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Smooth neighberhood structures

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Abstract

In this paper, we introduce the notion of smooth neighborhoods in smooth topological spaces and investigate some of their properties. In particular, we can obtain some smooth topologies from a smooth neighborhood system.

Key Words : Smooth topological spaces, Smooth neighborhood structures, Smooth continuous maps

1. Introduction

Shostak [11] introduced the fuzzy topology as an extension of Chang's fuzzy topology [1]. In 1992, the same concept under the name of gradation of openness was rediscovered by Chattopadhyay et.al.[2]. Ramadan and his colleagues [5,6,9] called it smooth topology. It has been developed in many directions [3,4,8]. Demirci [4] and Ramadan [10] introduced smooth neighborhood structures in other viewpoints.

In this paper, we introduce the notion of smooth neighborhoods in smooth topological spaces with a different viewpoint in [4,10] and investigate some of their properties. In particular, we can obtain some smooth topologies from a smooth neighborhood system.

Throughout this paper, let X be a non-empty set and $I=[0,1]$ be an unit interval. The family I^X denotes the set of all fuzzy subsets of a given set X. For each $\alpha\in I$, let $\underline{\alpha}$ denote the constant fuzzy subset of X with value α. All the other notations and the other definitions are standard in fuzzy set theory.

2. Preliminaries

Definition 2.1 [9,11] A mapping $\tau: I^X\to I$ is called a smooth topology on X if it satisfies the following conditions:

(O1) $\tau(\underline{0})=\tau(\underline{1})=1$,

(O2) for each $\lambda_1,\lambda_2\in I^X$, $\tau(\lambda_1\wedge\lambda_2)\geq\tau(\lambda_1)\wedge\tau(\lambda_2)$,

(O3) $\tau(\bigvee_{i\in\Gamma}\lambda_i)\geq\bigwedge_{i\in\Gamma}\tau(\lambda_i)$ for each $\{\lambda_i \mid i\in\Gamma\}\subset I^X$.

The pair (X,τ) is called a smooth topological space.

Remark 2.2 If a smooth topology τ on X satisfies the following property $\tau(I^X)\subseteq\{0,1\}$, then there is a one-to-one correspondence between a smooth topology τ and a Chang's fuzzy topology [1].

Definition 2.3 [11] Let (X,τ_1) and (Y,τ_2) be smooth topological spaces. A map $f:(X,\tau_1)\to(Y,\tau_2)$ is called smooth continuous iff for every $\mu\in I^Y$, we have

$$\tau_1(f^{-1}(\mu))\geq\tau_2(\mu).$$

3. Smooth neighborhood structures

We define a smooth neighborhood system and we give some of its properties.

Definition 3.1 Let (X,τ) be a smooth topological space.

(1) A fuzzy set $\lambda\in I^X$ is a smooth neighborhood of a point $x\in X$ iff there exists $\mu\in I^X$ with $\tau(\mu)>0$ such that $\mu\leq\lambda$, $x\in supp(\mu)$, where $supp(\mu)=\{x\in X \mid \mu(x)>0\}$.

(2) A mapping $N_x^\tau: I^X\to I$ is called a smooth neighborhood system of $x\in X$ with respect to τ if

$$N_x^\tau(\lambda)=\begin{cases}\bigvee\{\tau(\mu) \mid \mu\leq\lambda,\ x\in supp(\mu),\ \tau(\mu)>0\}\\ 0, \text{ otherwise.}\end{cases}$$

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Theorem 3.2 Let (X, τ) be a smooth topological space. For each $x \in X$, the smooth neighborhood system N_x^τ satisfies the following properties:

(1) $N_x^\tau(\underline{1}) = 1$, for each $x \in X$,

(2) if $N_x^\tau(\lambda) > 0$ for each $\lambda \in I^X$, then $x \in supp(\lambda)$,

(3) if $\lambda \le \mu$ for each $\lambda, \mu \in I^X$, then $N_x^\tau(\lambda) \le N_x^\tau(\mu)$,

(4) for each $\lambda, \mu \in I^X$,

$$N_x^\tau(\lambda \wedge \mu) \ge N_x^\tau(\lambda) \wedge N_x^\tau(\mu),$$

(5) for each $\lambda \in I^X$,

$$N_x^\tau(\lambda) \le \bigvee \{N_x^\tau(\mu) \wedge (\bigwedge_{y \in supp(\mu)} N_y^\tau(\mu)) \mid \mu \le \lambda\}.$$

Proof. (1),(2) and (3) are easily proved from the definition of N_x^τ.

(4) Suppose there exist $\lambda, \mu \in I^X$ and $x \in X$ such that

$$N_x^\tau(\lambda \wedge \mu) \not\ge N_x^\tau(\lambda) \wedge N_x^\tau(\mu).$$

Then there exists $t \in (0,1)$ such that

$$N_x^\tau(\lambda \wedge \mu) < t < N_x^\tau(\lambda) \wedge N_x^\tau(\mu).$$

Since $N_x^\tau(\lambda) > t$ and $N_x^\tau(\mu) > t$, there exist $\lambda_1, \mu_1 \in I^X$ with $x \in supp(\lambda_1)$, $x \in supp(\mu_1)$, $\lambda_1 \le \lambda$ and $\mu_1 \le \mu$ such that $\tau(\lambda_1) > t$, $\tau(\mu_1) > t$. It implies $x \in supp(\lambda_1 \wedge \mu_1)$, $\lambda_1 \wedge \mu_1 \le \lambda \wedge \mu$ such that $\tau(\lambda_1 \wedge \mu_1) \ge \tau(\lambda_1) \wedge \tau(\mu_1) > t$.
So, $N_x^\tau(\lambda \wedge \mu) > t$. It is a contradiction.

(5) Suppose

$$N_x^\tau(\lambda) > t > \bigvee \{N_x^\tau(\mu) \wedge (\bigwedge_{y \in supp(\mu)} N_y^\tau(\mu)) \mid \mu \le \lambda\}.$$

Since $N_x^\tau(\lambda) > t$, there exists $\mu \in I^X$ with $x \in supp(\mu)$ and $\mu \le \lambda$ such that $\tau(\mu) > t$. Furthermore, $y \in supp(\mu)$ and $\mu \le \mu$ such that

$$\bigwedge_{y \in supp(\mu)} N_y^\tau(\mu) \ge \tau(\mu) > t.$$

Hence,

$$\bigvee \{N_x^\tau(\mu) \wedge (\bigwedge_{y \in supp(\mu)} N_y^\tau(\mu)) \mid \mu \le \lambda\} > t.$$

It is a contradiction.

Definition 3.3 A mapping $N: X \to I^{I^X}$ is called a smooth neighborhood system on X iff it satisfies:

(N1) $N_x(\underline{1}) = 1$, for each $x \in X$,

(N2) if $N_x(\lambda) > 0$ for each $\lambda \in I^X$, then $x \in supp(\lambda)$,

(N3) if $\lambda \le \mu$ for each $\lambda, \mu \in I^X$, then $N_x(\lambda) \le N_x(\mu)$,

(N4) for each $\lambda, \mu \in I^X$,

$$N_x(\lambda \wedge \mu) \ge N_x(\lambda) \wedge N_x(\mu),$$

(N5) for each $\lambda \in I^X$,

$$N_x(\lambda) \le \bigvee \{N_x(\mu) \wedge (\bigwedge_{y \in supp(\mu)} N_y(\mu)) \mid \mu \le \lambda\}.$$

Theorem 3.4 Let $N: X \to I^{I^X}$ be a mapping satisfying conditions (N1)-(N4) in Definition 3.3. We define a mapping $\tau_N: I^X \to I$ by

$$\tau_N(\lambda) = \begin{cases} \bigwedge_{x \in supp(\lambda)} N_x(\lambda) & \text{if } \lambda \ne \underline{0} \\ 1, & \text{if } \lambda = \underline{0}. \end{cases}$$

Then :

(1) τ_N is a smooth topology on X.

(2) If the mapping N is a smooth neighborhood system on X, then $N_x(\lambda) = N_x^{\tau_N}(\lambda)$, for each $x \in X$ and $\lambda \in I^X$.

Proof (1) (O1) It is trivial from the definition of τ_N.

(O2)
$$\begin{aligned} &\tau_N(\lambda \wedge \mu) \\ &= \bigwedge_{x \in supp(\lambda \wedge \mu)} N_x(\lambda \wedge \mu) \\ &\ge \bigwedge_{x \in supp(\lambda \wedge \mu)} N_x(\lambda) \wedge N_x(\mu) \\ &= \bigwedge_{x \in supp(\lambda \wedge \mu)} N_x(\lambda) \wedge (\bigwedge_{x \in supp(\lambda \wedge \mu)} N_x(\mu)) \\ &\ge \bigwedge_{x \in supp(\lambda)} N_x(\lambda) \wedge (\bigwedge_{x \in supp(\mu)} N_x(\mu)) \\ &= \tau_N(\lambda) \wedge \tau_N(\mu). \end{aligned}$$

(O3) Let $\{\lambda_i \mid i \in J\} \subset I^X$.

If $\bigvee_{i \in J} \lambda_i = \underline{0}$, then it is obvious that

$$\tau_N(\bigvee_{i \in J} \lambda_i) = 1 \ge \bigwedge_{i \in J} \tau_N(\lambda_i).$$

If $\bigvee_{i \in J} \lambda_i \ne \underline{0}$, then there exists $x \in supp(\bigvee_{i \in J} \lambda_i)$ which there exists $k \in J$ such that $x \in supp(\lambda_k)$.
Thus,

$$N_x(\bigvee_{i \in J} \lambda_i) \ge N_x(\lambda_k) \ge \bigwedge_{x \in supp(\lambda_k)} N_x(\lambda_k) = \tau_N(\lambda_k).$$

Hence

$$\tau_N(\bigvee_{i \in J} \lambda_i) = \bigwedge_{x \in supp(\bigvee_{i \in J} \lambda_i)} N_x(\bigvee_{i \in J} \lambda_i) \ge \bigwedge_{i \in J} \tau_N(\lambda_i).$$

Hence τ_N is a smooth topology on X.

(2) Let N be a smooth neighborhood system on X. We have

$$\begin{aligned} N_x^{\tau_N}(\lambda) &= \bigvee \{\tau_N(\mu) \mid x \in supp(\mu),\ \mu \le \lambda\} \\ &= \bigvee \{\bigwedge_{y \in supp(\mu)} N_y(\mu) \mid x \in supp(\mu),\ \mu \le \lambda\} \\ &\le N_x(\lambda). \quad (\text{since } N_x(\mu) \le N_x(\lambda)) \end{aligned}$$

Hence, $N_x^{\tau_N} \le N_x$.

Conversely, by (N5) of Definition 3.3,

$$\begin{aligned} &N_x(\lambda) \\ &\le \bigvee \{N_x(\mu) \wedge (\bigwedge_{y \in supp(\mu)} N_y(\mu)) \mid \mu \le \lambda\} \\ &= \bigvee \{N_x(\mu) \wedge (\bigwedge_{y \in supp(\mu)} N_y(\mu)) \mid x \in supp(\mu),\ \mu \le \lambda\} \\ &\le \bigvee \{\bigwedge_{y \in supp(\mu)} N_y(\mu) \mid x \in supp(\mu),\ \mu \le \lambda\} \\ &= \bigvee \{\tau_N(\mu) \mid x \in supp(\mu),\ \mu \le \lambda\} \\ &= N_x^{\tau_N}(\lambda). \end{aligned}$$

Theorem 3.5 Let (X, τ) be a smooth topological space. Then $\tau_{N^\tau}(\lambda) \geq \tau(\lambda)$ for each $\lambda \in I^X$.

Proof If $\tau(\lambda)=0$ or $\lambda=\underline{0}$, it is trivial.
If $\tau(\lambda)>0$ and $\lambda \neq \underline{0}$, then there exists $x \in X$ such that $N^\tau_x(\lambda) \geq \tau(\lambda)$. It implies

$$\tau_{N^\tau}(\lambda) = \bigwedge_{x \in supp(\lambda)} N^\tau_x(\lambda) \geq \tau(\lambda).$$

Theorem 3.6 Let (X, τ) and (Y, η) be two smooth topological spaces. If $f:(X, \tau) \to (Y, \eta)$ is smooth continuous, then $N^\eta_{f(x)}(\lambda) \leq N^\tau_x(f^{-1}(\lambda))$ for all $x \in X$ and $\lambda \in I^Y$.

Proof. For any $x \in X$ and $\lambda \in I^Y$, we have
$N^\eta_{f(x)}(\lambda)$
$= \bigvee\{\eta(\mu) \mid f(x) \in supp(\mu), \mu \leq \lambda\}$
$\leq \bigvee\{\tau(f^{-1}(\mu)) \mid x \in supp(f^{-1}(\mu)), f^{-1}(\mu) \leq f^{-1}(\lambda)\}$
$\leq N^\tau_x(f^{-1}(\lambda))$.

Example 3.7 Let $X=\{a, b\}$ be a set and $\mu \in I^X$ as follows:

$$\mu(a)=0.6, \quad \mu(b)=0.3.$$

(1) We define a smooth fuzzy topology

$$\tau(\lambda)=\begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda=\mu \\ 0, & \text{otherwise.} \end{cases}$$

From Definition 3.1(2), we obtain $N^\tau_a, N^\tau_b : I^X \to I$ as follows:

$$N^\tau_a(\lambda)=\begin{cases} 1, & \text{if } \lambda=\underline{1}, \\ \frac{1}{2}, & \text{if } \underline{1} \neq \lambda \geq \mu \\ 0, & \text{otherwise.} \end{cases}$$

$$N^\tau_b(\lambda)=\begin{cases} 1, & \text{if } \lambda=\underline{1}, \\ \frac{1}{2}, & \text{if } \underline{1} \neq \lambda \geq \mu \\ 0, & \text{otherwise.} \end{cases}$$

From Theorem 3.4, we have

$$\tau_{N^\tau}(\lambda)=\begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \underline{1} \neq \lambda \geq \mu, \\ 0, & \text{otherwise.} \end{cases}$$

Thus, $\tau_{N^\tau}(\lambda) \geq \tau(\lambda)$ for each $\lambda \in I^X$.

(2) We define $N_a, N_b: I^X \to I$ as follows:

$$N_a(\lambda)=\begin{cases} 1, & \text{if } \lambda=\underline{1}, \\ \frac{1}{2}, & \text{if } \underline{1} \neq \lambda \geq \mu \\ 0, & \text{otherwise.} \end{cases}$$

$$N_b(\lambda)=\begin{cases} 1, & \text{if } \lambda=\underline{1}, \\ \frac{1}{3}, & \text{if } \underline{1} \neq \lambda \geq \mu \\ 0, & \text{otherwise.} \end{cases}$$

Since

$$\frac{1}{2}=N_a(\mu) > N_a(\mu) \wedge \Big(\bigwedge_{y \in supp(\mu)} N_y(\mu)\Big)=\frac{1}{3},$$

it does not satisfy the condition (N5) of Definition3.3. By Theorem 3.4, we have

$$\tau_N(\lambda)=\begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{3}, & \text{if } \underline{1} \neq \lambda \geq \mu, \\ 0, & \text{otherwise.} \end{cases}$$

We obtaine $N^{\tau_N}_a, N^{\tau_N}_b: I^X \to I$ as follows:

$$N^{\tau_N}_a(\lambda)=\begin{cases} 1, & \text{if } \lambda=\underline{1}, \\ \frac{1}{3}, & \text{if } \underline{1} \neq \lambda \geq \mu \\ 0, & \text{otherwise.} \end{cases}$$

$$N^{\tau_N}_b(\lambda)=\begin{cases} 1, & \text{if } \lambda=\underline{1}, \\ \frac{1}{3}, & \text{if } \underline{1} \neq \lambda \geq \mu \\ 0, & \text{otherwise.} \end{cases}$$

In general, $N_x \neq N^{\tau_N}_x$.

We can obtain another smooth topology from a smooth neighborhood system.

Theorem 3.8 Let $N: X \to I^{I^X}$ be a mapping satisfying conditions (N1)-(N4) in Definition 3.3. We define a mapping $T_N: I^X \to I$ by

$$T_N(\lambda)=\begin{cases} 1 & \text{if } \lambda=\underline{0} \\ 1, & \text{if } N_x(\lambda)>0 \text{ for each } x \in supp(\lambda) \\ 0 & \text{otherwise} \end{cases}$$

Then
(1) T_N is a smooth topology on X.
(2) If the mapping N is a smooth neighborhood system, then $N_x(\lambda) \leq N^{T_N}_x(\lambda)$, for each $x \in X$ and $\lambda \in I^X$.
Furthermore, $supp(N_x)=supp(N^{T_N}_x)$.

Proof (1) (O1) Obvious.
(O2) We show that $T_N(\lambda_1 \wedge \lambda_2) \geq T_N(\lambda_1) \wedge T_N(\lambda_2)$.
If $T_N(\lambda_1)=0$ or $T_N(\lambda_2)=0$, it is trivial.
If $T_N(\lambda_1)=1$ and $T_N(\lambda_2)=1$, for each $x \in supp(\lambda_1 \wedge \lambda_2)$,

we have

$$N_x(\lambda_1 \wedge \lambda_2) \geq N_x(\lambda_1) \wedge N_x(\lambda_2) > 0.$$

It implies $T_N(\lambda_1 \wedge \lambda_2) = 1$.

(O3) We show that

$$T_N(\bigvee_{i\in\Gamma} \lambda_i) \geq \bigwedge_{i\in\Gamma} T_N(\lambda_i).$$

If $T_N(\lambda_i) = 0$ for some $i \in \Gamma$, it is trivial.
If $T_N(\lambda_i) = 1$ for all $i \in \Gamma$, for each $x \in supp(\bigvee_{i\in\Gamma} \lambda_i)$, there exists $j \in \Gamma$ with

$$x \in supp(\lambda_j) \text{ and } N_x(\lambda_j) > 0.$$

So,

$$N_x(\bigvee_{i\in\Gamma} \lambda_i) \geq N_x(\lambda_j) > 0.$$

$$T_N(\bigvee_{i\in\Gamma} \lambda_i) = 1.$$

(2). If $N_x(\lambda) = 0$, it is trivial. Let $N_x(\lambda) > 0$.
Since, by (N5) of Definition 3.3,

$$N_x(\lambda) \leq \bigvee\{N_x(\mu) \wedge (\bigwedge_{y\in supp(\mu)} N_y(\mu)) \mid \mu \leq \lambda\},$$

there exists $\mu \in I^X$ with $\mu \leq \lambda$ such that

$$N_x(\mu) \wedge (\bigwedge_{y\in supp(\mu)} N_y(\mu)) > 0.$$

Since $\bigwedge_{y\in supp(\mu)} N_y(\mu) > 0$, that is, for each

$$y \in supp(\mu), \quad N_y(\mu) > 0, \text{ then } T_N(\mu) = 1.$$

Thus,

$$N_x^{T_N}(\lambda) = \bigvee\{T_N(\mu) \mid x \in supp(\mu),\ \mu \leq \lambda\} = 1.$$

Thus,

$$N_x(\lambda) \leq N_x^{T_N}(\lambda), \text{ for each } x \in X \text{ and } \lambda \in I^X.$$

We will show that

$$supp(N_x) = supp(N_x^{T_N}).$$

Since

$$N_x(\lambda) \leq N_x^{T_N}(\lambda),\ supp(N_x) \subset supp(N_x^{T_N}).$$

Let $\lambda \in supp(N_x^{T_N})$. Then

$$0 < N_x^{T_N}(\lambda) = \bigvee\{T_N(\mu) \mid x \in supp(\mu),\ \mu \leq \lambda\}.$$

There exists $\mu \in I^X$ with $x \in supp(\mu)$, $\mu \leq \lambda$ such that $T_N(\mu) = 1$.
Hence $N_x(\lambda) \geq N_x(\mu) > 0$.
Thus $\lambda \in supp(N_x)$. Thus, $supp(N_x^{T_N}) \subset supp(N_x)$.

Theorem 3.9 Let (X, τ) be a smooth topological space. Then

$$T_{N^\tau}(\lambda) \geq \tau(\lambda) \text{ for each } \lambda \in I^X.$$

Proof If $\tau(\lambda) = 0$ or $\lambda = \underline{0}$, it is trivial.
If $\tau(\lambda) > 0$ and $\lambda \neq \underline{0}$, then there exists $x \in X$ such that $N_x^\tau(\lambda) \geq \tau(\lambda)$.
It implies $T_{N^\tau}(\lambda) = 1 \geq \tau(\lambda)$.

Example 3.10 Let X, μ and τ as in Example 3.7. By Theorem3.8, we obtain

$$T_{N^\tau}(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ 1, & \text{if } \underline{1} \neq \lambda \geq \mu, \\ 0, & \text{otherwise.} \end{cases}$$

Hence $T_{N^\tau}(\lambda) \geq \tau(\lambda)$ for each $\lambda \in I^X$.

Let $\rho \in I^X$ as follows:

$$\rho(a) = 0.5, \quad \rho(b) = 0.$$

We define $N_a, N_b : I^X \to I$ as follows :

$$N_a(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1}, \\ \frac{1}{3}, & \text{if } \underline{1} \neq \lambda \geq \rho \\ 0, & \text{otherwise.} \end{cases}$$

$$N_b(\lambda) = \begin{cases} 1, & \text{if } \lambda = \underline{1}, \\ \frac{1}{2}, & \text{if } \underline{1} \neq \lambda \geq \rho \\ 0, & \text{otherwise.} \end{cases}$$

For each $\rho \leq \lambda$, since $a \in supp(\lambda)$ and

$$N_a(\lambda) = \frac{1}{3}, \quad T_N(\lambda) = 1.$$

Hence, we obtain

$$T_N(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ 1, & \text{if } \underline{1} \neq \lambda \geq \rho, \\ 0, & \text{otherwise.} \end{cases}$$

Since

$$\frac{1}{2} = N_b(\rho) > N_b(\rho) \wedge (\bigwedge_{y\in supp(\rho)} N_y(\rho)) = \frac{1}{3},$$

it does not satisfy the condition (N5) of Definition3.3.

$$\text{So, } \frac{1}{2} = N_b(\rho) > N_b^{T_N}(\rho) = 0.$$

In general,

$$N_x(\lambda) \nleq N_x^{T_N}(\lambda).$$

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