of 0.998. Then it is possible to express the relation as

$$\sigma = Cu^{0.651} \tag{6}$$

where $Cu = D_{60}/D_{10}$.

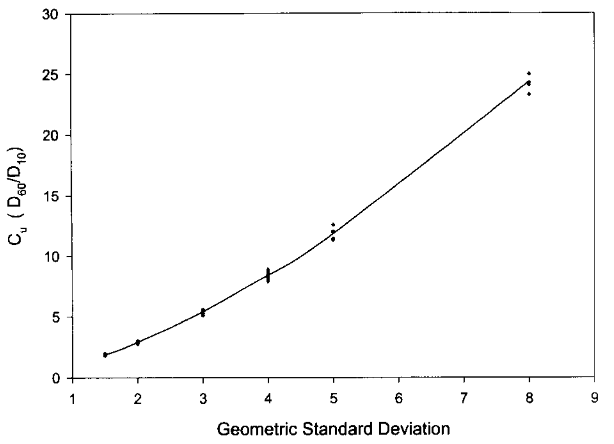


Fig. 7. Relationship between GSD and uniformity coefficient, C_u

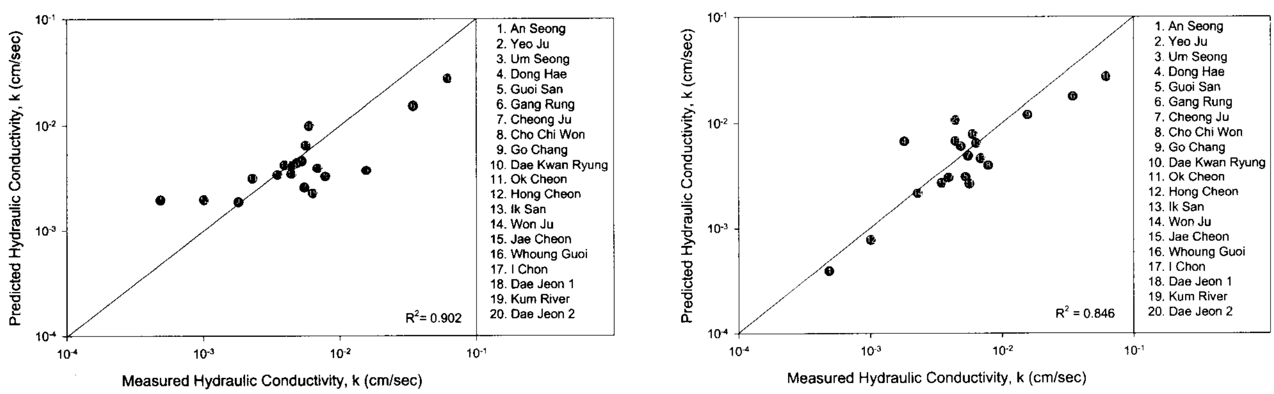
And Substitution of D_{50} for μ, and substitution of preceding equation into equation (5) yields

$$k = 0.3357 \times (D_{50})^{2.077} \times Cu^{-2.693} \fallingdotseq \frac{D_{50}^2 \times Cu^{-2.7}}{3} \qquad (7)$$

This equation has R^2 of 0.998. Equation (7) is basically based upon equation (5). However, statistic parameters would be necessary to obtain hydraulic conductivity when using equation (5), where the grain-size distribution curve parameters would be needed when using equation (7).

5.4 Performance of the Model

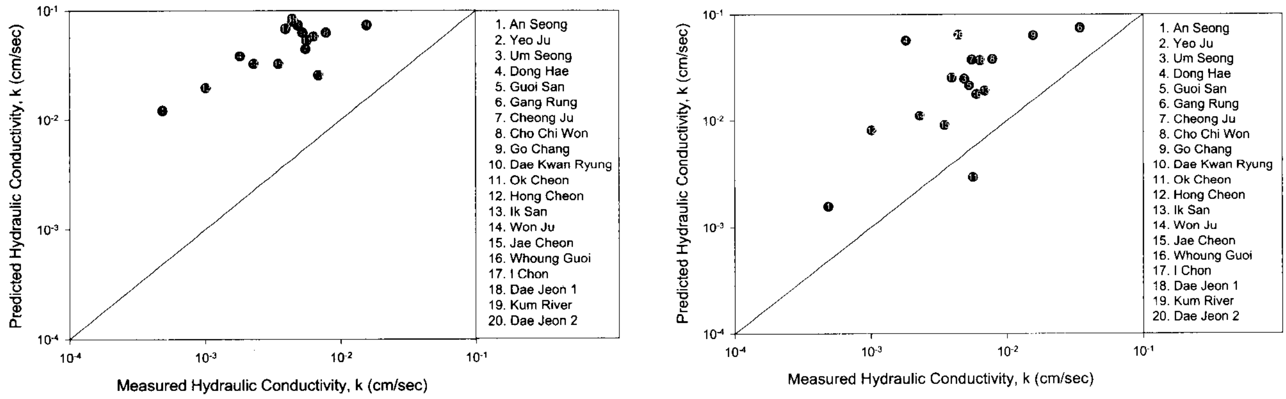
Soil samples obtained from 20 locations including An-Seong, Dong-Hae, Cheong-Ju, Go-Chang, Kum-River within the Korean Peninsula were tested using constant-head permeameter to assure the validity of the proposed models. Soil samples are classified as SP or SW. Comparisons of the measured hydraulic conductivity with the predicted one by equation (5), as well as (7) are presented in Fig. 8. The figure shows that proposed models are in good agreement with experimental results. The coefficients of correlation were 0.902 and 0.846 respectively and a straight line means perfect equality in



(a) Predicted by Eq. (5)

(b) Predicted by Eq. (7)

Fig. 8. Measured hydraulic conductivity k versus computed value k



(a) Predicted by hazen model (1911)

(b) Predicted by alyamani and sen model (1993)

Fig. 9. Measured hydraulic conductivity k versus computed value k

Fig. 8. Thus suggested models would be helpful to estimating k for the common soils, at least, within Korea. Equation (7) basically originats from equation (5) as mentioned in section 5.3. The variation between predicted values from equation (5) and those from equation (7) may be due to using graphical parameters, in lieu of mathematical values. Hydraulic conductivity data shown in Fig. 8 should not be used for representative values of k in the area, because it is not for providing information on k, but for demon- strating applicability of proposed models.

Comparisons of the measured hydraulic conductivity with that predicted by Hazen, Alyamani and Sen (1993) are also presented in Figs. 9 (a) and (b) respectively. It has been observed that Hazen Model overpredicts k within a particular range of k, 1×10^{-1}cm/sec to 1×10^{-2}cm /sec, and Alyamani and Sen (1993) Model does not so overpredict k as Hazen do. However, this may be controversial, because k could vary due to the conditions on which hydraulic conductivity tests are conducted.

6. Conclusions

Both laboratory and field procedures involved in estimating hydraulic conductivity of soils are tedious, time consuming, and expensive (Boadu 2000). Hence alternative methods such as using empirical formula would be effective for predicting k with ease and simplicity. This research has focused on investigating a interrelation between hydraulic conductivity of sands and statistic parameters as well as grain-size distribution curve parameters. Two types of methods to predict k have been presented in this paper. Soil samples from 20 locations within Korea were tested using constant-head permeability apparatus to assure the validity of the proposed models. It was found that hydraulic conductivity predicted by proposed model is in good agreement with the laboratory measurements. The presented models may be suggested as useful alternative to laboratory tests. The abilities of the developed models to predict k are also compared with those of existing models. Two models, Hazen and Alyamani-Sen appeared to overpredict k of the samples.

However, all the field soil samples tested in this work showed well graded or uniform-graded. When the soil samples are gap-graded, it may be implausible to get satisfactory agreement. Hence, further research would be needed to develope a model fitting gap-graded soils, and to extend the range of particle size analyzed to predict hydraulic conductivity of soil with more fines content.

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